Bed-Load Sediment Transport on Large Slopes

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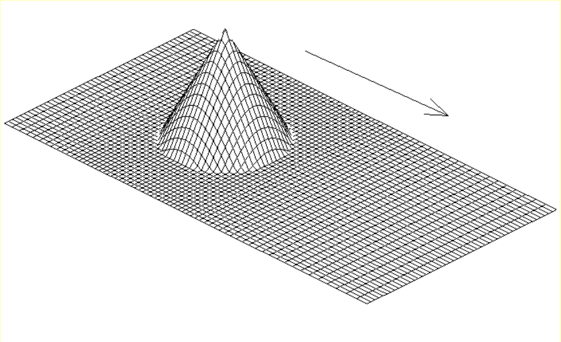
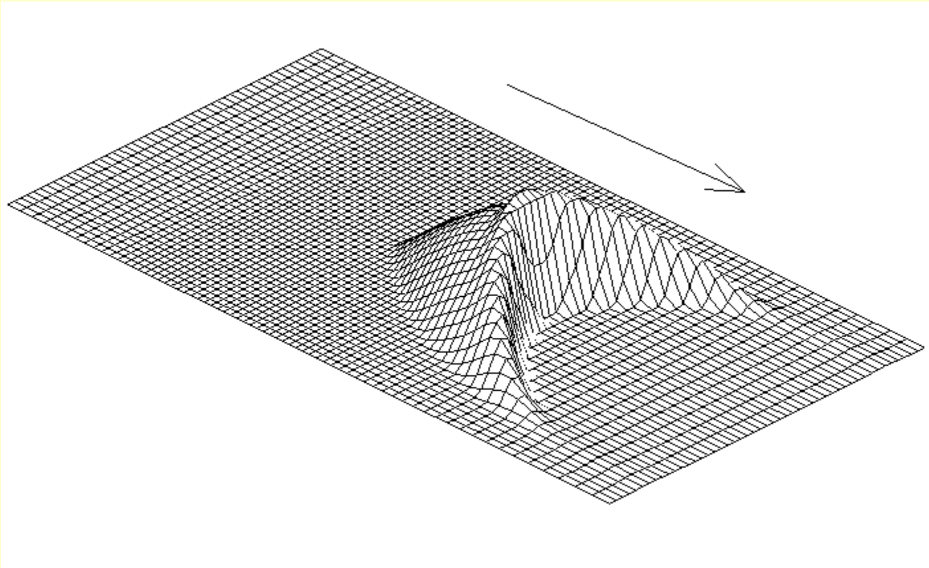
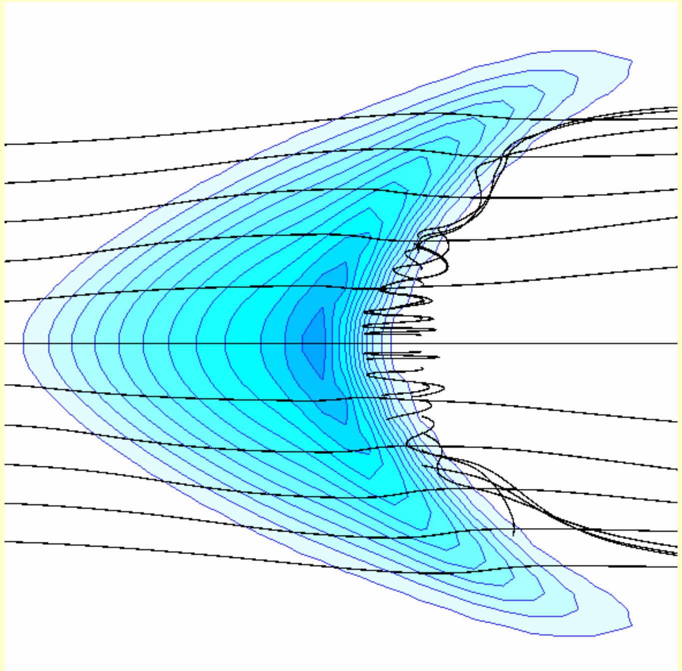
Motivation

- Movement of sand mounds







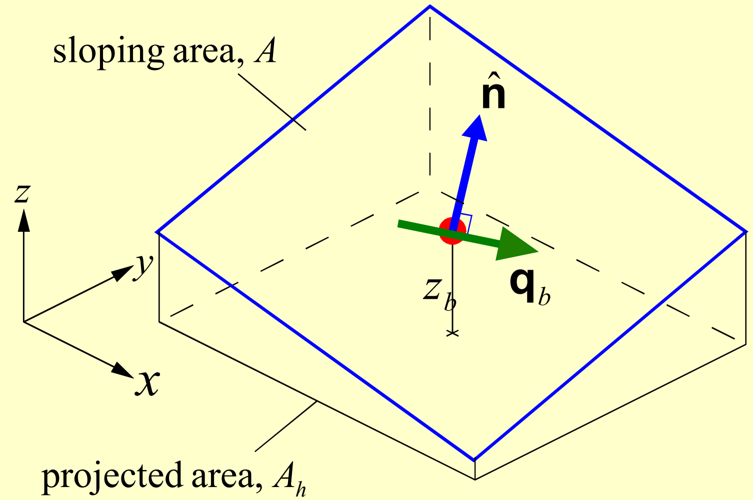
Main Issues

- Modification of sediment-transport formulae for large slopes
- Implementation of morphodynamics equation in a RANS solver

Sediment Morphodynamics

Change of volume = - net outward flux

$$(1-p)A_h \frac{\Delta z_b}{\Delta t} = -\oint_{\partial A} \mathbf{q}_b \bullet \mathrm{d}\mathbf{s} \wedge \hat{\mathbf{n}}$$



(Bed-load) sediment flux

$$\mathbf{q}_b = f\,(surface\ shear\ stress,\ surface\ orientation)$$

Surface shear stress predicted by RANS code with an advanced wall-function treatment

Sediment-Transport Formulae

$$q^* = f(\tau^*, d^*)$$ $$\tau^*_{crit} = f(d^*)$$

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}$$ dimensionless bed flux

$$\tau^* = \frac{\tau_w}{\rho(s-1)gd}$$ dimensionless shear stress ("*Shields stress*")

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$ dimensionless particle diameter

Sediment-Transport Formulae

Meyer-Peter and Müller (1948)	$q^* = 8(\tau^* - \tau^*_{crit})^{3/2}$
Einstein-Brown (Brown, 1950)	$q^* = \begin{cases} \dfrac{K\exp(-0.391/\tau^*)}{0.465} & \tau^* < 0.182 \\ 40K{\tau^*}^3 & \tau^* \geq 0.182 \end{cases}$
Yalin (1963)	$q^* = 0.635r\sqrt{\tau^*}[1 - \frac{1}{\sigma r}\ln(1 + \sigma r)]$
Van Rijn (1984)	$q^* = \frac{0.053}{{d^*}^{0.3}}(\frac{\tau^*}{\tau^*_{crit}} - 1)^{2.1}$
Nielsen (1992)	$q^* = 12(\tau^* - \tau^*_{crit})\sqrt{\tau^*}$

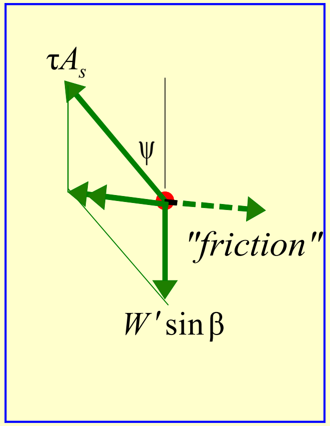
Soulsby (1997) $\tau^*_{crit} = \frac{0.30}{1 + 1.2d^*} + 0.055\,[1 - \exp(-0.020d^*)]$

Slope Modifications to Sediment-Transport Formulae

1. **Effective stress** τ_{eff} combining fluid stress and downslope component of weight

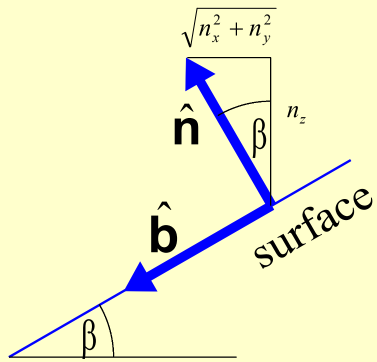
$$\boldsymbol{\tau}_{eff} A = \boldsymbol{\tau} A + W' \sin\beta \, \hat{\mathbf{b}}$$

$$\Rightarrow \quad \boldsymbol{\tau}^*_{eff} = \boldsymbol{\tau}^* + D_0 \sin\beta \, \hat{\mathbf{b}} \qquad D_0 = \frac{\tau^*_{crit,0}}{\tan\phi}$$



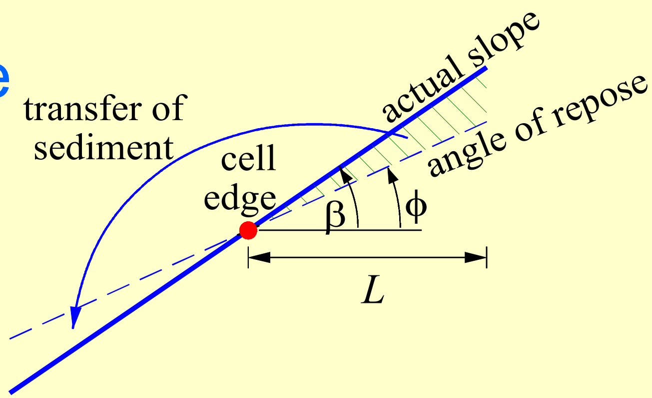
2. **Critical effective stress reduced** in proportion to the slope-normal component of gravity

$$\tau^*_{eff,crit} = \tau^*_{crit,0} \cos\beta$$



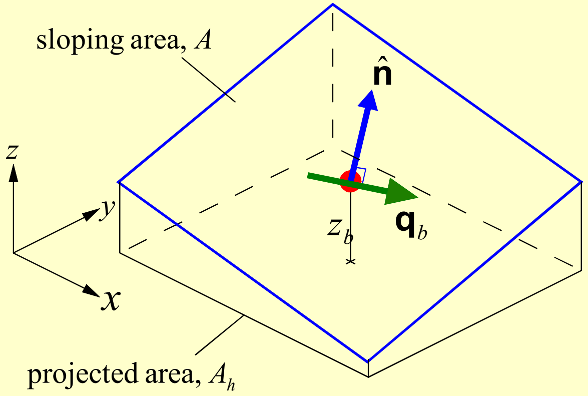
3. **Avalanche model** preventing the slope exceeding the angle of repose

$$q_{aval} = (1-p)\frac{\frac{1}{2}L^2(\tan\beta - \tan\phi)}{\cos\beta \, \Delta t}$$



Implementation of Morphodynamics Equation

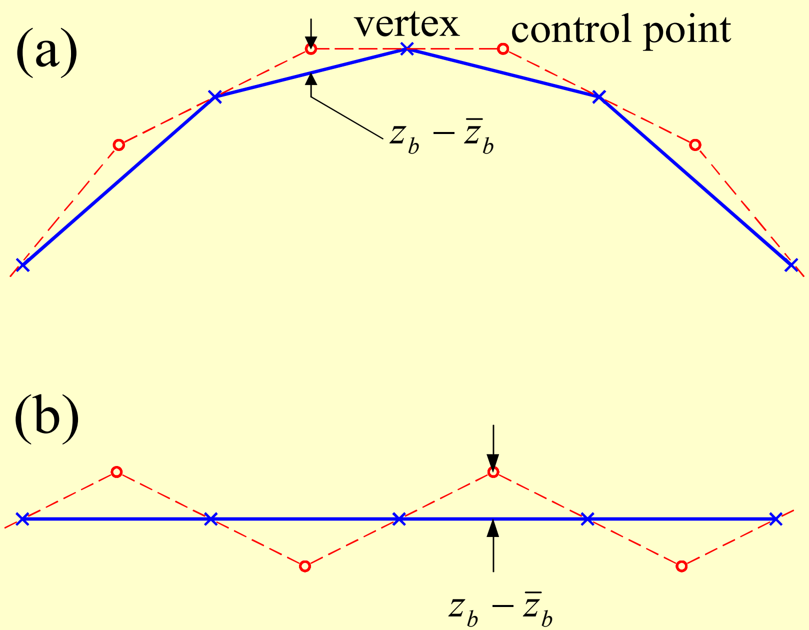
1. **Arbitrary-orientation surface when computing flux divergence**



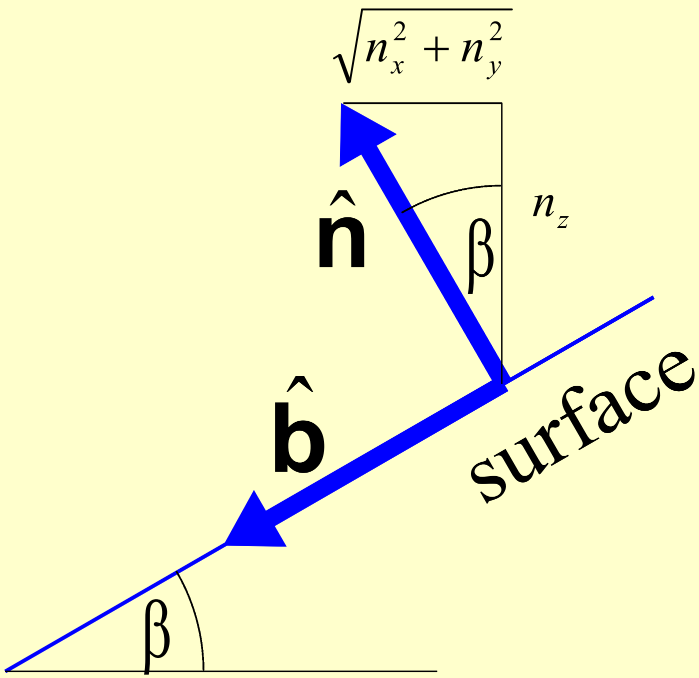
$$\mathrm{DIV}(\mathbf{q}_b) \equiv \oint \mathbf{q} \bullet d\mathbf{s} \wedge \hat{\mathbf{n}} \;\rightarrow\; \sum_{edges} \mathbf{q}_{b,edge} \bullet \Delta \mathbf{s} \wedge \hat{\mathbf{n}}_{edge}$$

2. **Smoothing algorithm based on surface curvature**

$$q_b \rightarrow q_b + \frac{10|q_b|}{\Delta l} \times \left[(z_b - \bar{z}_b)_L - (z_b - \bar{z}_b)_R \right]$$



Surface Orientation

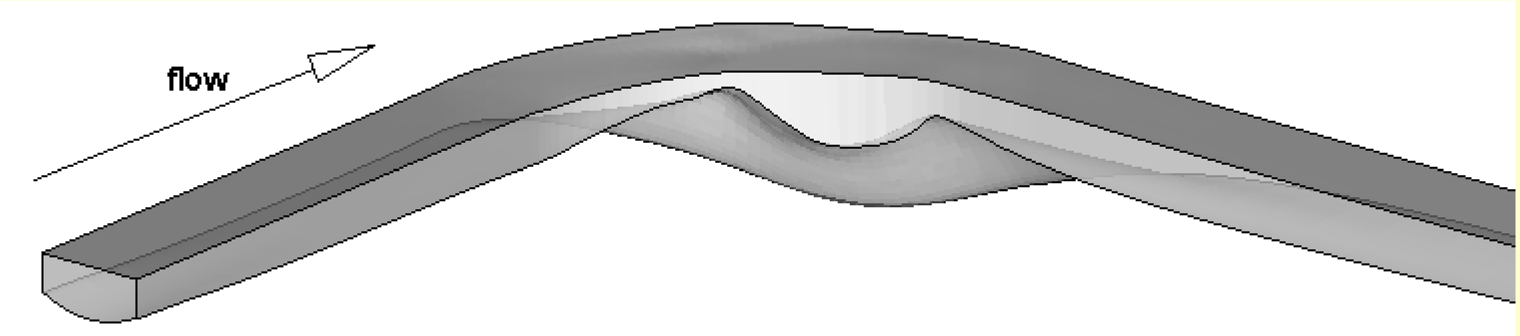


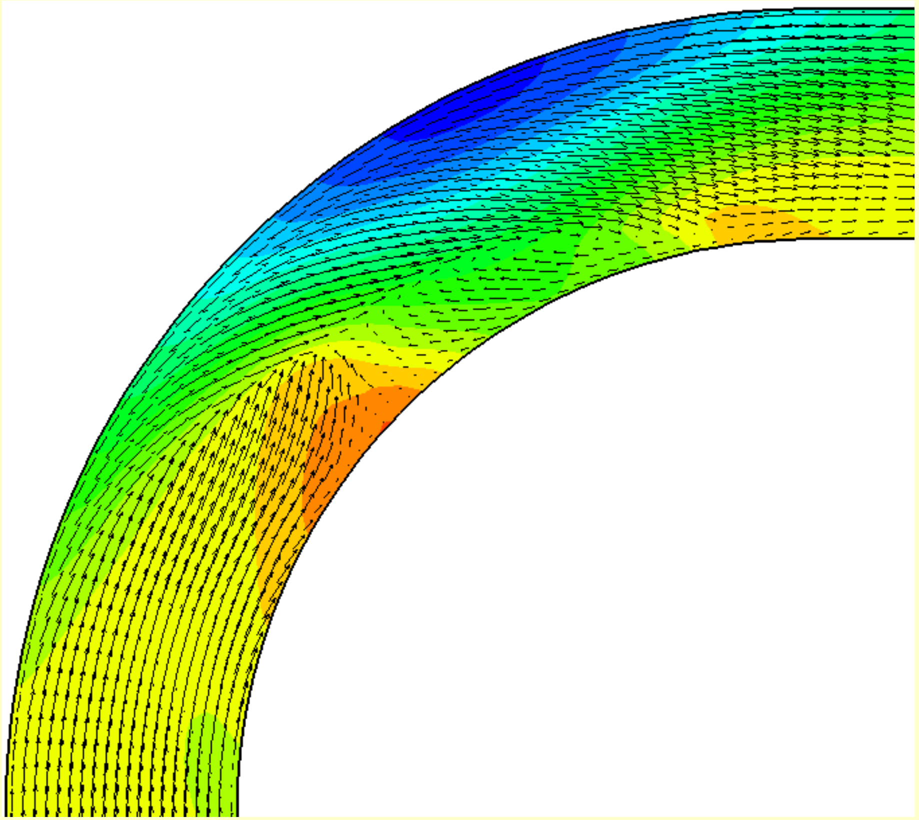
$$\mathbf{n} = \frac{1}{\sqrt{1 + \left|\nabla z_b\right|^2}}\left(-\frac{\partial z_b}{\partial x}, -\frac{\partial z_b}{\partial y}, 1\right)$$

$$\cos\beta = n_z$$

$$\sin\beta = \sqrt{n_x^2 + n_y^2}$$

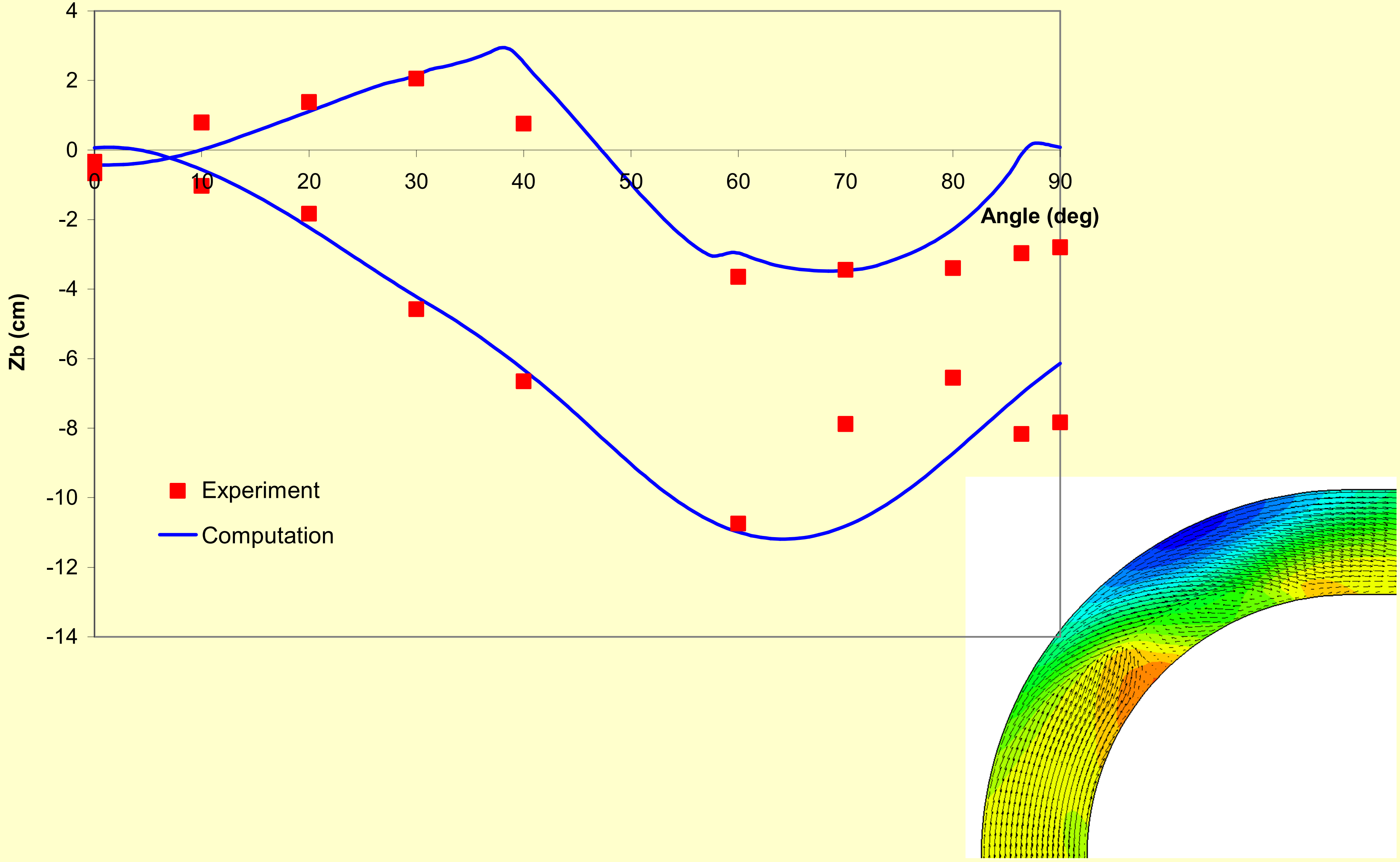
Scour in a Channel Bend



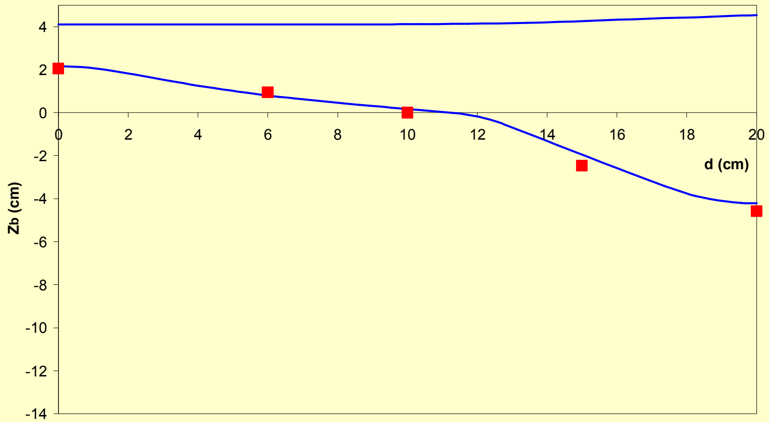


Data: Kawai and Julien, 1996

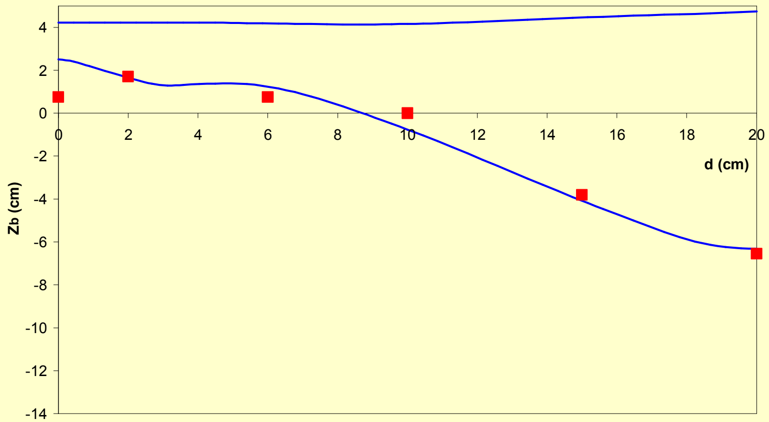
Bed Profiles



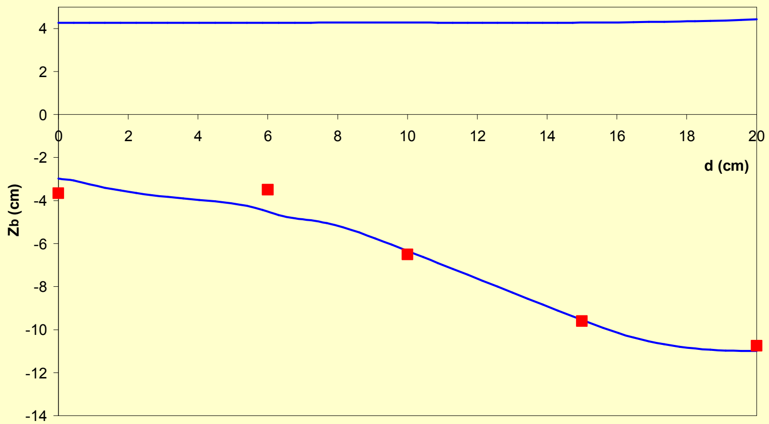
30º

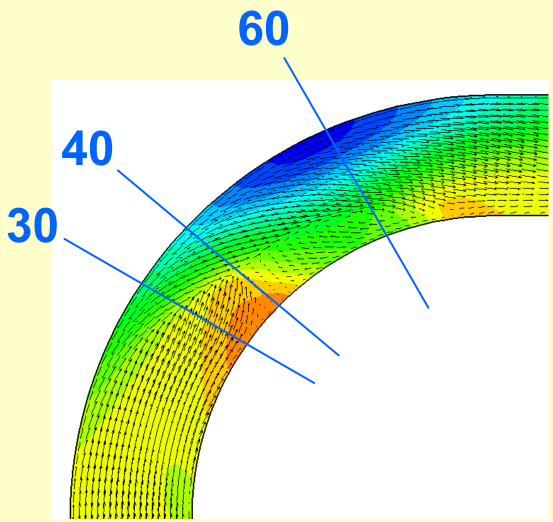


40º

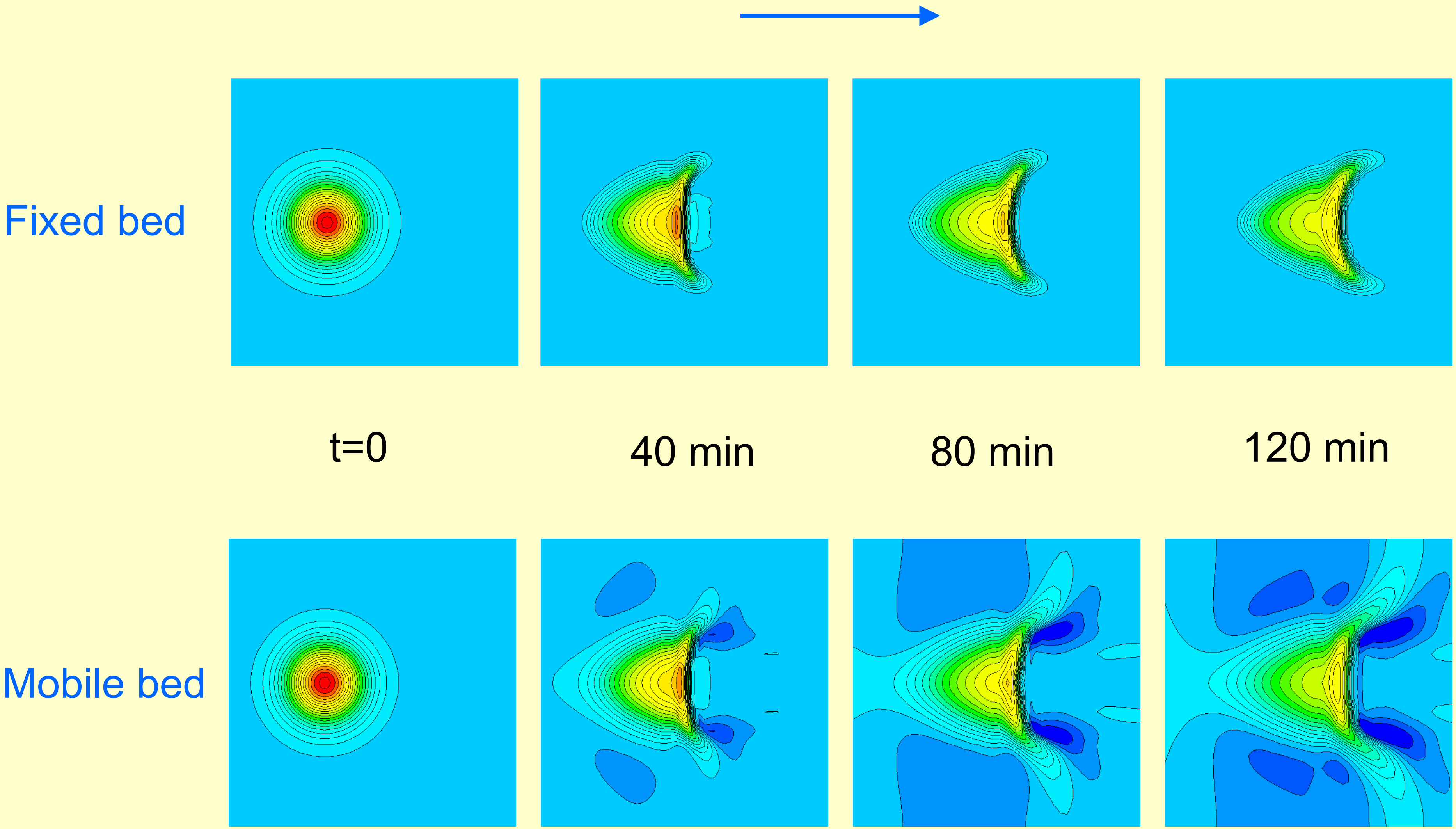


60º



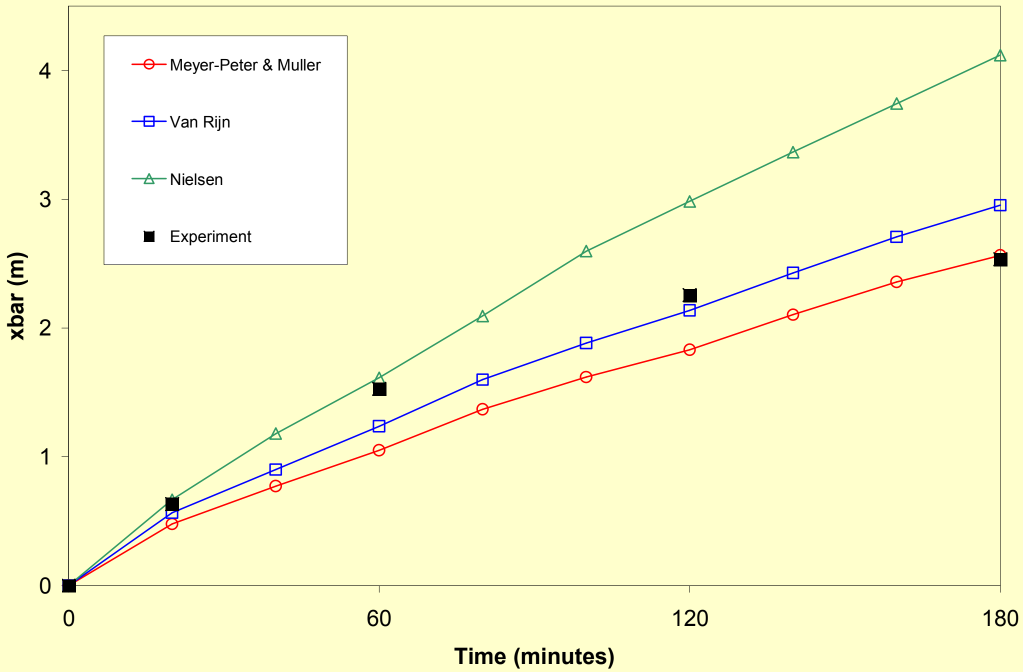


Transport of a Gaussian Mound

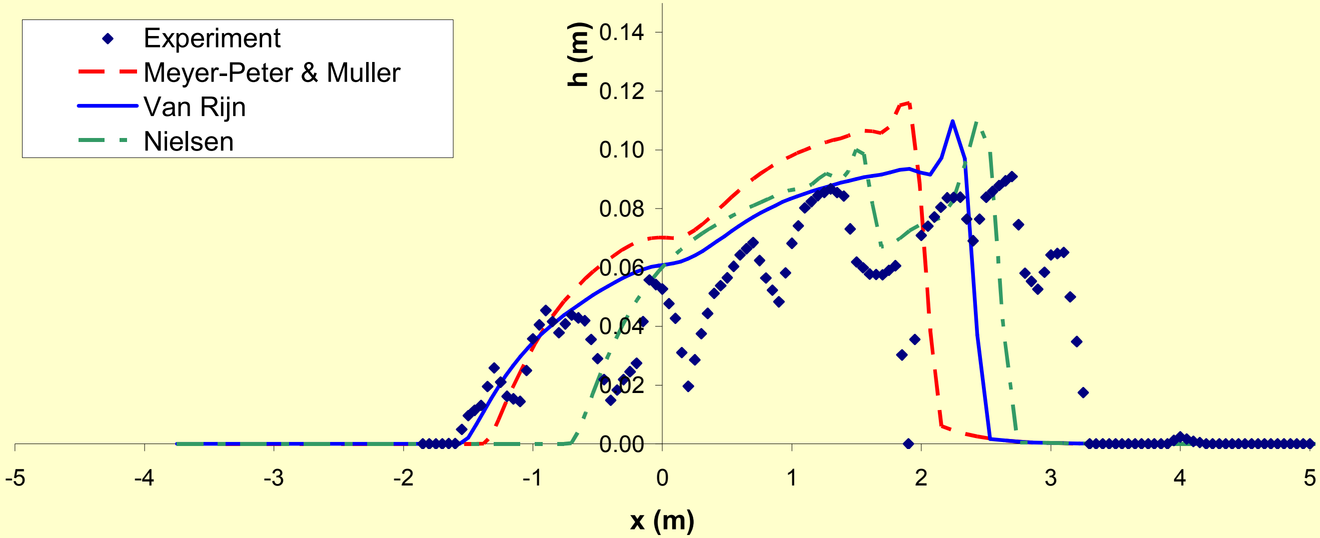


Transport of a Gaussian Mound

Motion of centroid



Centreline profile (1 hour)



Summary of Key Elements

- **Fully-vectorial formulation** for arbitrary surface orientation
- **Effective stress** replaces fluid stress in bed-load formulae
- **Critical effective stress** reduced in proportion to normal reaction
- **Avalanche model** to limit slopes to angle of repose
- **Smoothing algorithm** to eliminate surface wiggles

References

- Apsley, D.D., 2007, CFD calculation of turbulent flow with arbitrary wall roughness, *Flow, Turbulence and Combustion*, **78**, 153-175.

- Apsley, D.D. and Stansby, P.K., Bed-load sediment transport on large slopes: model formulation and implementation within a RANS flow solver, *submitted to ASCE Journal of Hydraulic Engineering*.