

Q1

(a) Intersect the trajectory of the ball with the equation of the plane.

To keep things positive, take the origin at the bottom of the plane, directly below the point of release, with x horizontal and y vertically upward, “ $s = s_0 + ut + \frac{1}{2}at^2$ ” in each direction, initial $y_0 = 9 \tan 25^\circ = 4.197$ m, giving, in m-s units:

$$x = 12t$$

$$y = 4.197 - \frac{1}{2}gt^2$$

and hence, eliminating t , a trajectory equation

$$y = 4.197 - 0.03406x^2$$

Intersect this with the equation of the plane

$$y = x \tan 25^\circ = 0.4663x$$

giving

$$0.4663x = 4.197 - 0.03406x^2$$

or, in standard form for a quadratic,

$$0.03406x^2 + 0.4663x - 4.197 = 0$$

The positive root is

$$x_B = \frac{-0.4663 + \sqrt{0.4663^2 + 4 \times 0.03406 \times 4.197}}{2 \times 0.03406} = 6.196 \text{ m}$$

The horizontal distance from the wall is then

$$9 - 6.196 = 2.804 \text{ m}$$

and hence the slopewise distance is

$$\frac{2.804}{\cos 25^\circ} = 3.094 \text{ m}$$

Answer: 3.094 m

(b) The time to the bounce is

$$t_B = \frac{x_B}{V_0} = \frac{6.196}{12} = 0.5163 \text{ s}$$

and so the cartesian velocity components at this point are

$$v_x = 12 \text{ m s}^{-1}$$

$$v_y = u_y - gt_B = 0 - 9.81 \times 0.5163 = -5.065 \text{ m s}^{-1}$$

Taking components parallel (or tangential, t , positive up-plane) and perpendicular (or normal, n , positive away from the plane):

$$v_t = v_x \cos 25^\circ + v_y \sin 25^\circ = 8.735 \text{ m s}^{-1}$$

$$v_n = -v_x \sin 25^\circ + v_y \cos 25^\circ = -9.662 \text{ m s}^{-1} \quad (\text{note that } v_y \text{ is negative})$$

Answer: 8.74 m s^{-1} ; -9.662 m s^{-1}

(c) After the bounce the tangential component is unchanged, whilst the normal component is reversed in direction and multiplied by e :

$$v'_t = v_t = 8.735 \text{ m s}^{-1}$$

$$v'_n = -e v_n = -0.75 \times (-9.662) = 7.247 \text{ m s}^{-1}$$

Answer: 8.74 m s^{-1} ; 7.25 m s^{-1}

(d) Transform back to cartesian components to start the final stage of motion:

$$u_x = v'_t \cos 25^\circ - v'_n \sin 25^\circ = 4.854 \text{ m s}^{-1}$$

$$u_y = v'_t \sin 25^\circ + v'_n \cos 25^\circ = 10.26 \text{ m s}^{-1}$$

Finally, move the origin of space and time to the bounce. The time taken to traverse the remaining horizontal distance 2.804 m to the wall is given by

$$2.804 = 4.854t$$

or

$$t = 0.5777 \text{ s}$$

The vertical height change is

$$y = u_y t - \frac{1}{2} g t^2 = 10.26 \times 0.5777 - \frac{1}{2} \times 9.81 \times 0.5777^2 = 4.290$$

and hence the distance above the end of the plane is

$$y - 2.804 \tan 25^\circ = 2.982 \text{ m}$$

Answer: 2.98 m

Q2.

Let L ($= 6$ m) be the unstretched length of the string, which is also the horizontal distance of the tether point from the pole. Let h ($= 4$ m) be the height of the pole.

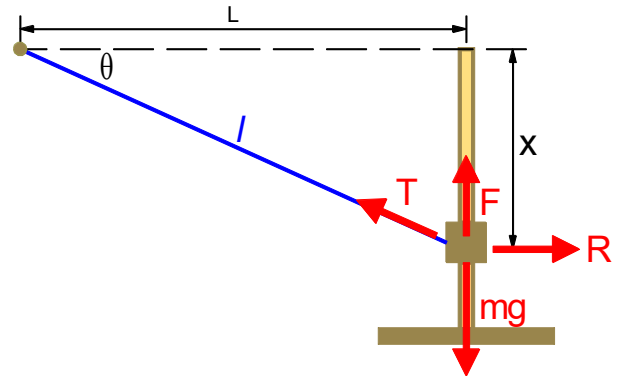
Let m ($= 3$ kg) be the mass of the collar and k ($= 90$ N m⁻¹) be the stiffness of the string.

(a) When the collar has descended a distance x we have, from Pythagoras:
length of string:

$$l = \sqrt{L^2 + x^2}$$

extension of string:

$$e = l - L = \sqrt{L^2 + x^2} - L$$



From the energy principle, with no friction, at $x = h$:

change in (KE + GPE + EPE) = 0

$$\frac{1}{2}mv^2 - mgh + \frac{1}{2}ke^2 = 0$$

$$\begin{aligned} v^2 &= 2gh - \frac{k}{m}(\sqrt{L^2 + h^2} - L)^2 \\ &= 2 \times 9.81 \times 4 - \frac{90}{3} \times (\sqrt{6^2 + 4^2} - 6)^2 \\ &= 34.48 \text{ (m s}^{-1}\text{)}^2 \end{aligned}$$

giving

$$v = 5.872 \text{ m s}^{-1}$$

Answer: 5.87 m s⁻¹

(b) If there is friction then the mechanical energy principle becomes

change in (KE + GPE + EPE) = work done by friction

where the RHS is, noting that friction acts in the opposite direction to motion so does *negative* work,

$$W_f = - \int_0^h F \, dx$$

For the friction force, first find the tension in the cable:

$$T = ke = k(\sqrt{L^2 + x^2} - L)$$

The normal reaction R balances the horizontal component of this:

$$R = T \cos \theta = k(\sqrt{L^2 + x^2} - L) \times \frac{L}{\sqrt{L^2 + x^2}} = kL \left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right)$$

Then, for a moving collar, where friction is maximal:

$$F = \mu R = \mu kL \left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right)$$

(Friction is maximal because the collar must move: when $x = 0$ there is no tension or friction force to counter the downward weight.)

Hence, the work done by friction is

$$W_f = - \int_0^h F dx = -\mu kL \int_0^h \left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right) dx = -\mu kL \left\{h - L \sinh^{-1}\left(\frac{h}{L}\right)\right\}$$

(The last part comes from the standard integral

$$\int_0^x \frac{1}{\sqrt{L^2 + x^2}} dx = \sinh^{-1}\left(\frac{x}{L}\right)$$

where $\sinh x$ is the hyperbolic sine function or, from first principles, by substitution $x = L \sinh \theta$).

Substituting in the energy equation:

$$\frac{1}{2}mv^2 - mgh + \frac{1}{2}k(\sqrt{L^2 + h^2} - L)^2 = -\mu kL \left\{h - L \sinh^{-1}\left(\frac{h}{L}\right)\right\}$$

Hence,

$$\begin{aligned}v^2 &= 2gh - \frac{k}{m}(\sqrt{L^2 + h^2} - L)^2 - 2\frac{\mu kL}{m} \left\{h - L \sinh^{-1}\left(\frac{h}{L}\right)\right\} \\&= 78.48 - 44.00 - 26.91 \\&= 7.57 \text{ (m s}^{-1}\text{)}^2\end{aligned}$$

giving

$$v = 2.751 \text{ m s}^{-1}$$

Answer: 2.75 m s^{-1}