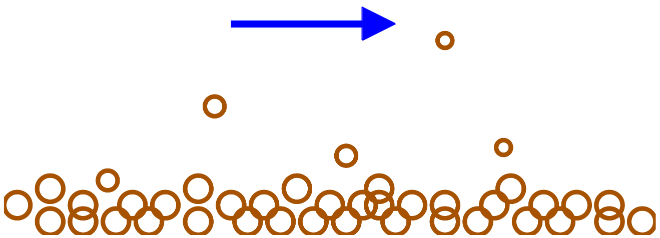
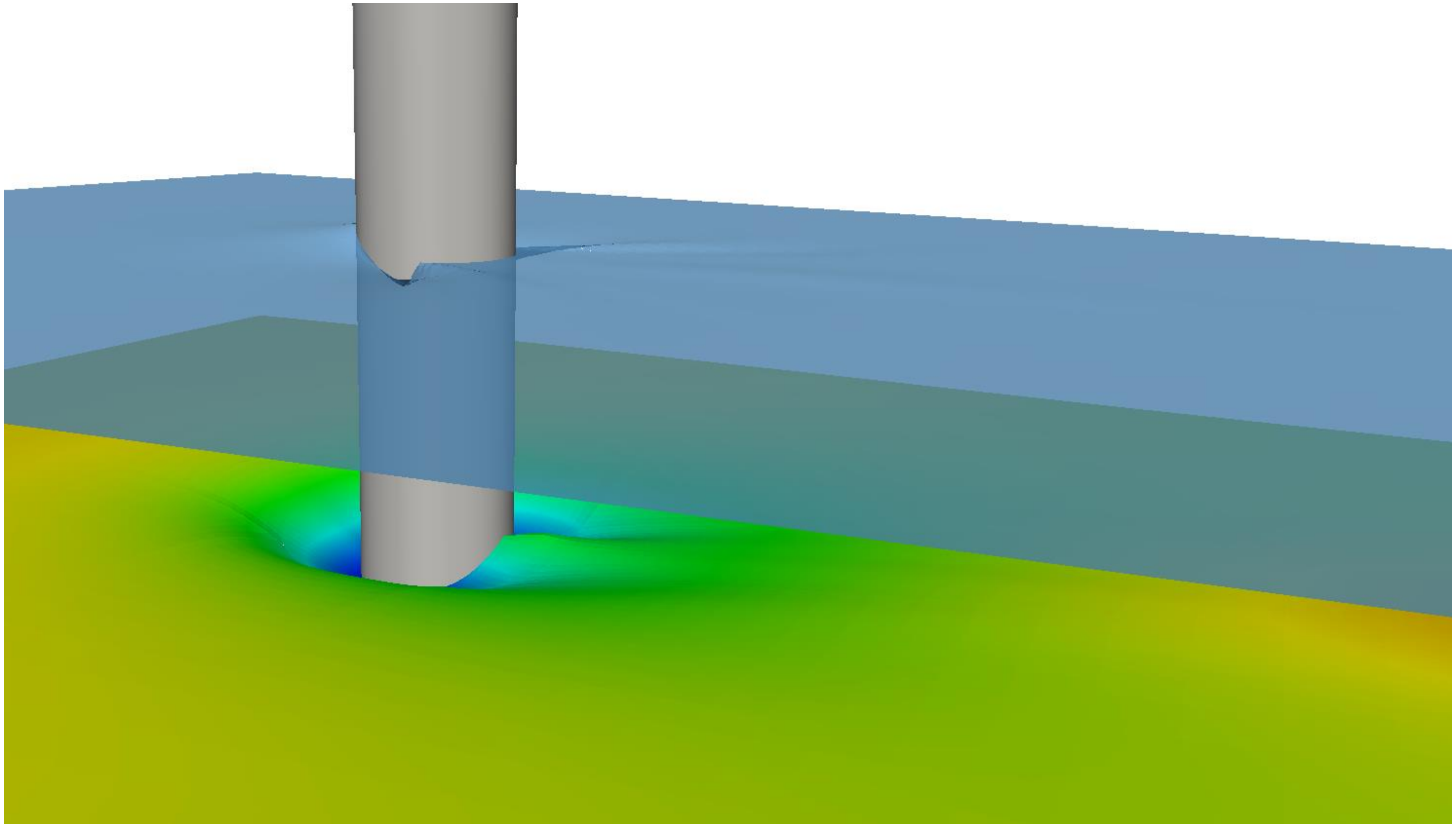
Sediment Transport



- Most natural channels have **mobile** beds
- Sufficiently vigorous fluid motions can stir these into motion
- Many rivers carry a large **sediment load** (e.g. Yellow River)
- Most mobile beds are in **dynamic equilibrium**:
  - sediment in = sediment out … on average
- This equilibrium can be seriously disturbed by
  - short-term extreme events (storms; floods)
  - man-made infrastructure (dams; coastal defences)

Bridge Scour



Sediment Transport

Basic questions:

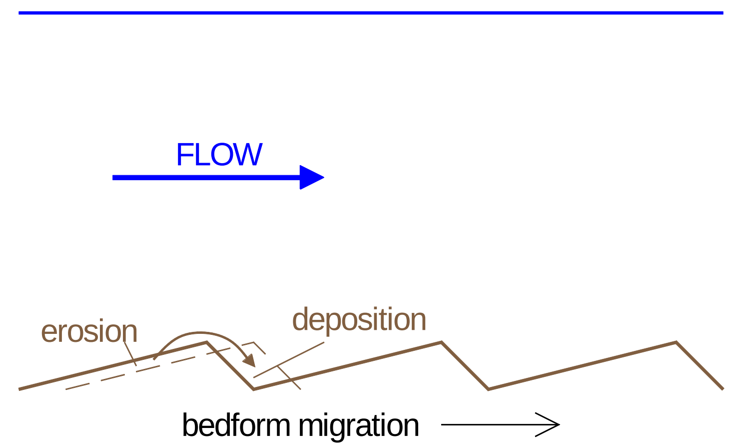
- Does sediment transport occur? (**Threshold of motion**).
- If so, then at what rate? (**Sediment load**)
- What net effect does it have on the bed? (**Scour/accretion**)

Main types of sediment load (volume of sediment per second):

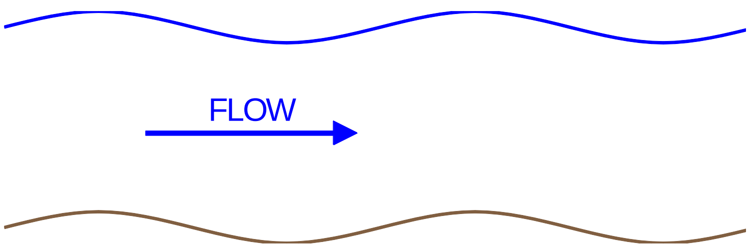
- **Bed load**
- **Suspended load**

The combination is **total load**

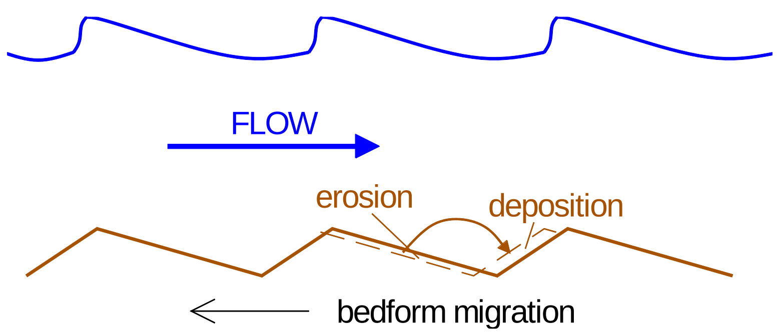
Classical Bedforms



Dunes



Standing waves



Antidunes

Relevant Properties

- **Particle**
  - Diameter, d
  - Specific gravity, $s = \rho_s/\rho$
  - Settling velocity, w_s
  - Porosity, P
  - Angle of repose, ϕ
- **Fluid**
  - Density, ρ
  - Kinematic viscosity, ν
- **Flow**
  - Bed shear stress, τ_b
  - Mean-velocity profile, $U(z)$
  - Eddy-viscosity profile, $\nu_t(z)$

Inception of Motion

- Inception and magnitude of **bed-load** depends on:
  - bed shear stress τ_b
  - particle diameter d and specific gravity s

- Inception of **suspended load** depends on ratio of:
  - settling velocity w_s
  - typical turbulent velocity (friction velocity u_τ)

Particle Properties: Diameter d

Various types:

- sieve diameter
- sedimentation diameter
- nominal diameter

Type	Diameter
Boulders	> 256 mm
Cobbles	64 mm – 256 mm
Gravel	2 mm – 64 mm
Sand	0.06 mm – 2 mm
Silt	0.002 mm – 0.06 mm
Clay	< 0.002 mm (cohesive)

In practice, there is a range of diameters (typically, lognormally distributed).

Particle Properties: Specific Gravity s

$$s = \frac{\rho_s}{\rho}$$

Quartz-like: $\rho_s \approx 2650 \text{ kg m}^{-3}$, $s \approx 2.65$

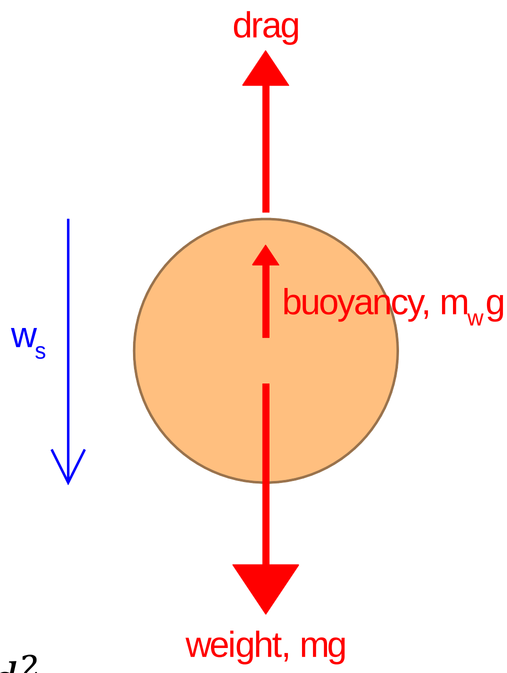
Anthracite: $\rho_s \approx 1500 \text{ kg m}^{-3}$, $s \approx 1.50$

Particle Properties: Settling Velocity w_s

- Used to determine onset and distribution of suspended load.
- Terminal velocity in still fluid.

drag = weight – buoyancy

$$c_D\left(\frac{1}{2}\rho w_s^2\right)\left(\frac{\pi d^2}{4}\right) = (\rho_s - \rho) g \frac{\pi d^3}{6}$$



$$w_s = \left(\frac{4}{3}\frac{(s-1)gd}{c_D}\right)^{1/2} \qquad c_D = f(\mathrm{Re}), \qquad \mathrm{Re} = \frac{w_s d}{\nu}$$

Small particles (Stokes' Law): $$c_D = \frac{24}{\mathrm{Re}} \qquad w_s = \frac{1}{18}\frac{(s-1)gd^2}{\nu}$$

$$\frac{w_s d}{\nu} = \frac{1}{18}\frac{(s-1)gd^3}{\nu^2} = \frac{1}{18} d^{*3} \qquad d^* = d\left(\frac{(s-1)g}{\nu^2}\right)^{1/3}$$

Realistic sizes and shapes: $$\frac{w_s d}{\nu} = \left[(25 + 1.2 d^{*2})^{1/2} - 5\right]^{3/2}$$

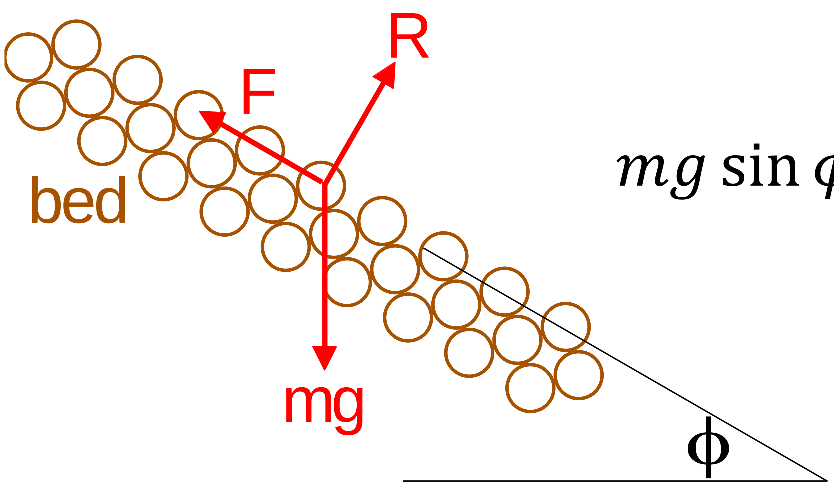
Particle Properties: Porosity P

Porosity = fraction of voids (by volume)

Typical uncompacted sediment: $P \approx 0.4$.

Particle Properties: Angle of Repose ϕ

Angle of repose = limiting angle of slope (in still fluid)



$$mg \sin\phi = \mu_f (mg \cos\phi)$$

Effective **coefficient of friction** $\mu_f = \tan\phi$

Can be used to estimate the effect of slopes on incipient motion

Flow Properties: Bed Friction

Bed shear stress τ_b

- Drag (per unit area) of flow on granular bed.
- Determines inception and magnitude of **bed load**.

Friction velocity u_τ

- Defined (on dimensional grounds) by:

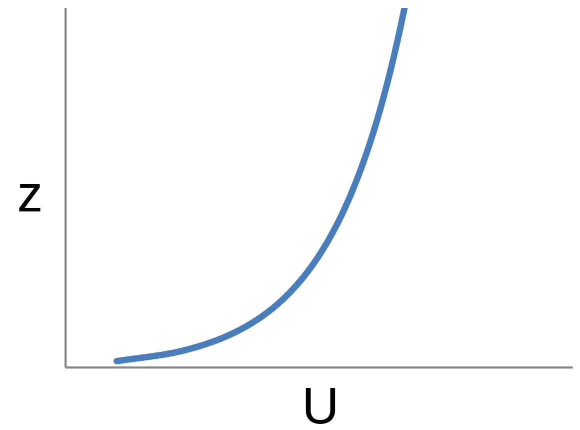
$$\tau_b = \rho u_\tau^2 \qquad \text{or} \qquad u_\tau = \sqrt{\tau_b/\rho}$$

- Determines inception and magnitude of **suspended load**.

Flow Properties: Mean-Velocity Profile

For a **rough** boundary:

$$U(z) = \frac{u_\tau}{\kappa} \ln(33 \frac{z}{k_s})$$



u_τ = friction velocity;

κ = von Kármán's constant (≈ 0.41);

z = distance from boundary;

k_s = roughness height (1.0 to 2.5 times particle diameter).

Flow Properties: Eddy-Viscosity Profile

A model for the effective shear stress τ in a turbulent flow:

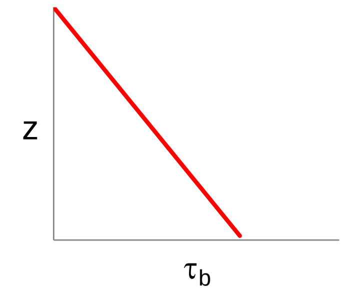
$$\tau = \mu_t \frac{\mathrm{d}U}{\mathrm{d}z} \quad \text{or} \quad \tau = \rho\nu_t \frac{\mathrm{d}U}{\mathrm{d}z}$$

μ_t and ν_t are the dynamic and kinematic **eddy viscosities**.

At the bed: $\tau = \tau_b \equiv \rho u_\tau^2$

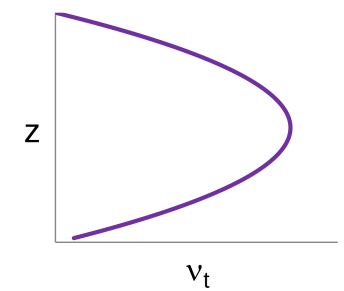
At the free surface: $\tau = 0$

Assuming linear: $\tau = \rho u_\tau^2 (1 - z/h)$



From stress and mean-velocity profiles: $\rho u_\tau^2 (1 - z/h) = \rho \nu_t \frac{u_\tau}{\kappa z}$

$$\nu_t = \kappa u_\tau z (1 - z/h)$$



Formulae For Bed Shear Stress

Normal flow: $\tau_b = \rho g R_h S$

Manning's formula: $V = \frac{1}{n} R_h^{2/3} S^{1/2}$ (gives R_h from V or Q)

Strickler's formula: $n = \frac{d^{1/6}}{21.1}$

Typical values: $n \approx 0.01$ to 0.035 m$^{-1/3}$ s

Via a **friction coefficient**: $\tau_b = c_f(\frac{1}{2}\rho V^2)$

Fully-developed boundary layer (log-law): $c_f = \frac{0.34}{[\ln(12h/k_s)]^2}$

Typical values: $c_f \approx 0.003$ to 0.01

Threshold of Motion

A mobile bed starts to move once the bed stress exceeds a **critical stress** τ_{crit}.

Simple model:

critical stress × representative area = friction coefficient × normal reaction

$$\tau_{\mathrm{crit}} \times c\frac{\pi d^2}{4} = \mu_{\mathrm{frict}} \times (\rho_s - \rho) g \frac{\pi d^3}{6}$$

$$\frac{\tau_{\mathrm{crit}}}{(\rho_s - \rho) g d} = \frac{2\mu_{\mathrm{frict}}}{3c}$$

$$\frac{\tau_{\mathrm{crit}}}{(\rho_s - \rho) g d} = \text{dimensionless function of particle shape and size}$$

Shields parameter

Shields' Diagram (A.F. Shields, 1936)

Dimensionless groups:

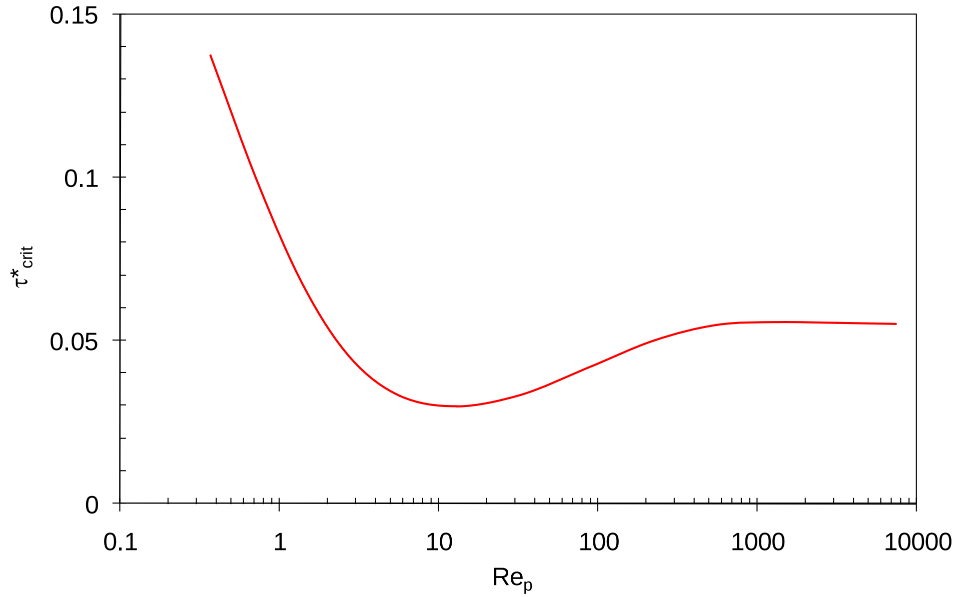
$$\tau^* = \frac{\tau_b}{(\rho_s - \rho) g d}$$

Shields parameter or **Shields stress**

$$\mathrm{Re}_p = \frac{u_\tau d}{\nu}$$

particle Reynolds number

$$\tau^*_{\mathrm{crit}} = f(\mathrm{Re}_p)$$



But this is not helpful in finding the critical τ ... because that also appears on the RHS (via u_τ)

Critical Stress

2 dimensionless groups:

$$\Pi_1 = \frac{\tau_b}{(\rho_s - \rho) g d} \qquad (\tau^*)$$

$$\Pi_2 = \frac{u_\tau d}{\nu} = \frac{(\sqrt{\tau_b/\rho}) d}{\nu} \qquad (Re_p)$$

Replace Π_2 by

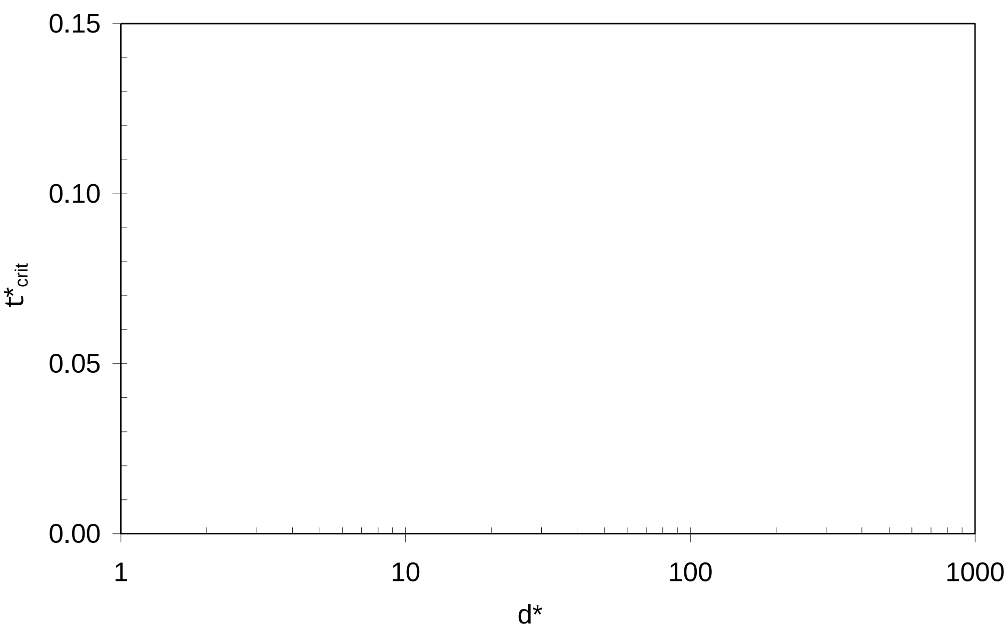
$$\Pi_2' = \left(\frac{\Pi_2^2}{\Pi_1}\right)^{1/3} = \left[\frac{(s-1)g}{\nu^2}\right]^{1/3} d$$

... call this d^*

$$\tau^*_{\text{crit}} = f(d^*)$$

$$\tau^* = \frac{\tau_b}{(\rho_s - \rho) g d}$$ **Shields parameter**

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} \qquad (s = \rho_s/\rho)$$

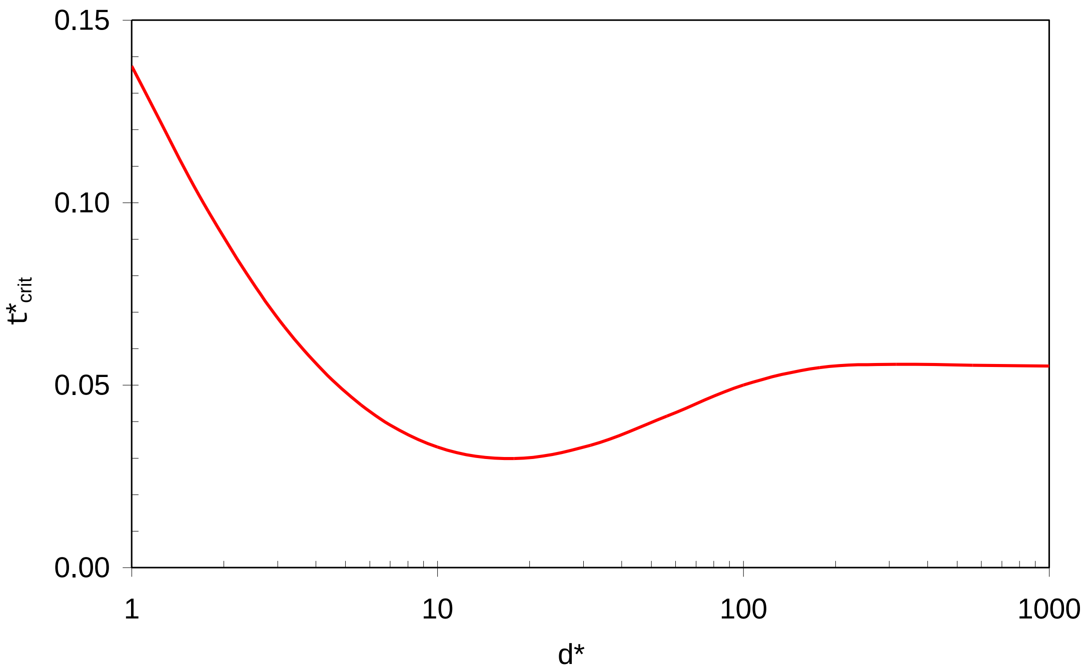


Finding the Threshold of Motion

$$\tau^*_{\text{crit}} = f(d^*)$$

$$\tau^* = \frac{\tau_b}{(\rho_s - \rho)gd}$$

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$



Curve fit (Soulsby):

$$\tau^*_{\text{crit}} = \frac{0.30}{1 + 1.2d^*} + 0.055\,[1 - \exp(-0.020d^*)]$$

Example

An undershot sluice is placed in a channel with a horizontal bed covered by gravel with a median diameter of 5 cm and density 2650 kg m^{-3}. The flow rate is 4 m^3 s^{-1} per metre width and initially the depth below the sluice is 0.5 m.

Assuming a critical Shields parameter τ^*_{crit} of 0.06 and friction coefficient c_f of 0.01:

(a) find the depth just upstream of the sluice and show that the bed there is stationary;

(b) show that the bed below the sluice will erode and determine the depth of scour.

Example

A long rectangular channel contracts smoothly from 5 m width to 3 m width for a region near its midpoint. The bed is composed of gravel with a median grain size of 10 mm and density 2650 kg m^{-3}. The friction coefficient c_f is 0.01 and the critical Shields parameter is 0.05. If the flow rate is 5 m^3 s^{-1} and the upstream depth is 1 m:

(a) show that the bed is stationary in the 5 m width and mobile in the 3 m width by assuming that the bed is initially flat;

(b) determine the depth of the scour hole which results just after the contraction.

Inception of Motion in Normal Flow

Assume:

coarse sediment: $$\frac{\tau_b}{(\rho_s - \rho)gd} > 0.056$$

normal flow: $$\tau_b = \rho g R_h S$$

Mobile if $$\frac{R_h S}{(\rho_s/\rho - 1)} > 0.056\, d$$

For sand: $$d < 10.8 R_h S$$

Effect of Slopes

Gravitational forces may oppose or assist the initiation of motion

$$\tau_{\text{crit}} = \frac{\sin(\phi + \beta)}{\sin\phi}\tau_{\text{crit},0}$$

(β positive for upslope flow)

On side slopes the gravitational force always favours motion:

$$\tau_{\text{crit}} = \cos\beta\sqrt{1 - \frac{\tan^2\beta}{\tan^2\phi}}\;\tau_{\text{crit},0}$$

Bed Load

- **Bed load** consists of particles sliding, rolling or saltating, but remaining essentially in contact with the bed

- It is the dominant form of sediment transport for larger particles (settling velocity too large for suspension)

- The **bed-load flux** q_b is the volume of non-suspended sediment crossing unit width of bed per unit time.

Dimensional Analysis

Variable	Symbol	Dimensions
Bed load flux	q_b	L^2T^{-1}
Bed shear stress	τ_b	$ML^{-1}T^{-2}$
Reduced gravity	$(s-1)g$	LT^{-2}
Particle diameter	d	L
Density	ρ	ML^{-3}
Kinematic viscosity	ν	L^2T^{-1}

6 variables, 3 dimensions $\Rightarrow$ 3 dimensionless groups

3 scales: ρ, $(s-1)g$, d

$$\Pi_1 = \frac{q_b}{\sqrt{(s-1)gd^3}} \qquad \Pi_2 = \frac{\tau_b}{\rho(s-1)gd} \qquad \Pi_3 = \frac{\nu}{\sqrt{(s-1)gd^3}}$$

$$\Pi_3' = \frac{1}{\Pi_3^{2/3}} = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

Dimensionless Groups

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}$$

dimensionless bed-load flux

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd}$$

dimensionless bed shear stress (Shields parameter)

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

dimensionless particle diameter

Bed-Load Models

Reference	Formula
Meyer-Peter and Müller (1948)	$q^* = 8(\tau^* - \tau^*_{\text{crit}})^{3/2}$
Nielsen (1992)	$q^* = 12(\tau^* - \tau^*_{\text{crit}})\sqrt{\tau^*}$
Van Rijn (1984)	$q^* = \frac{0.053}{d^{*0.3}}(\frac{\tau^*}{\tau^*_{\text{crit}}} - 1)^{2.1}$
Einstein-Brown (Brown, 1950)	$q^* = \begin{cases} \frac{K\exp(-0.391/\tau^*)}{0.465} & \tau^* < 0.182 \\ 40K{\tau^*}^3 & \tau^* \geq 0.182 \end{cases}$
Yalin (1963)	$q^* = 0.635r\sqrt{\tau^*}[1 - \frac{1}{\sigma r}\ln(1 + \sigma r)]$

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}} \qquad \tau^* = \frac{\tau_b}{\rho(s-1)gd} \qquad d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

Calculating Bed Load

$$q^* = f(\tau^*, d^*)$$

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}$$ dimensionless bed-load flux

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd}$$ dimensionless bed shear stress

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$ dimensionless particle diameter

To find bed-load flux:

- from particle and fluid properties, find d^*
- from formula or graph, find τ^*_{crit}
- from flow hydraulics, find τ_b and hence τ^*
- if $\tau^* > \tau^*_{\text{crit}}$ (or $\tau > \tau_{\text{crit}}$), find q^* by chosen model
- invert to get absolute bed-load flux per unit width, q_b
- if required, multiply by width to get bed-load flux, Q_b

Suspended Load

- **Suspended load** consists of finer particles carried in suspension by turbulent fluid flow.

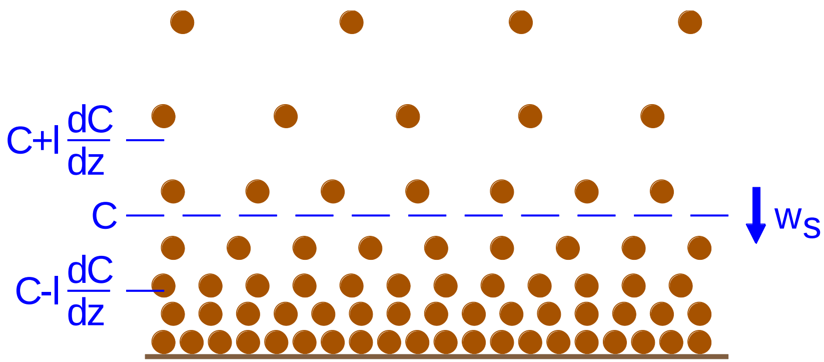
- Significant suspended load only occurs if turbulent velocity fluctuations are larger than the settling velocity:

$$\frac{u_\tau}{w_s} > 1$$

- For coarser sediment, suspended load does not occur and all sediment motion is bed load.

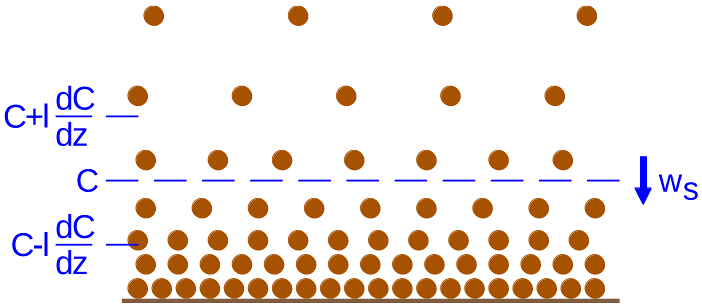
Concentration

- **Concentration** C is the volume of sediment per total volume



- Sediment settles, so concentrations are larger near the bed.
- Hence, upward-moving eddies tend to carry more sediment than downward-moving ones.
- This leads to a net upward diffusion of material.
- Equilibrium when **downward settling = upward diffusion**.

Fluxes



Sediment flux = volume flux × amount per unit volume
= (velocity × area) × concentration

$$C(z+\delta z) = C(z) + \delta z\frac{\mathrm{d}C}{\mathrm{d}z} + \cdots$$

Diffusion:

Net upward flux of sediment through a horizontal area *A*:

$$\frac{1}{2}u'A(C - l\frac{\mathrm{d}C}{\mathrm{d}z}) - \frac{1}{2}u'A(C + l\frac{\mathrm{d}C}{\mathrm{d}z}) = -u'l\frac{\mathrm{d}C}{\mathrm{d}z}A$$

Upward diffusive flux: $-K\frac{\mathrm{d}C}{\mathrm{d}z}A$ K = **eddy diffusivity**

Settling:

Downward settling flux: $(w_s A)C$

Diffusion Equation For Concentration (1)

A dynamic equilibrium exists when the net **upward flux** due to diffusion equals the net **downward flux** due to settling:

$$-K\frac{\mathrm{d}C}{\mathrm{d}z} = w_s C$$

Eddy Diffusivity

Diffusive flux of sediment (per unit area): $-K\frac{\mathrm{d}C}{\mathrm{d}z}$

Sediment is diffused by the same turbulent eddies as those that transfer momentum:

$$\tau = \mu_t \frac{\mathrm{d}U}{\mathrm{d}z} = \nu_t \frac{\mathrm{d}(\rho U)}{\mathrm{d}z}$$

flux of momentum (per unit area)

momentum (per unit volume)

Hence, common to take $K = \nu_t = \kappa u_\tau z(1 - z/h)$

Diffusion Equation For Concentration (2)

$$-K\frac{\mathrm{d}C}{\mathrm{d}z} = w_s C$$

where

$$K = \nu_t = \kappa u_\tau z(1 - z/h)$$

$$-\kappa u_\tau z(1 - z/h)\frac{\mathrm{d}C}{\mathrm{d}z} = w_s C$$

$$\frac{\mathrm{d}C}{C} = -\frac{w_s}{\kappa u_\tau}\frac{1}{z(1 - z/h)}\mathrm{d}z$$

$$\int_{C_{\mathrm{ref}}}^{C}\frac{\mathrm{d}C}{C} = -\frac{w_s}{\kappa u_\tau}\int_{z_{\mathrm{ref}}}^{z}\left(\frac{1}{z} + \frac{1}{h - z}\right)\mathrm{d}z$$

Semi-empirical formulae for C_{ref} and z_{ref}

Solution

$$[\ln C]_{C_{\text{ref}}}^{C} = -\frac{w_s}{\kappa u_\tau}[\ln z - \ln(h - z)]_{z_{\text{ref}}}^{z}$$

$$\ln C - \ln C_{\text{ref}} = -\frac{w_s}{\kappa u_\tau}\left(\ln\frac{z}{h - z} - \ln\frac{z_{\text{ref}}}{h - z_{\text{ref}}}\right)$$

$$\ln C - \ln C_{\text{ref}} = -\frac{w_s}{\kappa u_\tau}\left(\ln\frac{1}{h/z - 1} - \ln\frac{1}{h/z_{\text{ref}} - 1}\right)$$

$$\ln C - \ln C_{\text{ref}} = \frac{w_s}{\kappa u_\tau}(\ln(h/z - 1) - \ln(h/z_{\text{ref}} - 1))$$

$$\ln\frac{C}{C_{\text{ref}}} = \ln\left(\frac{h/z - 1}{h/z_{\text{ref}} - 1}\right)^{\frac{w_s}{\kappa u_\tau}}$$

$$\ln A - \ln B = \ln\frac{A}{B}$$

$$n \ln A = \ln A^n$$

$$\ln\frac{1}{A} = -\ln A$$

Rouse profile: $$\frac{C}{C_{\text{ref}}} = \left(\frac{h/z - 1}{h/z_{\text{ref}} - 1}\right)^{\frac{w_s}{\kappa u_\tau}}$$

Rouse number: $$\frac{w_s}{\kappa u_\tau}$$

Rouse Distribution

Rouse profile:

$$\frac{C}{C_{\text{ref}}} = \left(\frac{h/z - 1}{h/z_{\text{ref}} - 1}\right)^{\frac{w_s}{\kappa u_\tau}}$$

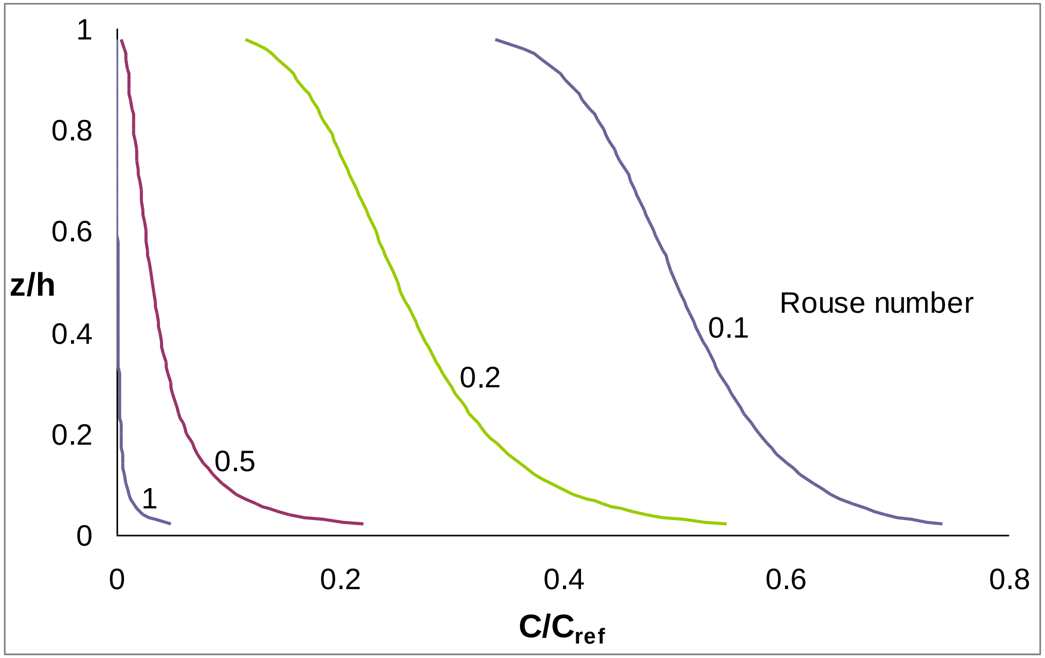
Rouse number:

$$\frac{w_s}{\kappa u_\tau}$$

w_s = settling velocity of the particle

u_τ = friction velocity of the flow $= \sqrt{\tau_b/\rho}$

κ =von Kármán's constant (≈ 0.41)



Calculation of Suspended Load

Volume flow rate of water: $u \, \mathrm{d}A$

per unit span: $u \, \mathrm{d}z$ (through depth dz)

Volume flux of sediment = concentration × volume flux of water

$$= Cu \, \mathrm{d}z$$

Suspended load:

$$q_s = \int_{z_{\mathrm{ref}}}^{h} Cu \, \mathrm{d}z$$

$$u(z) = \frac{u_\tau}{\kappa} \ln\left(33 \frac{z}{k_s}\right)$$

$$\frac{C}{C_{\mathrm{ref}}} = \left(\frac{h/z - 1}{h/z_{\mathrm{ref}} - 1}\right)^{\frac{w_s}{\kappa u_\tau}}$$

Example Sheet, Q1

Using Cheng's formula estimate the settling velocity of a sand particle of diameter 1 mm in

(a) air;

(b) water.

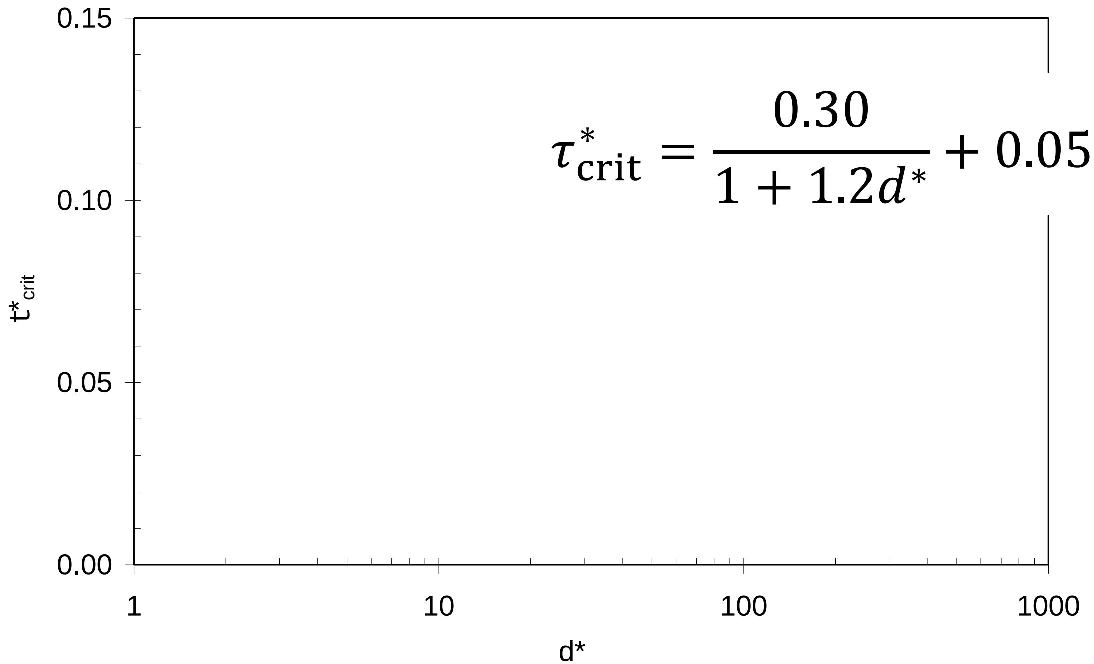
Cheng's formula for settling velocity:

$$\frac{w_s d}{\nu} = \left[(25 + 1.2 {d^*}^2)^{1/2} - 5\right]^{3/2} \qquad d^* = d\left(\frac{(s-1)g}{\nu^2}\right)^{1/3}$$

Example Sheet, Q2

Find the critical Shields parameter and critical absolute stress τ_{crit} for a sand particle of diameter 1 mm in water.

Critical shear stress



$$\tau^*_{crit} = \frac{0.30}{1 + 1.2d^*} + 0.055\ [1 - \exp(-0.020d^*)]$$

$$\tau^* = \frac{\tau_b}{(\rho_s - \rho)gd}$$

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

Example Sheet, Q3

Assuming a friction coefficient c_f = 0.005, estimate the velocities V at which

(a) incipient motion;

(b) incipient suspended load;

occur in water for sand particles of density 2650 kg m^{-3} and diameter 1 mm.

$$\tau^* = \frac{\tau_b}{(\rho_s - \rho)gd}$$

Example Sheet, Q9

A wide channel of slope 1:800 has a fine sandy bed with $d_{50} = 0.5\ \mathrm{mm}$. The discharge is 5 $m^3\ s^{-1}$ per metre width. The specific gravity of the bed material is 2.65.

(a) Estimate Manning's n using Strickler's formula.

(b) Find the depth of flow; (assume normal flow).

(c) Find the bed shear stress.

(d) Show that the bed is mobile and calculate the bed-load flux (per metre width) using: (i) Meyer-Peter and Müller; (ii) Nielsen models.

(e) Find the particle settling velocity and show that suspended load will occur.

(f) Estimate the suspended-load flux (per metre width), explaining your method and stating any assumptions made.

Bed-Load Formulae

Meyer-Peter and Müller (1948)	$q^* = 8(\tau^* - \tau^*_{\text{crit}})^{3/2}$
Nielsen (1992)	$q^* = 12(\tau^* - \tau^*_{\text{crit}})\sqrt{\tau^*}$
Van Rijn (1984)	$q^* = \frac{0.053}{{d^*}^{0.3}}(\frac{\tau^*}{\tau^*_{\text{crit}}} - 1)^{2.1}$

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}$$ (dimensionless bed-load flux)

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd}$$ (dimensionless bed shear stress)

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$ (dimensionless particle diameter)