Waves

2. Wave Transformation

Wave Transformation

- **Refraction**
  - change of **direction** on moving into shallower water
- **Shoaling**
  - change of **height** on moving into shallower water
- **Breaking**
  - collapse of waves after steepening
- **Diffraction**
  - spreading of waves into geometric shadow
- **Reflection**
  - reversal of direction at boundary

Wave Transformation

2. WAVE TRANSFORMATION

2.1 Refraction

2.2 Shoaling

2.3 Breaking

2.4 Diffraction

2.5 Reflection

Refraction

As waves move into shallower water:

- **period** remains constant;
- **speed** decreases (because depth decreases).

Hence:

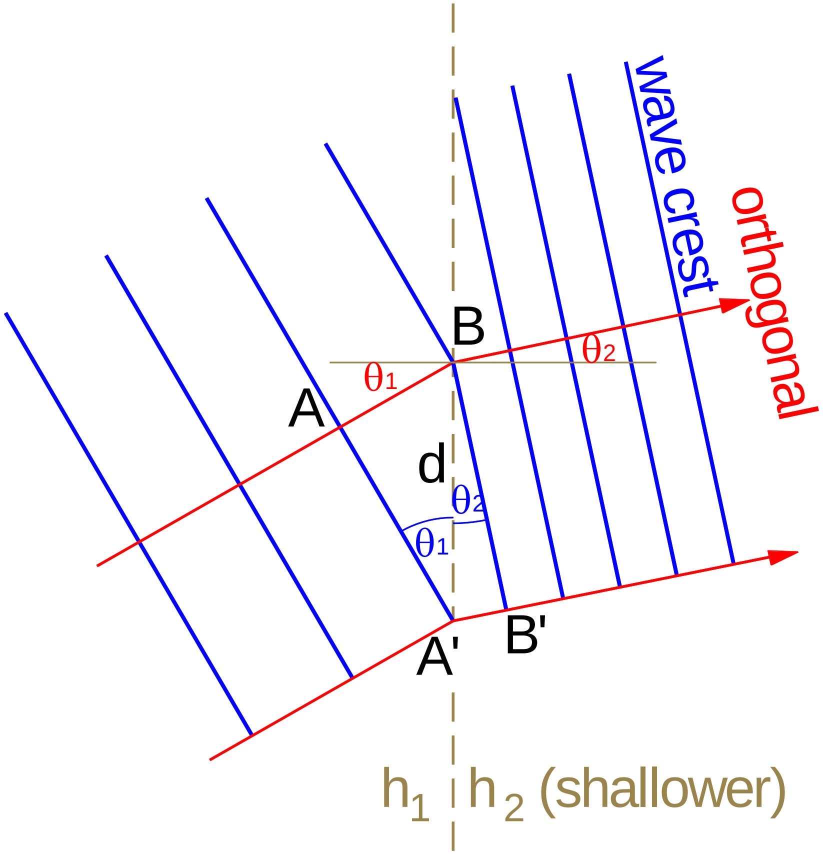
- for oblique waves, **direction** changes.

Refraction is the change in **direction** with wave speed. For water waves, this speed change is due to a change in depth.

The change in direction is governed by **Snell's Law**.

Snell's Law

$$T = \frac{\text{distance}}{\text{speed}} = \frac{d\sin\theta_1}{c_1} = \frac{d\sin\theta_2}{c_2}$$

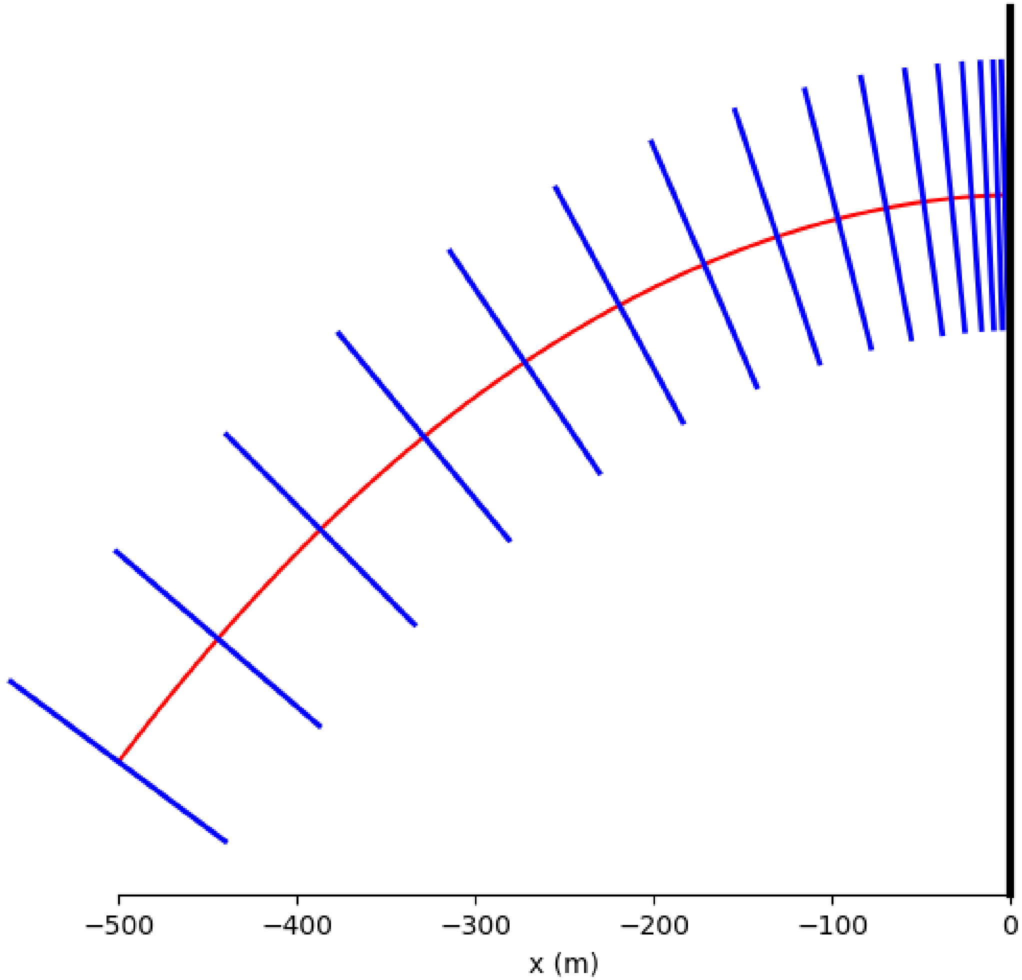


$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2}$$

$$\frac{k_1\sin\theta_1}{\omega_1} = \frac{k_2\sin\theta_2}{\omega_2} \qquad \omega_1 = \omega_2$$

$$(k\sin\theta)_1 = (k\sin\theta)_2$$

Refraction



Example

A straight coastline borders a uniformly-sloping sea bed. Regular waves are observed to cross the 8 m depth contour at an angle of 14° to the coastline-normal, with wavelength 45 m. Find:

(a) the wave period;

(b) the wavelength in deep water;

(c) the direction in deep water.

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(b) the wavelength in deep water;
(c) the direction in deep water.

Inshore:

$$h = 8\text{ m}$$
$$\theta = 14°$$
$$L = 45\text{ m}$$

$$k = \frac{2\pi}{L} = 0.1396\text{ m}^{-1}$$

$$\omega^2 = gk\tanh kh \qquad \omega = 1.051\text{ rad s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \mathbf{5.978\ s}$$

Deep water:

$$L_0 = \frac{gT^2}{2\pi} = \mathbf{55.80\ m}$$

Snell's Law:

$$(k\sin\theta)_0 = (k\sin\theta)_{8\text{ m}} \qquad k_0 = \frac{2\pi}{L_0} = 0.1126\text{ m}^{-1}$$

$$0.1126\sin\theta_0 = 0.1396\sin 14°$$

$$\boldsymbol{\theta_0 = 17.45°}$$

Wave Transformation

2. WAVE TRANSFORMATION

2.1 Refraction

2.2 Shoaling

2.3 Breaking

2.4 Diffraction

2.5 Reflection

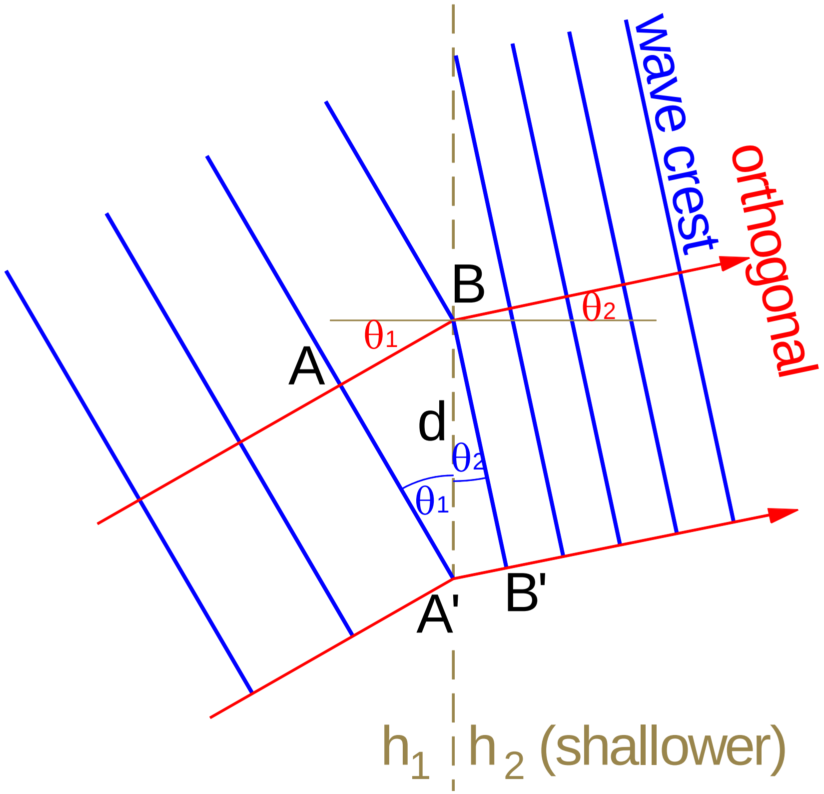
Shoaling

If there is no energy dissipation:

- **shoreward component of power** remains constant;

or

- **power between orthogonals** is constant.



$$P \cos \theta = \text{constant}$$

Shoaling

$$P\cos\theta = \text{constant}$$

Energy: $E = \frac{1}{2}\rho g A^2 \quad = \frac{1}{8}\rho g H^2$ (per unit area)

Power: $P = E c_g \quad = (\frac{1}{8}\rho g H^2)(nc)$ (per unit length of crest)

$$(H^2 nc\cos\theta)_1 = (H^2 nc\cos\theta)_2$$

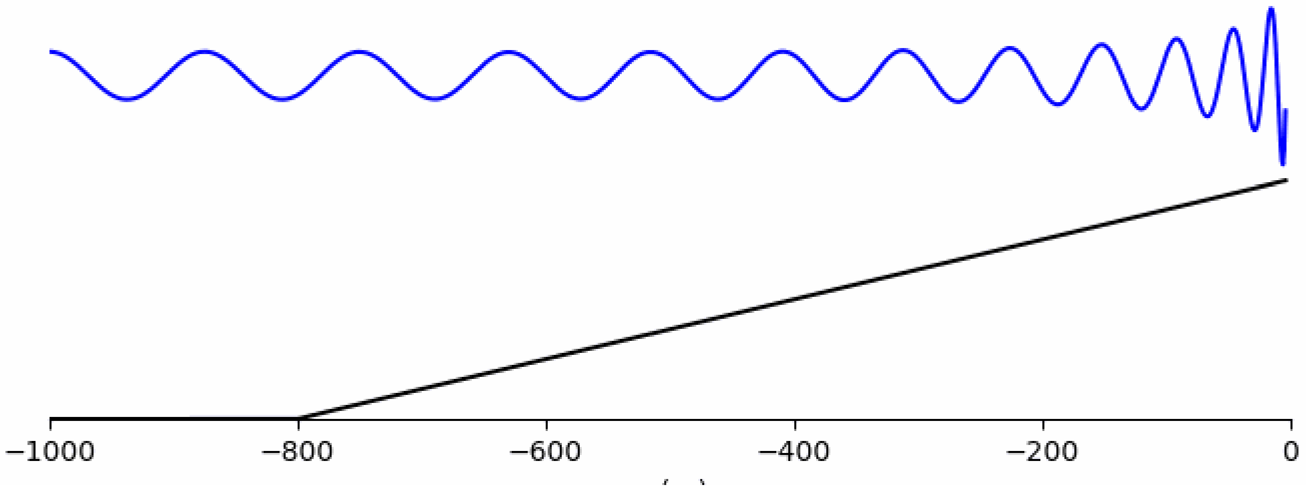
$$n = \frac{1}{2}\left[1 + \frac{2kh}{\sinh 2kh}\right]$$

$$c = \frac{\omega}{k}$$

Shoaling

$$(H^2 nc\cos\theta)_1 = (H^2 nc\cos\theta)_2$$

$$H_2 = H_1 \underbrace{\left(\frac{\cos\theta_1}{\cos\theta_2}\right)^{1/2}}_{\substack{\text{refraction}\\ \text{coefficient}\\ K_R}} \underbrace{\left(\frac{(nc)_1}{(nc)_2}\right)^{1/2}}_{\substack{\text{shoaling}\\ \text{coefficient}\\ K_S}}$$



Example

Waves propagate towards a long straight coastline that has a very gradual bed slope normal to the coast. In water depth of 20 m, regular waves propagate at heading $\theta = 40°$ relative to the bed slope.

(a) Sketch the shape of a wave ray from the 20 m depth contour to the 5 m depth contour for a wave that is of deep-water type in both depths and, separately, for a wave that is of shallow water type in both depths. Calculations are not required.

(b) For a wave with period $T = 8$ s and height 1.2 m at 20 m depth, calculate the wave heading and wave height at the 5 m depth contour.

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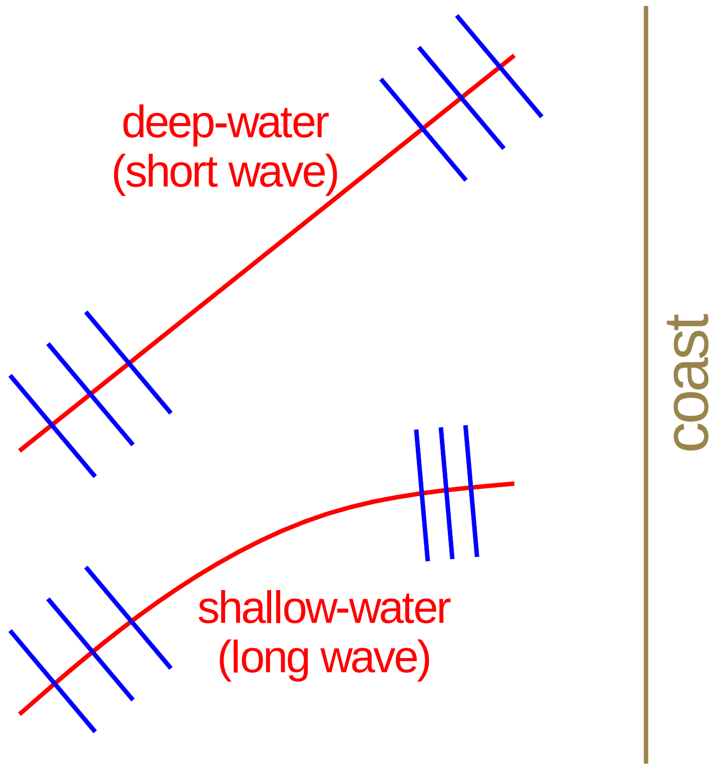

Deep-water (or **short-wavelength**) waves, by definition, do not feel the effect of the bed. Hence they propagate undisturbed.

Shallow-water (or **long-wavelength**) waves, reduce in wavelength and slow down. Hence they bend toward the normal.

Waves propagate towards a long straight coastline that has a very gradual bed slope normal to the coast. In water depth of 20 m, regular waves propagate at heading $\theta = 40°$ relative to the bed slope.

(b) For a wave with period $T = 8$ s and height 1.2 m at 20 m depth, calculate the wave heading and wave height at the 5 m depth contour.

$$T = 8\text{ s} \qquad \text{(both depths)}$$

$$\omega = \frac{2\pi}{T} = 0.7854\text{ rad s}^{-1}$$

Dispersion:

$$\omega^2 = gk \tanh kh$$

$$\frac{\omega^2 h}{g} = kh \tanh kh$$

$$kh = \frac{\omega^2 h/g}{\tanh kh} \qquad \text{or} \qquad kh = \frac{1}{2}\left(kh + \frac{\omega^2 h/g}{\tanh kh}\right)$$

Refraction (Snell's Law):

$$(k \sin\theta)_{5\text{ m}} = (k \sin\theta)_{20\text{ m}}$$

Shoaling:

$$(H^2 nc \cos\theta)_{5\text{ m}} = (H^2 nc \cos\theta)_{20\text{ m}}$$

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(b) For a wave with period $T = 8$ s and height 1.2 m at 20 m depth, calculate the wave heading and wave height at the 5 m depth contour.

$\omega = 0.7854 \text{ rad s}^{-1}$

	$h = 5$ m	$h = 20$ m
$\omega^2 h/g$	0.3144	1.258
Iteration:	$kh = \frac{1}{2}\left(kh + \frac{0.3144}{\tanh kh}\right)$	$kh = \frac{1.258}{\tanh kh}$
kh	0.5918	1.416
k	0.1184 m^{-1}	0.07080 m^{-1}
$c = \frac{\omega}{k}$	6.633 m s^{-1}	11.09 m s^{-1}
$n = \frac{1}{2}\left[1 + \frac{2kh}{\sinh 2kh}\right]$	0.8999	0.6674
θ	---	40°
H	---	1.2 m

Refraction: $(k \sin\theta)_{5\text{ m}} = (k \sin\theta)_{20\text{ m}}$

$0.1184 \sin\theta_{5\text{m}} = 0.07080 \sin 40°$ $\qquad$ $\boldsymbol{\theta_{5\text{m}} = 22.60°}$

Shoaling: $(H^2 nc \cos\theta)_{5\text{ m}} = (H^2 nc \cos\theta)_{20\text{ m}}$ $\qquad$ $\boldsymbol{H_{5\text{m}} = 1.217\text{ m}}$

$H_{5\text{m}}^2 \times 0.8999 \times 6.633 \times \cos 22.60° = 1.2^2 \times 0.6674 \times 11.09 \times \cos 40°$

Wave Transformation

2. WAVE TRANSFORMATION

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Breaking – Miche Criterion

$$\left(\frac{H}{L}\right)_b = 0.14\tanh(kh)_b$$

Idea: breaking occurs when $u_{\max}/c$ exceeds a critical value

$$\frac{u_{\max}}{c} = \frac{(Agk/\omega)}{(\omega/k)} = \frac{Agk^2}{\omega^2} = \frac{Ak}{\tanh kh} = \frac{\pi H/L}{\tanh kh}$$

At breaking: $\frac{H}{L} = \text{constant} \times \tanh(kh)$

Deep water: $\left(\frac{H}{L}\right)_b = 0.14 \approx \frac{1}{7}$

Shallow water: $\left(\frac{H}{L}\right)_b = 0.14 \times \frac{2\pi h}{L}$ $\left(\frac{H}{h}\right)_b = 0.88$

Breaker Height Index

Breaker height index:

$$\Omega_b = \frac{H_b}{H_0} \quad \left(\frac{\text{wave height at breaking}}{\text{deep-water wave height}}\right)$$

e.g.

$$\Omega_b = 0.56\left(\frac{H_0}{L_0}\right)^{-1/5}$$

Deep-water quantities extrapolated from height and period measured at one point:

H_0 from shoaling: $$(H^2 nc\cos\theta)_0 = (H^2 nc\cos\theta)_{\text{measured}}$$

L_0 from: $$L_0 = \frac{gT^2}{2\pi}$$

Breaker Depth Index

Breaker depth index:

$$\gamma_b = \left(\frac{H}{h}\right)_b \qquad \left(\frac{\text{wave height at breaking}}{\text{water depth at breaking}}\right)$$

On a mild slope:

$$\gamma_b = 0.78$$

On a beach of slope m:

$$\gamma_b = b - a\frac{H_b}{gT^2} \qquad = b - a\frac{H_b}{2\pi L_0}$$

$$a = 43.8(1 - e^{-19m}), \qquad b = \frac{1.56}{1 + e^{-19.5m}}$$

Types of Breaker

Irribarren Number (aka **surf-similarity parameter**):

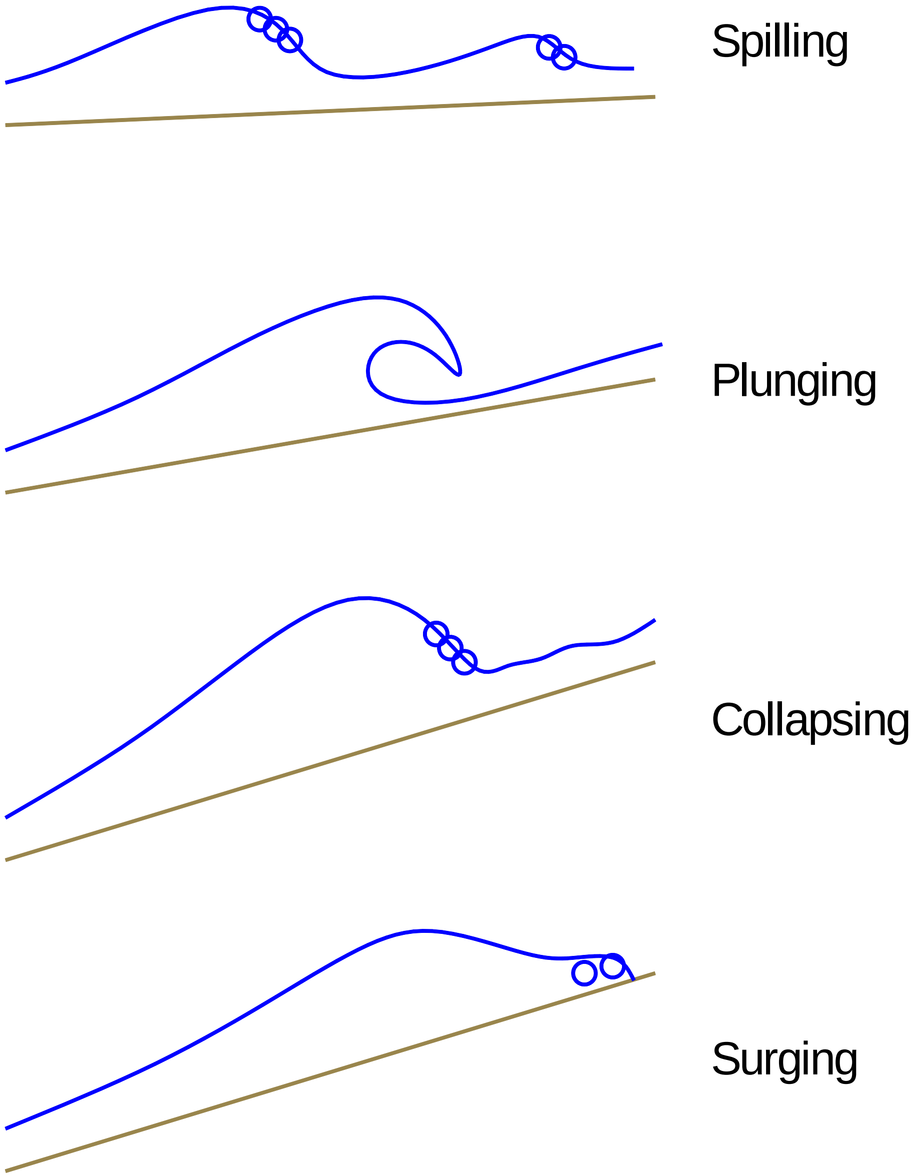
$$\left(\frac{\text{beach slope}}{\sqrt{\text{wave steepness}}}\right)$$

$$\xi_0 = \frac{m}{\sqrt{H_0/L_0}}$$

$$\xi_b = \frac{m}{\sqrt{H_b/L_0}}$$

$\xi_0 < 0.5$	spilling breakers
$0.5 < \xi_0 < 3.3$	plunging breakers
$3.3 < \xi_0$	surging or collapsing breakers

Types of Breaker



Example

Waves propagate towards a long straight coastline that has a constant bed slope of 1 in 100. Consider the *x*-axis to be normal to the coastline and the *y*-axis parallel to the coastline. Waves propagate at an angle θ to the *x*-axis.

(a) A wave with period 7 s and height 1.2 m crosses the 36 m depth contour at angle $\theta = 22°$.

  (i) Determine the direction, height and power per metre width of wave crest at the 4 m depth contour.

  (ii) Explain how height changes between these depths.

(b) A wave with period 7 s and height 1.0 m crosses the 4 m depth contour at angle $\theta = 0°$. Determine the breaking wave height and breaking depth from their corresponding indices and identify the type of breaker expected.

(c) Further along the coast, waves propagate over the outflow of a river. In water depth of 14 m, measurements indicate a period of 7 s and depth-averaged flow velocity of 0.8 m s^{-1} against the wave direction. Determine the wavelength.

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(i) Determine the direction, height and power per metre width of wave crest at the 4 m depth contour.

$T = 7\ \mathrm{s}$ (both depths)

$$\omega = \frac{2\pi}{T} = 0.8976\ \mathrm{rad\ s^{-1}}$$

Dispersion:

$$\omega^2 = gk \tanh kh$$

$$\frac{\omega^2 h}{g} = kh \tanh kh$$

$$kh = \frac{\omega^2 h/g}{\tanh kh} \quad \text{or} \quad kh = \frac{1}{2}\left(kh + \frac{\omega^2 h/g}{\tanh kh}\right)$$

Refraction (Snell's Law):

$$(k \sin\theta)_{4\ \mathrm{m}} = (k \sin\theta)_{36\ \mathrm{m}}$$

Shoaling:

$$(H^2 nc \cos\theta)_{4\ \mathrm{m}} = (H^2 nc \cos\theta)_{36\ \mathrm{m}}$$

Power:

$$P = Ec_g \qquad E = \frac{1}{8}\rho g H^2 \qquad c_g = nc$$

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(i) Determine the direction, height and power per metre width of wave crest at the 4 m depth contour.

$\omega = 0.8976 \text{ rad s}^{-1}$

	$h = 4$ m	$h = 36$ m
$\omega^2 h/g$	0.3285	2.957
Iteration:	$kh = \frac{1}{2}\left(kh + \frac{0.3285}{\tanh kh}\right)$	$kh = \frac{2.957}{\tanh kh}$
kh	0.6065	2.973
k	0.1516 m^{-1}	0.08258 m^{-1}
$c = \frac{\omega}{k}$	5.921 m s^{-1}	10.87 m s^{-1}
$n = \frac{1}{2}\left[1 + \frac{2kh}{\sinh 2kh}\right]$	0.8956	0.5156
θ	---	22°
H	---	1.2 m

Refraction: $(k \sin\theta)_{4\text{ m}} = (k\sin\theta)_{36\text{ m}}$

$$0.1516 \sin\theta_{4\text{ m}} = 0.08258 \sin 22° \qquad \boldsymbol{\theta_{4\text{m}} = 11.77°}$$

Shoaling: $(H^2 nc\cos\theta)_{4\text{ m}} = (H^2 nc\cos\theta)_{36\text{ m}}$ $\qquad \boldsymbol{H_{4\text{m}} = 1.201 \text{ m}}$

$$H_{4\text{ m}}^2 \times 0.8956 \times 5.921 \times \cos 11.77° = 1.2^2 \times 0.5156 \times 10.87 \times \cos 22°$$

Power: $P = Ec_g = \frac{1}{8}\rho g H^2 (nc) = \mathbf{9614 \text{ W m}^{-1}}$

(b) A wave with period 7 s and height 1.0 m crosses the 4 m depth contour at angle $\theta = 0°$. Determine the breaking wave height and breaking depth from their corresponding indices and identify the type of breaker expected.

$$H_b = 0.56 H_0 \left(\frac{H_0}{L_0}\right)^{-1/5}$$

$$\gamma_b \equiv \left(\frac{H}{h}\right)_b = b - a\frac{H_b}{gT^2}$$

$$a = 43.8(1 - e^{-19m})$$

$$b = \frac{1.56}{1 + e^{-19.5m}}$$

$$(H^2nc)_0 = (H^2nc)_{4\text{ m}}$$

$$T = 7\text{ s}$$

Depth 4 m	Deep water
$H = 1.0\text{ m}$	?
$n = 0.8956$	$n = 0.5$
$c = 5.921\text{ m s}^{-1}$	$c = \frac{gT}{2\pi} = 10.93\text{ m s}^{-1}$
	$L_0 = \frac{gT^2}{2\pi} = \mathbf{76.50\ m}$

Shoaling:

$$(H^2nc)_0 = (H^2nc)_{4\text{ m}}$$

$$H_0^2 \times 0.5 \times 10.93 = 1 \times 0.8956 \times 5.921$$

$$\mathbf{H_0 = 0.9851\ m}$$

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$$a = 43.8(1 - e^{-19m})$$

$$b = \frac{1.56}{1 + e^{-19.5m}}$$

$T = 7$ s

$H_0 = 0.9851$ m

$L_0 = 76.50$ m

$m = 0.01$

Breaking height:

$$\boldsymbol{H_b = 1.317\ \mathrm{m}}$$

Breaking depth:

$a = 7.579$

$b = 0.8558$

$\gamma_b = 0.8350$

$$\boldsymbol{h_b = 1.577\ \mathrm{m}}$$

Breaker type:

$$\xi_0 = \frac{m}{\sqrt{H_0/L_0}} = \mathbf{0.0881}$$

spilling breakers

(c) Further along the coast, waves propagate over the outflow of a river. In water depth of 14 m, measurements indicate a period of 7 s and depth-averaged flow velocity of 0.8 m s^{-1} against the wave direction. Determine the wavelength.

$$h = 14\ \mathrm{m}$$

$$T_a = 7\ \mathrm{s}$$

$$U = -0.8\ \mathrm{m\ s^{-1}}$$

$$\omega_a = \frac{2\pi}{T_a} = 0.8976\ \mathrm{rad\ s^{-1}}$$

$$(\omega_a - kU)^2 = \omega_r^2 = gk \tanh kh$$

$$k = \frac{(\omega_a - kU)^2}{g \tanh kh}$$

$$k = \frac{(0.8976 + 0.8k)^2}{9.81 \tanh 14k}$$

$$k = 0.1087\ \mathrm{m^{-1}}$$

$$L = \frac{2\pi}{k} = \mathbf{57.80\ m}$$

Example

Waves propagate towards a straight shoreline. The wave heading is equal to the angle formed between wave crests and the bed contours. The bed slope is less than 1 in 100. Waves are measured in 30 m depth and wave conditions at 6 m depth are required to inform design of nearshore structures.

Regular waves are measured with period of 7 s and height of 3 m.

(a) Determine the water depth in which waves with this period can be considered as deep-water waves.

(b) For 30 m depth, determine the breaking height by the Miche criterion and briefly describe this type of breaking wave.

(c) If the heading is zero degrees, calculate wave height in 6 m depth. State your assumptions.

(d) If the heading of the measured conditions is 30°, calculate the wave heading and height in 6 m depth. Hence calculate the change of wave power per unit width of wave crest (kW m^{-1}) between the two depths.

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(a) Determine the water depth in which waves with this period can be considered as deep-water waves.

$$T = 7\ \mathrm{s}$$

$$\omega = \frac{2\pi}{T} = 0.8976\ \mathrm{rad\ s^{-1}}$$

$$\omega^2 = gk \tanh kh$$

$$\frac{\omega^2 h}{g} = kh \tanh kh$$

Deep-water waves if $kh > \pi$:

$$0.08213h > \pi \tanh \pi$$

$$\boldsymbol{h > 38.11\ \mathrm{m}}$$

(b) For 30 m depth, determine the breaking height by the Miche criterion and briefly describe this type of breaking wave.

Miche criterion: $\frac{H_b}{L} = 0.14 \tanh kh$

Irribarren number: $\xi_b = \frac{m}{\sqrt{H_b/L_0}}$

Dispersion relation: $\omega^2 = gk \tanh kh$

$$\omega = 0.8976 \text{ rad s}^{-1}$$

$$h = 30 \text{ m}$$

$$\frac{\omega^2 h}{g} = kh \tanh kh$$

$$2.464 = kh \tanh kh$$

$$kh = \frac{2.464}{\tanh kh}$$

$$kh = 2.498$$

$$k = 0.08327 \text{ m}^{-1}$$

$$L = \frac{2\pi}{k} = 75.46 \text{ m}$$

Miche criterion:

$$\frac{H_b}{75.46} = 0.14 \tanh 2.498$$

$$\mathbf{H_b = 10.42 \text{ m}}$$

$$L_0 = \frac{gT^2}{2\pi} = 76.50 \text{ m}$$

$$m < 0.01$$

$$\xi_b < \frac{0.01}{\sqrt{10.42/76.50}} = 0.0271$$

spilling breakers

(c) If the heading is zero degrees, calculate wave height in 6 m depth. State your assumptions.

Shoaling (no refraction): $(H^2 nc)_{6\text{ m}} = (H^2 nc)_{30\text{ m}}$

$\omega = 0.8976 \text{ rad s}^{-1}$ $H_{30\text{ m}} = 3\text{ m}$

	6 m depth (exercise)	30 m depth (earlier)
kh	0.7651	2.498
k	0.1275 m^{-1}	0.08327 m^{-1}
$c = \dfrac{\omega}{k}$	7.04 m s^{-1}	10.78 m s^{-1}
$n = \dfrac{1}{2}\left[1 + \dfrac{2kh}{\sinh 2kh}\right]$	0.8476	0.5338
H	---	3 m

Shoaling:

$$H_{6\text{ m}}^2 \times 0.8476 \times 7.04 = 3^2 \times 0.5338 \times 10.78$$

$$\boldsymbol{H_{6\text{ m}} = 2.946\text{ m}}$$

(d) If the heading of the measured conditions is 30°, calculate the wave heading and height in 6 m depth. Hence calculate the change of wave power per unit width of wave crest (kW m^{-1}) between the two depths.

Refraction: $(k \sin\theta)_{6\text{ m}} = (k \sin\theta)_{30\text{ m}}$

Shoaling (with refraction): $(H^2 nc \cos\theta)_{6\text{ m}} = (H^2 nc \cos\theta)_{30\text{ m}}$

$\omega = 0.8976 \text{ rad s}^{-1}$

	6 m depth	30 m depth
kh	0.7651	2.498
k	0.1275 m^{-1}	0.08327 m^{-1}
$c = \dfrac{\omega}{k}$	7.04 m s^{-1}	10.78 m s^{-1}
$n = \dfrac{1}{2}\left[1 + \dfrac{2kh}{\sinh 2kh}\right]$	0.8476	0.5338
θ	---	$30°$
H	---	3 m

Refraction: $0.1275 \sin\theta_{6\text{ m}} = 0.08327 \sin 30°$ $\quad$ $\boldsymbol{\theta_{6\text{ m}} = 19.06°}$

Shoaling: $H_{6\text{ m}}^2 \times 0.8476 \times 7.040 \times \cos 19.06° = 3^2 \times 0.5338 \times 10.78 \times \cos 30°$

$$\boldsymbol{H_{6\text{ m}} = 2.820 \text{ m}}$$

(d) If the heading of the measured conditions is 30°, calculate the wave heading and height in 6 m depth. Hence calculate the change of wave power per unit width of wave crest (kW m^{-1}) between the two depths.

$$P = Ec_g$$

$$E = \frac{1}{8}\rho g H^2$$

$$c_g = nc$$

$$\Delta P = \left(\frac{1}{8}\rho g H^2 nc\right)_{30\ \mathrm{m}} - \left(\frac{1}{8}\rho g H^2 nc\right)_{6\ \mathrm{m}} \qquad = \mathbf{5.451\ kW\ m^{-1}}$$

Wave Transformation

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2.1 Refraction

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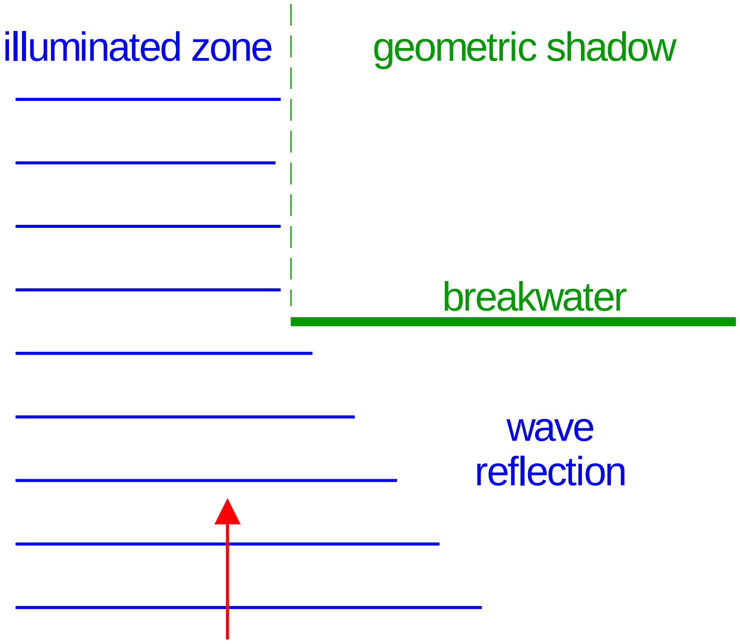
2.4 Diffraction

2.5 Reflection

Diffraction

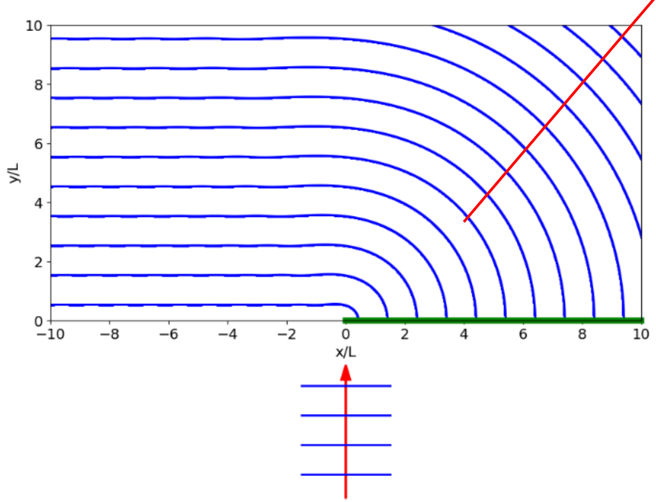
Diffraction is the spreading of waves into a region of geometric shadow.

It occurs because there cannot be discontinuities at the boundary of the illuminated zone.

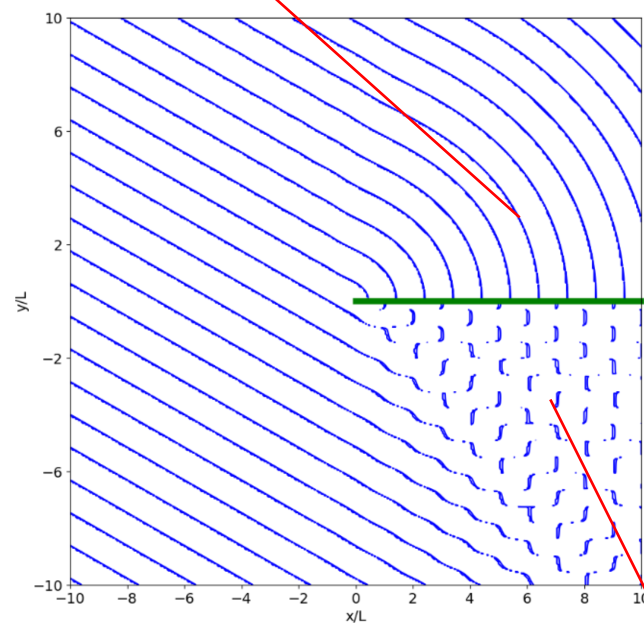


Diffraction – Semi-Infinite Breakwater

diffracted wave crests



Normal incidence



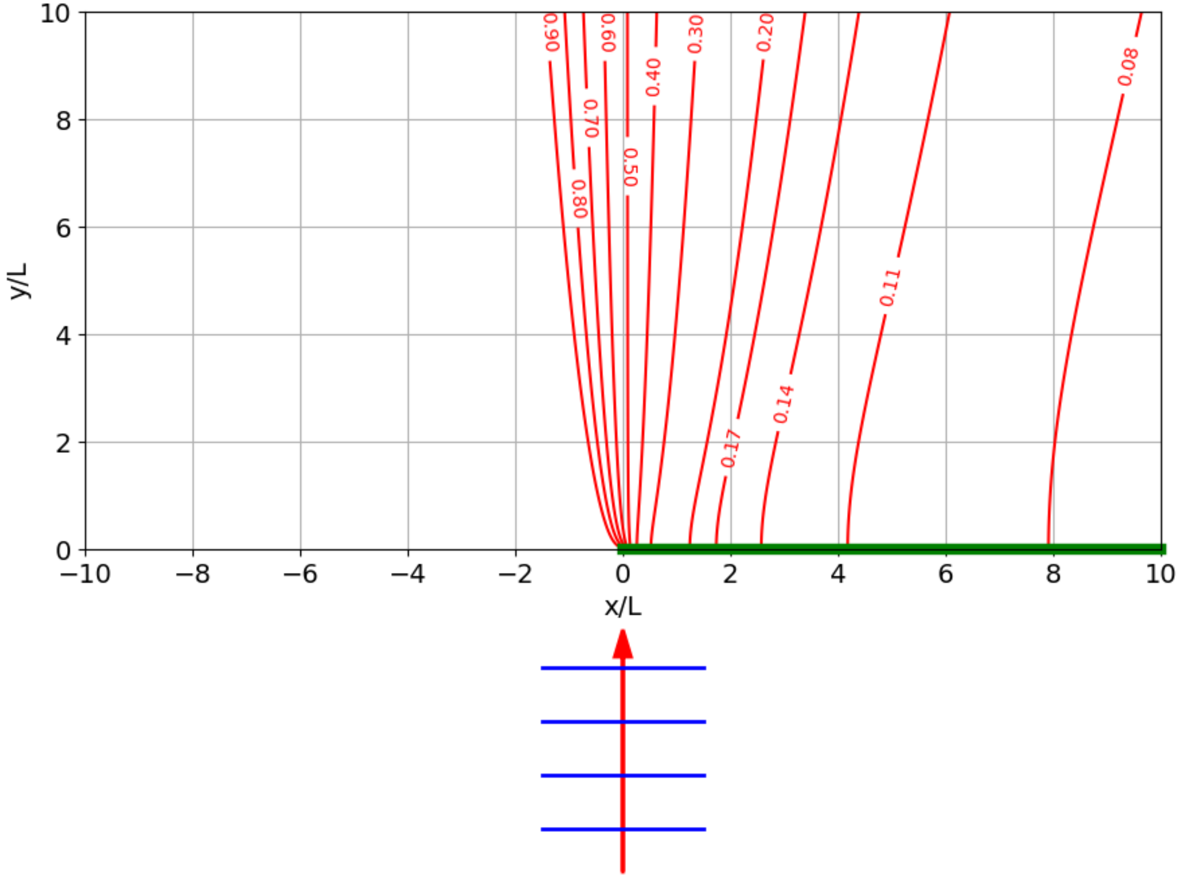
Oblique incidence

incident + reflected waves

Diffraction Coefficient

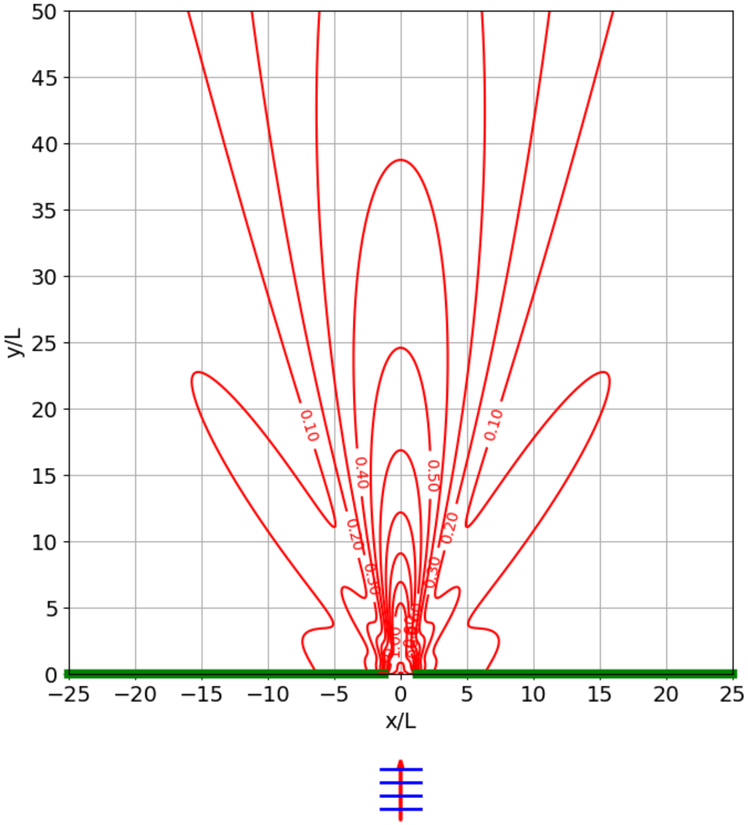
$$K_D(x) = \frac{H(x)}{H_0}$$

Semi-infinite plane breakwater; 90° wave incidence

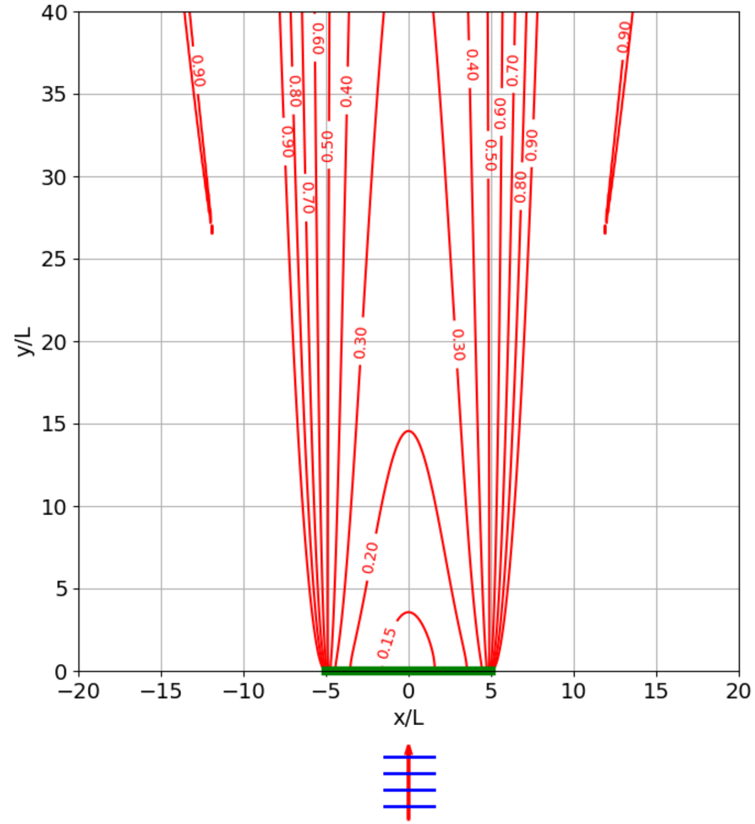


Diffraction Coefficient

Breakwater with a $2L$ gap

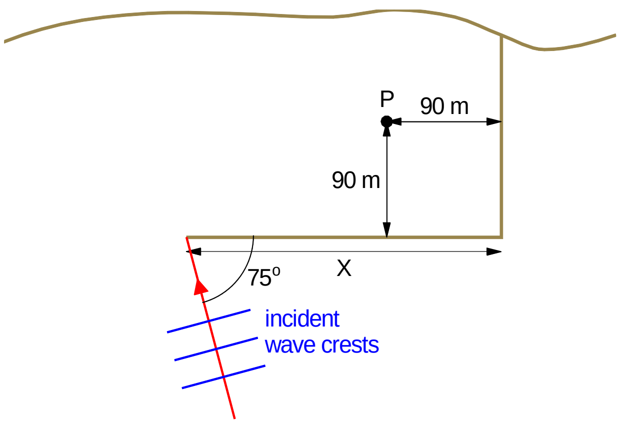


Offshore breakwater, length $10L$

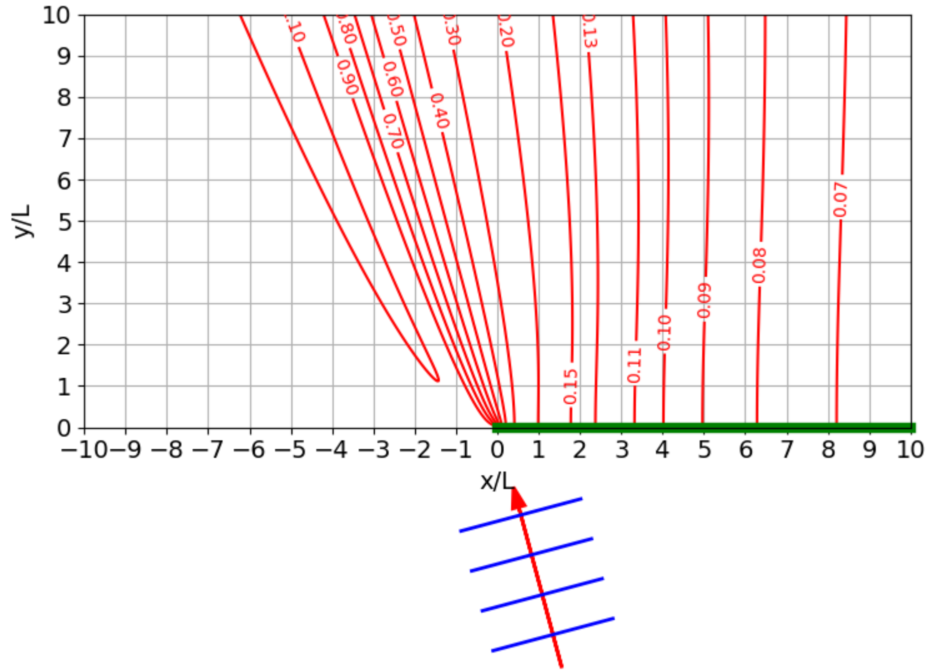


Example

A harbour is to be protected by an L-shaped breakwater as sketched. Determine the length X of the outer arm necessary for the wave height at point P to be 0.3 m when incident waves have a height of 3 m and a period of 5 s.



The depth is everywhere uniform at 5 m. The diffraction diagram for the appropriate approach angle is shown. Neglect reflections within the harbour.



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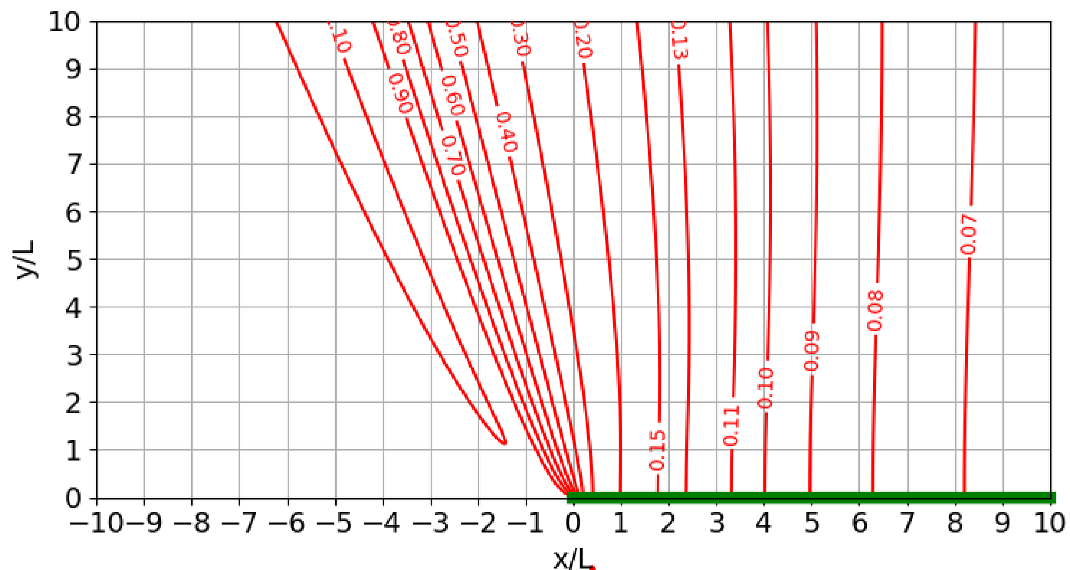
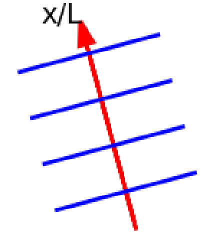
Require $K = \dfrac{0.3}{3} = 0.1$



From the diagram, this occurs at $x/L = 4.0$



The total arm length in m is $X = x + 90 \quad = 4L + 90$

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$h = 5 \text{ m}$

$T = 5 \text{ s}$

$$\omega = \frac{2\pi}{T} = 1.257 \text{ rad s}^{-1}$$

Arm length $X = 4L + 90$

$$\omega^2 = gk \tanh kh$$

$$\frac{\omega^2 h}{g} = kh \tanh kh$$

$kh \tanh kh = 0.8053$

$$kh = \frac{0.8053}{\tanh kh} \quad \text{or} \quad kh = \frac{1}{2}\left(kh + \frac{0.8053}{\tanh kh}\right)$$

$kh = 1.037$

$k = 0.2074 \text{ m}^{-1}$

$$L = \frac{2\pi}{k} = 30.30 \text{ m}$$

Arm length $X = 211$ m

Wave Transformation

2. WAVE TRANSFORMATION

2.1 Refraction

2.2 Shoaling

2.3 Breaking

2.4 Diffraction

2.5 Reflection

Reflection

For a rigid, impermeable boundary: $u = 0$ at the boundary

Superpose two **progressive waves** moving in opposite directions:

$$\eta = A\cos(kx - \omega t) + A\cos(-kx - \omega t)$$

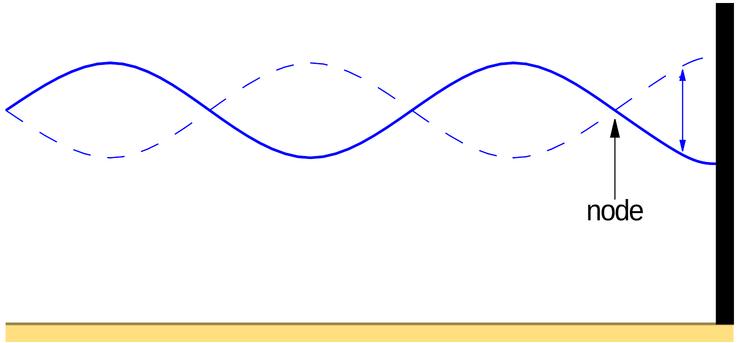
$$= A[\cos(kx - \omega t) + \cos(kx + \omega t)]$$

$$= 2A\cos kx \cos \omega t$$

This is a **standing wave**

... with **twice the amplitude**

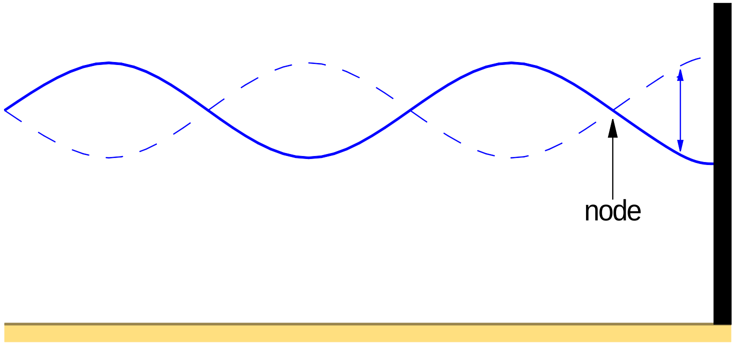
... and **nodes** every half-wavelength



Reflection

$$\eta = 2A \cos kx \cos \omega t$$

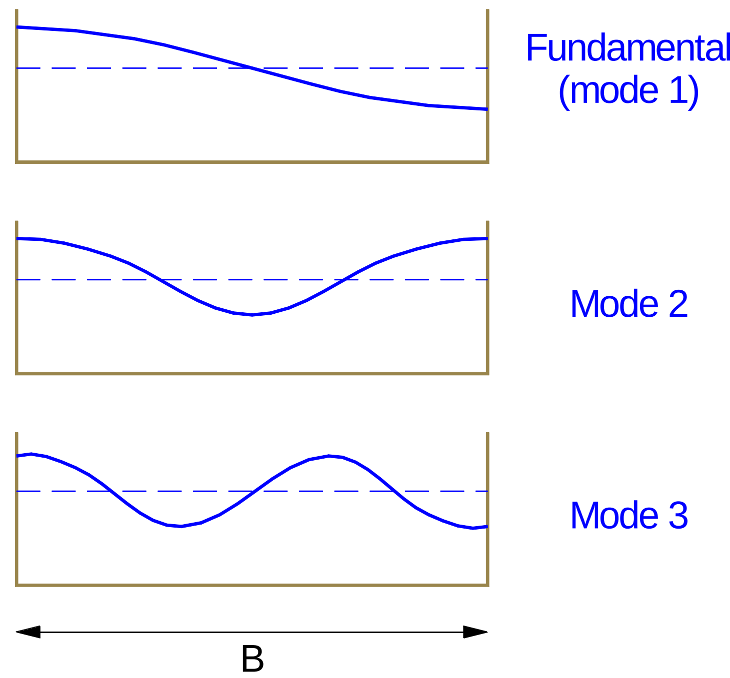
$$u = \frac{2Agk}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin kx \sin \omega t$$



- Reflection can be represented by the superposition of two equal and opposite progressive waves to form a standing wave.
- The standing wave has the same wavelength and frequency but twice the amplitude.
- There are surface nodes (i.e. zeroes of η) separated by half a wavelength, with velocity nodes intermediate between.
- The point of reflection corresponds to a point of zero velocity and double-amplitude displacement.

Seiching

Seiches are resonant standing waves set up in enclosed basins (lakes, harbours, ...)



The basin length is a whole number of half-wavelengths:

$$B = n_s\left(\tfrac{1}{2}L\right) \qquad L = \frac{2B}{n_s}$$

$$\text{period} = \frac{\text{wavelength}}{\text{speed}}$$

$$T = \frac{2B}{n_s\sqrt{gh}}$$

Reflection

Lake Baikal in Siberia contains about one fifth of the world's fresh-water resources. It is 636 km long, with an average depth of 744 m. Find the fundamental period for seiching.

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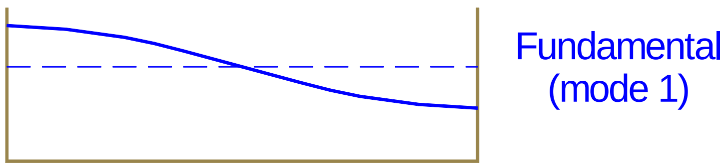

$$L = 2 \times \text{length of lake} \quad = 1.272 \times 10^6 \text{ m}$$

$$c = \sqrt{gh} \quad = 85.43 \text{ m s}^{-1}$$

$$T = \frac{L}{c} \quad = 14890 \text{ s} \qquad (\mathbf{4.1\ hours})$$