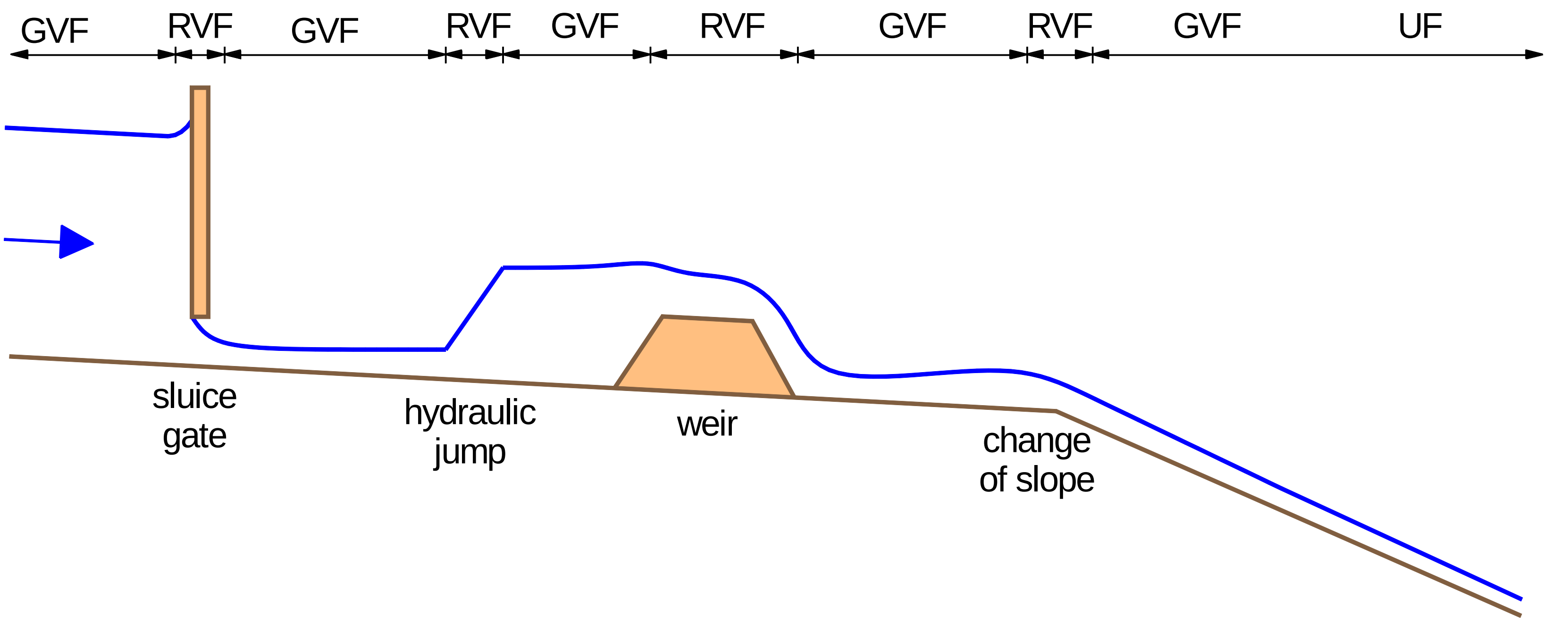
Open-Channel Flow

2. Rapidly-Varied Flow


GVF
RVF
GVF
RVF
GVF
RVF
GVF
RVF
GVF
UF
sluice gate
hydraulic jump
weir
change of slope

Rapidly-Varied Flow (RVF)

- Examples
  - hydraulic jump, weir, venturi, sluice, …
- Flow transitions between:
  - deep, slow flow (subcritical; $\mathrm{Fr} < 1$)
  - shallow, fast flow (supercritical; $\mathrm{Fr} > 1$)
- Changes over short distances (a few depths)
  - bed friction not important
  - total head approximately constant (except hydraulic jump)
- Either:
  - smooth transition (e.g. weir); negligible change in head
  - abrupt transition (hydraulic jump); significant head loss

Rapidly-Varied Flow

2. RAPIDLY-VARIED FLOW

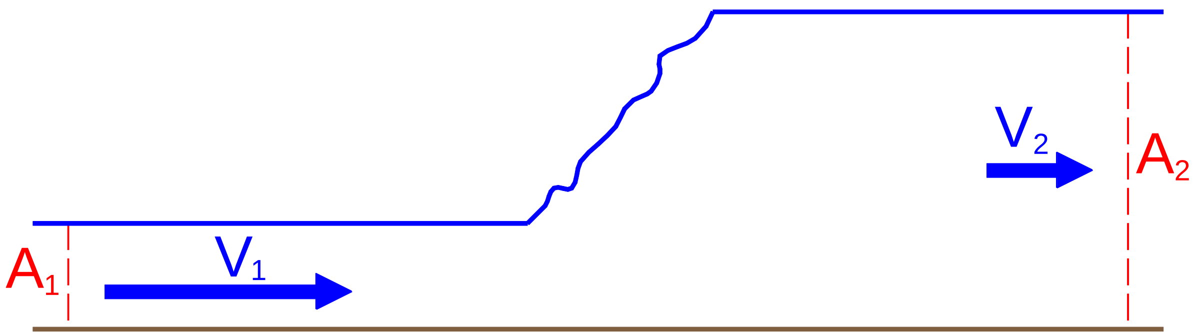
2.1 Hydraulic jump

2.2 Specific energy

2.3 Critical-flow devices

2.4 Forces on objects

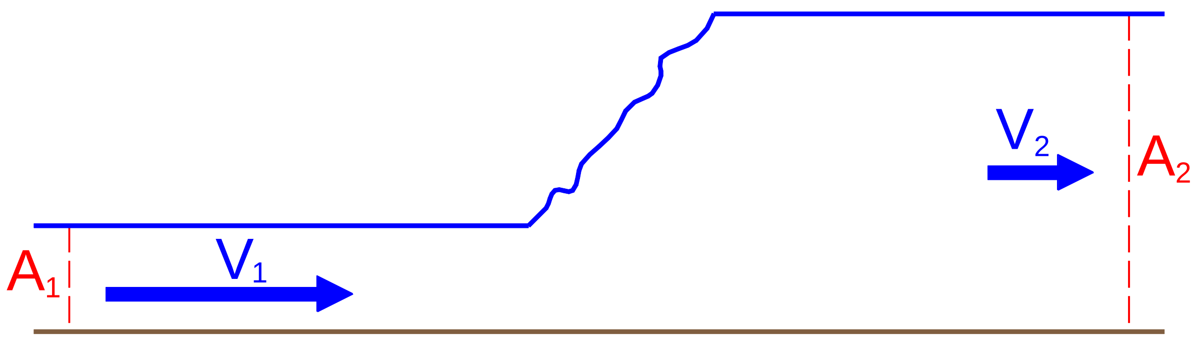
Hydraulic Jump



- Abrupt change from shallow ($\mathrm{Fr} > 1$) to deep ($\mathrm{Fr} < 1$)
- Occurs where up- and downstream depths are not compatible
- *Smooth* transition $\mathrm{Fr} > 1$ to $\mathrm{Fr} < 1$ not possible on a flat bed

$$h_2 = \frac{h_1}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right)$$

Hydraulic Jump: Assumptions

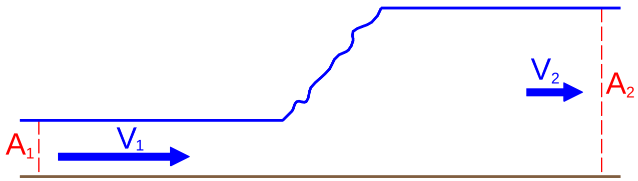


Assume (for now):

- uniform velocities upstream and downstream
- small slope (weight component not important)
- short extent (bed friction not important)
- wide or rectangular cross-section

Hydraulic Jump

Continuity: Flow rate $Q = VA$ constant



$$V = \frac{Q}{A}$$

Momentum: Net pressure force = change in momentum flux

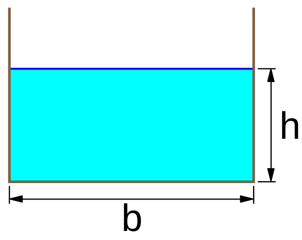
$$\bar{p}_1 A_1 - \bar{p}_2 A_2 = \rho Q (V_2 - V_1)$$

$$\bar{p} = \rho g \bar{d} \qquad V = \frac{Q}{A}$$

$$\rho g \bar{d}_1 A_1 - \rho g \bar{d}_2 A_2 = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

Restrict attention to a rectangular (or wide) channel

$$\bar{d} = \frac{1}{2} h \qquad A = hb \qquad Q = qb$$

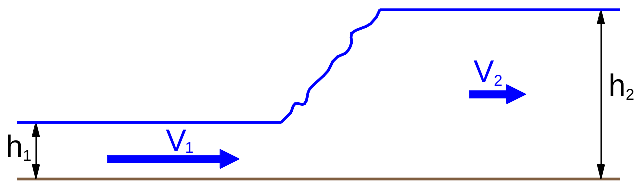


$$\tfrac{1}{2}\rho g h_1^2 b - \tfrac{1}{2}\rho g h_2^2 b = \rho q^2 b \left(\frac{1}{h_2} - \frac{1}{h_1}\right)$$

$$\tfrac{1}{2} g (h_1^2 - h_2^2) = q^2 \left(\frac{1}{h_2} - \frac{1}{h_1}\right)$$

Hydraulic Jump (Rectangular Channel)

$$\tfrac{1}{2}g(h_1^2 - h_2^2) = q^2\left(\frac{1}{h_2} - \frac{1}{h_1}\right)$$



$$\tfrac{1}{2}g(h_1 - h_2)(h_1 + h_2) = q^2\left(\frac{h_1 - h_2}{h_1 h_2}\right)$$

$$\tfrac{1}{2}h_1 h_2(h_1 + h_2) = \frac{q^2}{g}$$

$$\frac{1}{2}\frac{h_2}{h_1}\left(1 + \frac{h_2}{h_1}\right) = \frac{q^2}{g h_1^3}$$

$$\frac{q^2}{g h_1^3} = \frac{(q/h_1)^2}{g h_1} = \frac{V_1^2}{g h_1} = \mathrm{Fr}_1^2$$

$$\frac{1}{2}\frac{h_2}{h_1}\left(1 + \frac{h_2}{h_1}\right) = \mathrm{Fr}_1^2$$

$$\left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1} - 2\mathrm{Fr}_1^2 = 0$$

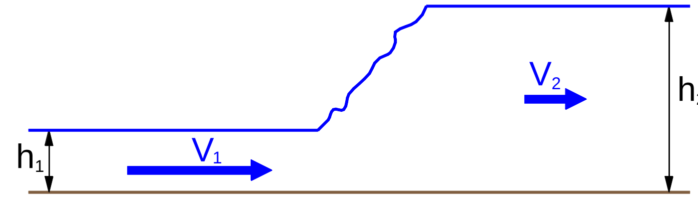
$$\frac{h_2}{h_1} = \frac{-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}}{2}$$

Hydraulic Jump (Rectangular Channel)

Mass and momentum:

$$\frac{1}{2}\frac{h_2}{h_1}(1+\frac{h_2}{h_1}) = \mathrm{Fr}_1^2$$

$$h_2 = \frac{h_1}{2}(-1+\sqrt{1+8\mathrm{Fr}_1^2})$$



So far:

1 and 2 could be either upstream or downstream;

Jump could be either shallow-to-deep or deep-to-shallow.

Energy:

Head loss: $$H_1 - H_2 = z_{s1} - z_{s2} + \frac{V_1^2 - V_2^2}{2g} \longleftarrow V = \frac{q}{h}$$

$$H_1 - H_2 = h_1 - h_2 + \frac{q^2}{2g}(\frac{1}{h_1^2} - \frac{1}{h_2^2}) \longleftarrow \frac{q^2}{g} = \frac{1}{2}h_1 h_2(h_1 + h_2)$$

$$H_1 - H_2 = \frac{(h_2 - h_1)^3}{4h_1 h_2}$$

Loss of mechanical energy $\Rightarrow$ $h_2 > h_1$ **Jump … from shallow to deep**

$\mathrm{Fr}_1 > 1$ and $\mathrm{Fr}_2 < 1$ **… supercritical to subcritical**

h_1 and h_2 are called **sequent depths**

Rapidly-Varied Flow

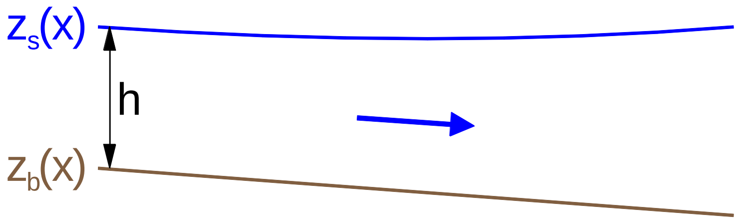
2. RAPIDLY-VARIED FLOW

- 2.1 Hydraulic jump
- **2.2 Specific energy**
- 2.3 Critical-flow devices
- 2.4 Forces on objects

Specific Energy

Total head: $$H = z_s + \frac{V^2}{2g}$$ (open channel, hydrostatic)

$$= z_b + h + \frac{V^2}{2g}$$



Specific energy E is the head **relative to the bed**:

$$E = h + \frac{V^2}{2g}$$

$$H = z_b + E$$

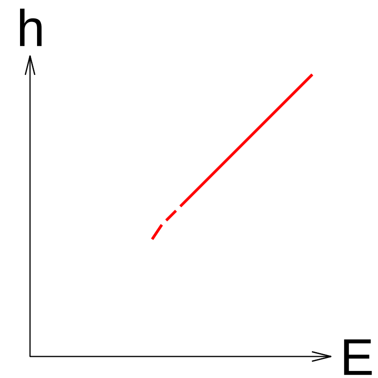
Increase in z_b $\leftrightarrow$ decrease in E

Rectangular (or Wide) Channel

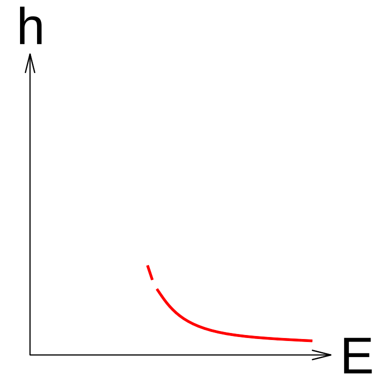
$$E = h + \frac{V^2}{2g} \longleftarrow V = \frac{q}{h}$$

$$E = h + \frac{q^2}{2gh^2}$$

Large h: $E \approx h$

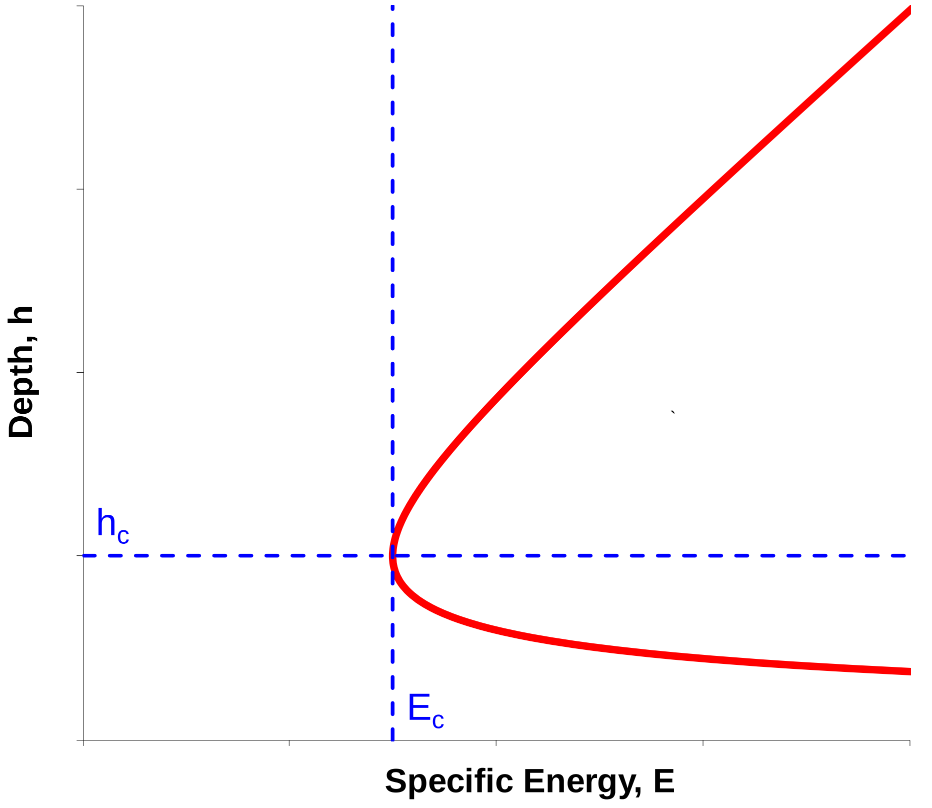


Small h: $E \approx \frac{\text{constant}}{h^2}$



Specific Energy in a Rectangular Channel

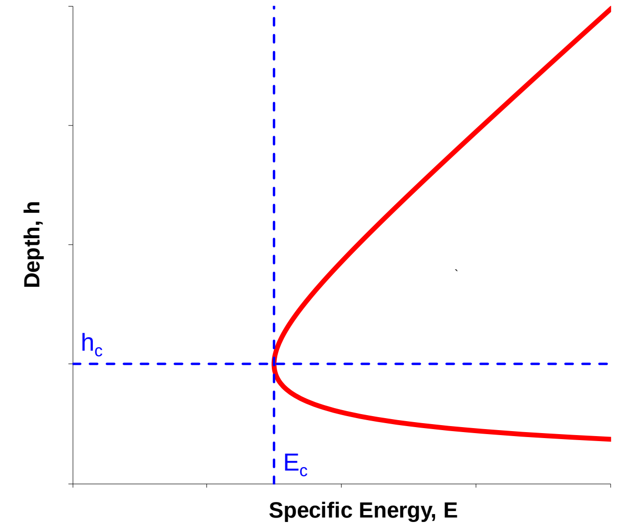
$$E = h + \frac{q^2}{2gh^2}$$



Minimum Specific Energy

$$E = h + \frac{q^2}{2gh^2} \qquad \Rightarrow \qquad \frac{\mathrm{d}E}{\mathrm{d}h} = 1 - \frac{q^2}{gh^3}$$

$$\frac{\mathrm{d}E}{\mathrm{d}h} = 0 \qquad \Leftrightarrow \qquad \frac{q^2}{gh^3} = 1 \qquad \Leftrightarrow \qquad h = \left(\frac{q^2}{g}\right)^{1/3}$$



$$E = h + \frac{q^2}{2gh^2} \qquad = h + \frac{1}{2}\left(\frac{q^2}{gh^3}\right)h \qquad = h + \frac{1}{2}h \qquad = \frac{3}{2}h$$

For a **rectangular** or **wide** channel:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad\qquad E_c = \frac{3}{2}h_c$$

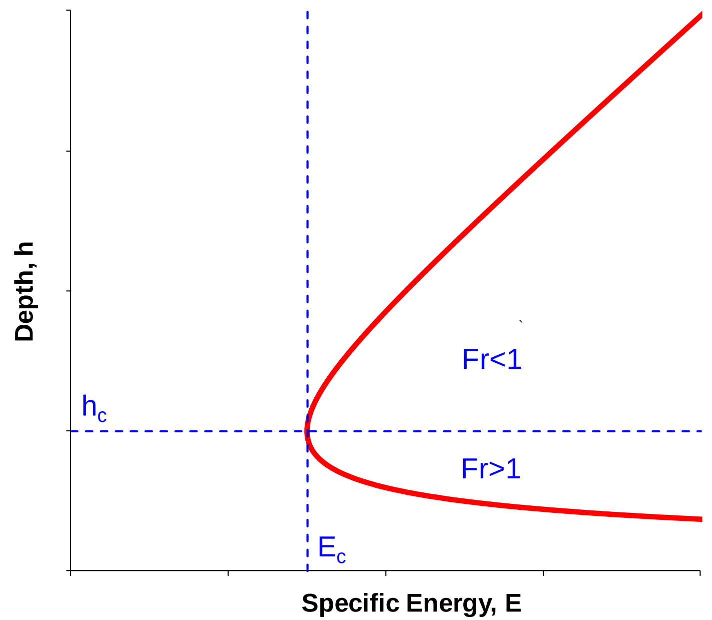
Critical Depth – Froude Number

Minimum E where $\frac{q^2}{gh^3} = 1$

$$\mathrm{Fr}^2 = \frac{V^2}{gh} \longleftarrow V = \frac{q}{h}$$

$$= \frac{q^2}{gh^3}$$

$$= 1$$



- For a given flow rate there is a (**strictly positive**) **minimum specific energy**, occurring at the critical depth where $\mathrm{Fr} = 1$.

- For any energy $E > E_c$ there are **two possible depths**:
  - a shallow ($h < h_c$), high-speed flow with $\mathrm{Fr} > 1$
  - a deep ($h > h_c$), low speed flow with $\mathrm{Fr} < 1$

These are called **alternate depths**.

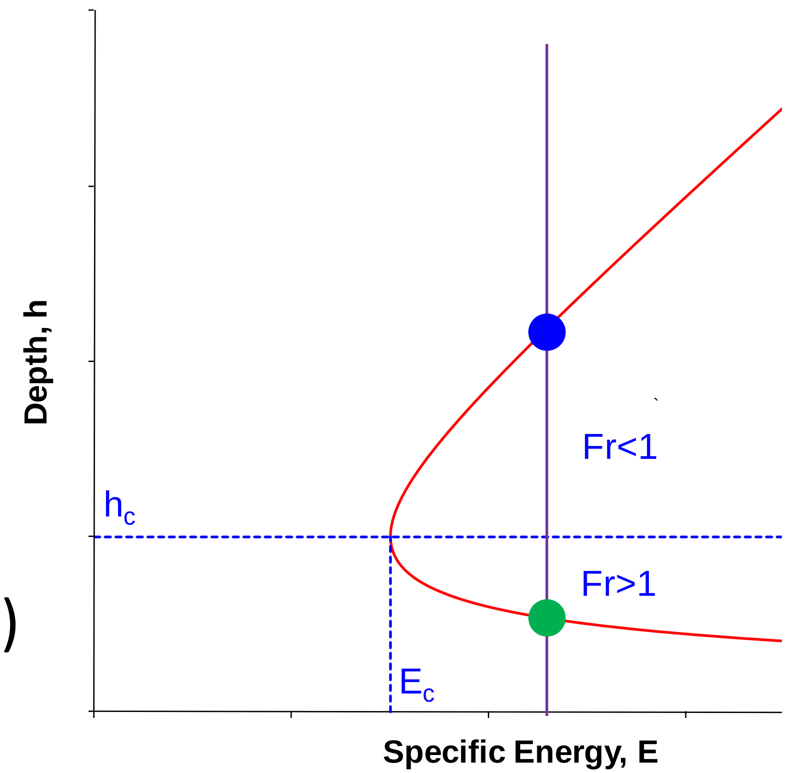
Calculating the Alternate Depths

$$E = h + \frac{V^2}{2g}$$

For a wide or rectangular channel:

$$E = h + \frac{q^2}{2gh^2}$$

(Specific energy = head, if bed height = 0)



Subcritical - rearrange for **deep** solution: $h = E - \frac{q^2}{2gh^2}$

Supercritical - rearrange for **shallow** solution: $h = \frac{q}{\sqrt{2g(E-h)}}$

Example

A 3 m wide channel carries a total discharge of 12 $m^3 s^{-1}$.
Calculate:

(a) the critical depth;
(b) the minimum specific energy;
(c) the alternate depths when $E = 4$ m.

A 3 m wide channel carries a total discharge of 12 m^3 s^{-1}. Calculate:

(a) the critical depth;

(b) the minimum specific energy;

(c) the alternate depths when $E = 4$ m.

$$\left.\begin{aligned} b &= 3\ \text{m} \\ Q &= 12\ \text{m}^3\ \text{s}^{-1} \end{aligned}\right\} \quad q \equiv \frac{Q}{b} \quad = 4\ \text{m}^2\ \text{s}^{-1}$$

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \mathbf{1.177\ m}$$

$$E_c = \frac{3}{2} h_c \quad = \mathbf{1.766\ m}$$

$$E = h + \frac{V^2}{2g} \qquad V = \frac{q}{h}$$

$$E = h + \frac{q^2}{2gh^2}$$

$$4 = h + \frac{0.8155}{h^2}$$

Deep: $h = 4 - \dfrac{0.8155}{h^2}$

4, 3.949, 3.948, … **3.948 m**

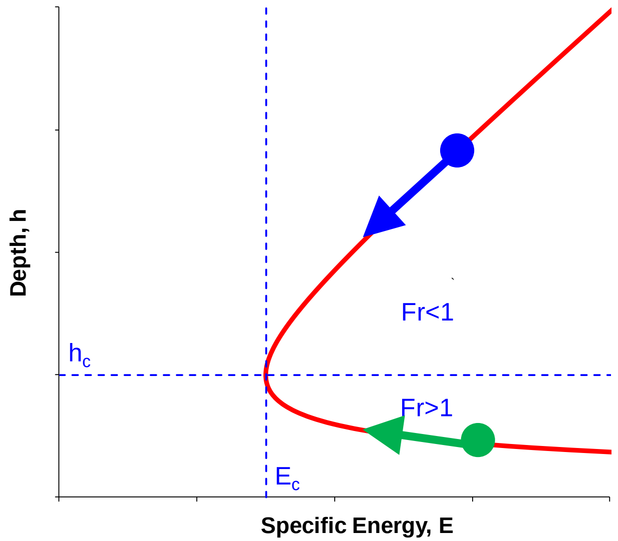
Shallow: $h = \sqrt{\dfrac{0.8155}{4 - h}}$

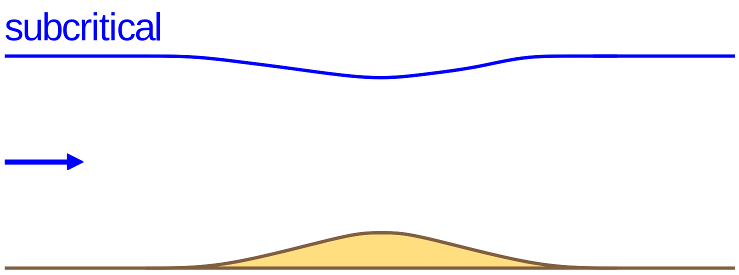
0, 0.4515, 0.4794, … **0.4814 m**

Flow Over a (Small) Bump

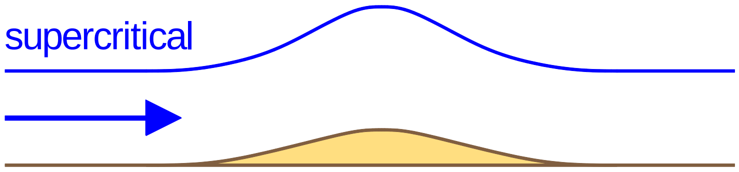
$$H = z_b + E \quad = \text{constant}$$

$$z_b \text{ increases} \quad \Leftrightarrow \quad E \text{ decreases}$$





Subcritical
– depth **decreases** over the bump.



Supercritical
– depth **increases** over the bump.

Surface Level (z_s) vs Depth (h)

$$H = z_s + \frac{V^2}{2g}$$

$$= z_s + \frac{q^2}{2gh^2} \qquad \text{(wide or rectangular channel)}$$

$$\mathrm{d}H = \mathrm{d}z_s - \frac{q^2}{gh^3}\mathrm{d}h$$

$$= \mathrm{d}z_s - \mathrm{Fr}^2\mathrm{d}h$$

For constant head ($\mathrm{d}H = 0$): $\mathrm{d}z_s = \mathrm{Fr}^2\mathrm{d}h$

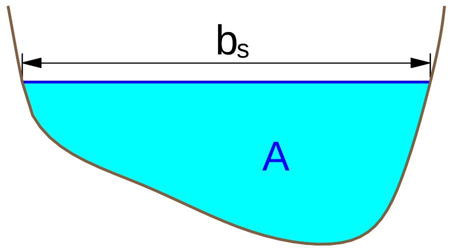
At constant head ...

(1) surface level changes in the same direction as depth;

(2) if Fr is very small, surface displacement is negligible.

Non-Rectangular Channel

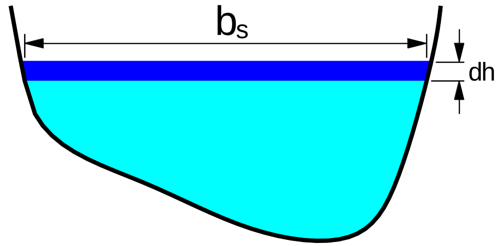
Total head: $H = z_s + \frac{V^2}{2g}$ ← $V = \frac{Q}{A}$ $\quad z_s = z_b + h$



$$H = z_b + E \qquad E = h + \frac{Q^2}{2gA^2}$$

Minimise specific energy: $\frac{\mathrm{d}E}{\mathrm{d}h} = 1 + \frac{\mathrm{d}}{\mathrm{d}A}\left(\frac{Q^2}{2gA^2}\right) \times \frac{\mathrm{d}A}{\mathrm{d}h} = 1 - \frac{Q^2}{gA^3}\frac{\mathrm{d}A}{\mathrm{d}h}$

$$\frac{\mathrm{d}E}{\mathrm{d}h} = 0 \quad \Rightarrow \quad \frac{Q^2}{gA^3}\frac{\mathrm{d}A}{\mathrm{d}h} = 1 \quad \leftarrow \quad \mathrm{d}A = b_s\,\mathrm{d}h$$



$$\frac{Q^2 b_s}{gA^3} = 1$$

$$\frac{(Q/A)^2}{g(A/b_s)} = 1$$

$$\frac{V^2}{g\bar{h}} = 1$$

$$\mathrm{Fr} = \frac{V}{\sqrt{g\bar{h}}}$$

Minimum specific energy occurs at $\mathrm{Fr} = 1$

Rapidly-Varied Flow

2. RAPIDLY-VARIED FLOW

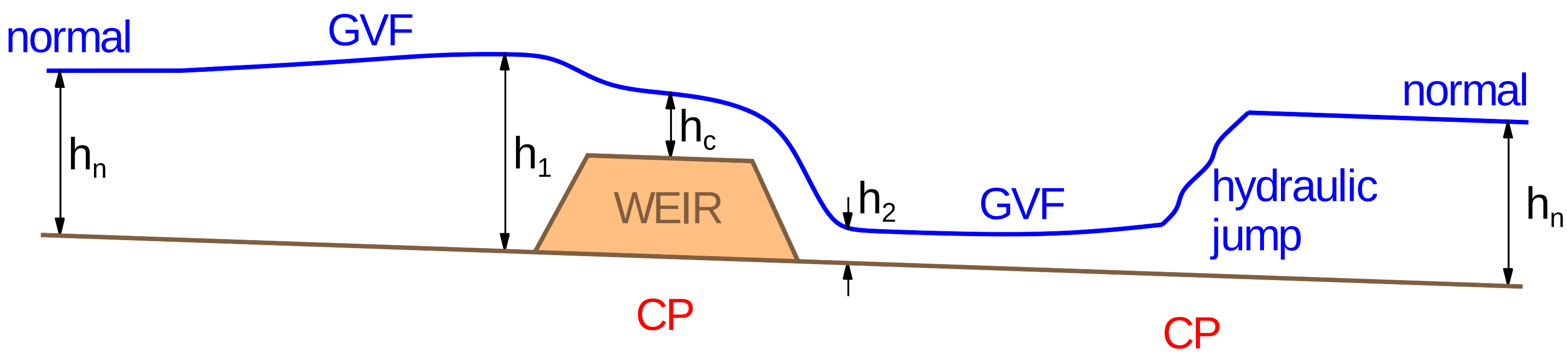
- 2.1 Hydraulic jump
- 2.2 Specific energy
- **2.3 Critical-flow devices**
- 2.4 Forces on objects

Critical-Flow Devices

- Broad-crested weir
- Venturi flume
- Sluice gate

- Free overfall

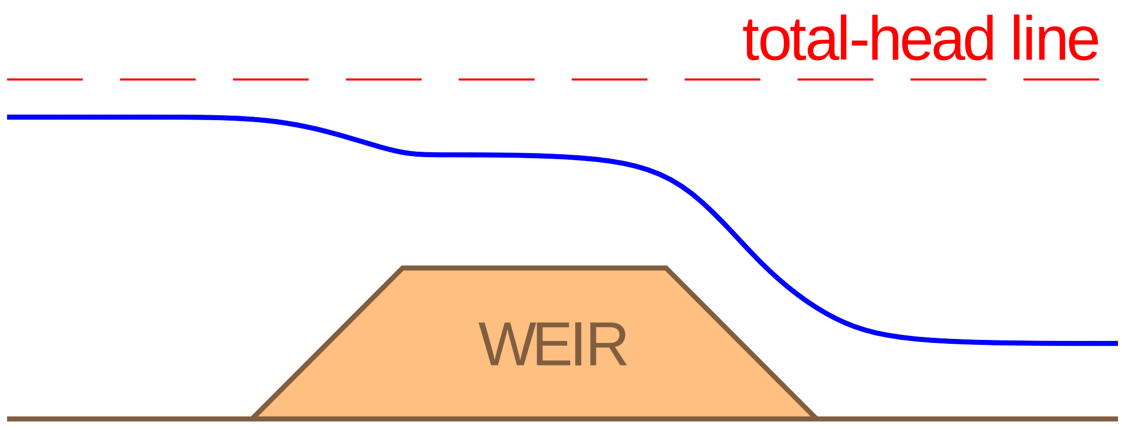
Critical-Flow Devices



For weir or venturi, given sufficient flow restriction:

- to attain the required minimum energy to pass the flow, the depth must increase just upstream; (i.e. the flow "**backs up**")
- the flow accelerates smoothly from sub- to supercritical, with **critical** conditions at the restriction;
- there is fixed relationship between **depth** ("stage") and **discharge**:
  - **measurement** of discharge
  - **control point** for GVF calculations

Critical-Flow Devices



Unlike a hydraulic jump:

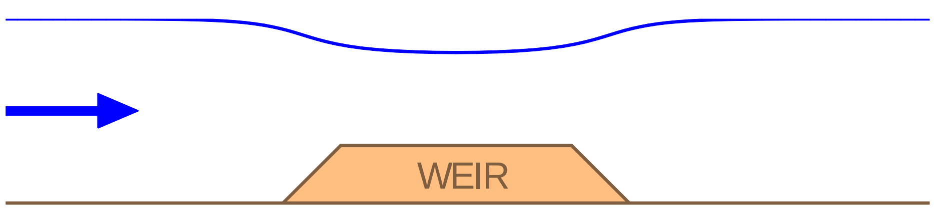
- **smooth** transition: no loss of head
- **sub- to supercritical** transition

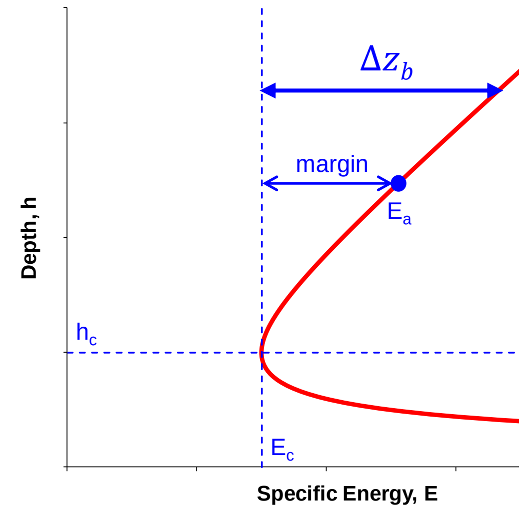
Weir

Broad-Crested Weir

Upstream: subcritical, with specific energy E_a .

Bed raised by Δz_b.
Specific energy reduced: $E \rightarrow E_a - \Delta z_b$.
Subcritical, so loss in depth.

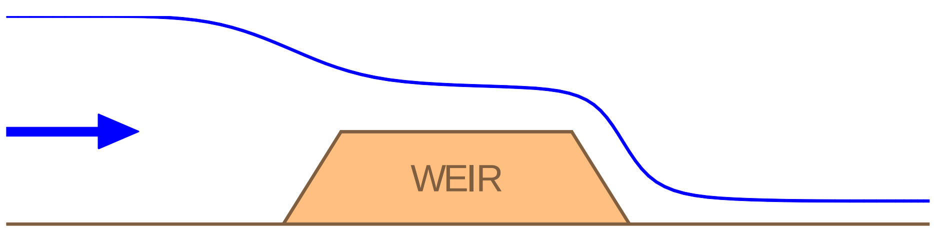




But E cannot be less than the critical value E_c at this discharge.

If Δz_b exceeds the allowed margin there must be an increase in depth immediately upstream to provide sufficient specific energy.

Increase in upstream depth is just sufficient to allow critical flow over the weir.



Broad-Crested Weir: Flow Depths

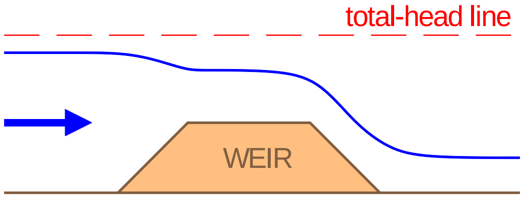
When the weir controls the flow:

- Smooth acceleration from sub- to supercritical flow
- Critical flow over the top:

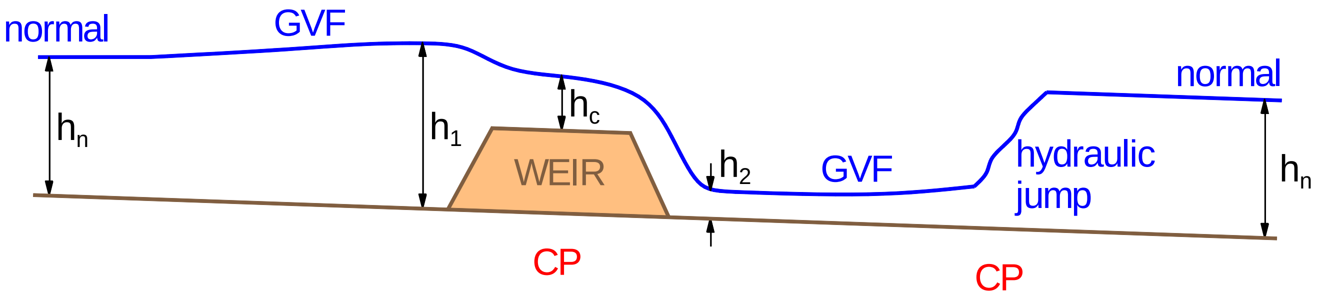
$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad E_c = \frac{3}{2} h_c$$

- The total head immediately up or downstream of the weir is the same as that over the top:

$$H = z_{\text{weir}} + E_c$$



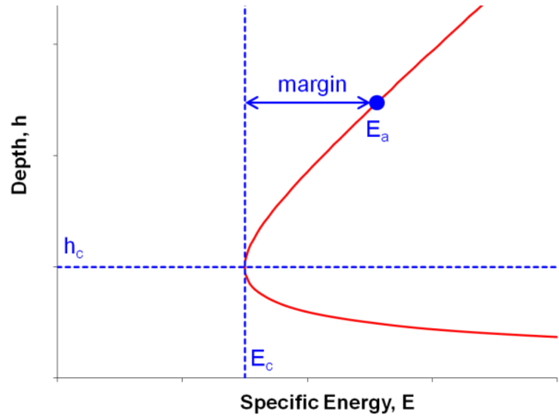
- Depths immediately up- or downstream of the weir can be found as the sub- and supercritical depths with this head.



Broad-Crested Weir: Test For Critical

First find, for the given discharge Q:

- **approach-flow** conditions (often normal): h_a and E_a
- **weir** critical conditions (h_c and E_c)



Then either:

Method 1

- Calculate specific energy following rise, $E_a - z_{\text{weir}}$, **assuming not critical**.
- If this is less than E_c then the flow must actually be critical over the weir.

Method 2 (my preference)

- Calculate total head over weir **assuming critical**; i.e. $H_c = z_{\text{weir}} + E_c$. (This is the minimum energy needed to get over the weir at this flow rate.)
- If this exceeds the available head, H_a, then critical conditions occur. (The depth just upstream must increase to supply the necessary head.)

Example

(a) Define:
   (i) specific energy
   (ii) Froude number
   for open-channel flow. What is special about these quantities in critical conditions?

A long, wide channel has a slope of 1:1000, a Manning's n of 0.015 $m^{-1/3}$ s and a discharge of 3 m^3 s^{-1} per metre width.

(b) Calculate the normal and critical depths.

(c) In a region of the channel the bed is raised by a height of 0.8 m over a length sufficient for the flow to be parallel to the bed over this length. Determine the depths upstream, downstream and over the raised bed, ignoring frictional losses. Sketch the key features of the flow, indicating *all* hydraulic transitions caused by the bed rise.

(d) In the same channel, the bed is lowered by 0.8 m from its original level. Determine the depths upstream, downstream and over the lowered bed, ignoring frictional losses. Sketch the flow.

(a) Define:

(i) specific energy

(ii) Froude number

for open-channel flow. What is special about these quantities in critical conditions?

Specific energy is head (energy per unit weight) relative to the bed of the channel

or

$$E = h + \frac{V^2}{2g}$$

The Froude number is

$$\mathrm{Fr} = \frac{V}{\sqrt{g\bar{h}}}$$

In critical conditions, $\mathrm{Fr} = 1$, and the specific energy is a minimum at the given discharge.

A long, wide channel has a slope of 1:1000, a Manning's n of 0.015 $m^{-1/3}$ s and a discharge of 3 m^3 s^{-1} per metre width.

(b) Calculate the normal and critical depths.

$$S = 0.001$$
$$n = 0.015 \text{ m}^{-1/3} \text{ s}$$
$$q = 3 \text{ m}^2 \text{ s}^{-1}$$

Normal:

$$q = Vh \qquad V = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad R_h = h \quad (\text{"wide"})$$

$$q = \frac{1}{n} h^{2/3} S^{1/2} h$$

$$q = \frac{h^{5/3}\sqrt{S}}{n}$$

$$h_n = \left(\frac{nq}{\sqrt{S}}\right)^{3/5} = \mathbf{1.236\ m}$$

Critical:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \mathbf{0.9717\ m}$$

(c) In a region of the channel the bed is raised by a height of 0.8 m over a length sufficient for the flow to be parallel to the bed over this length. Determine the depths upstream, downstream and over the raised bed, ignoring frictional losses. Sketch the key features of the flow, indicating *all* hydraulic transitions caused by the bed rise.

Minimum head required (critical conditions):

$$h_c = 0.9717 \text{ m}$$

$$E_c = \frac{3}{2} h_c = 1.458 \text{ m} \qquad z_b = 0.8 \text{ m}$$

$$H_c = z_b + E_c = 2.258 \text{ m}$$

Head available without backing up (normal flow):

$$H_a = 0 + E_a = h_n + \frac{V_n^2}{2g}$$

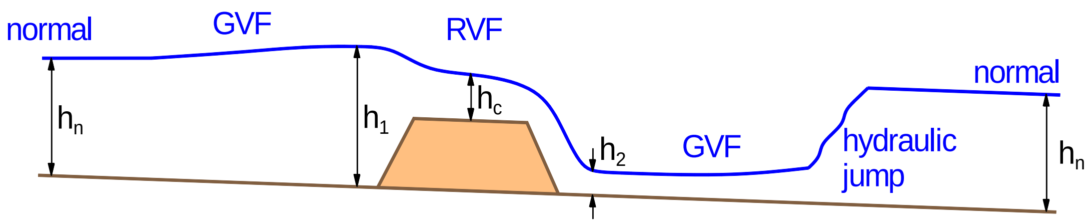
$$= h_n + \frac{q^2}{2g h_n^2} \qquad h_n = 1.236 \text{ m} \qquad q = 3 \text{ m}^2 \text{ s}^{-1}$$

$$= 1.536 \text{ m}$$

Available head (H_a) is less than the minimum required (H_c). Hence:

- the water depth must increase ("back up"), to raise the head immediately upstream;
- a hydraulic transition (subcritical to supercritical) must take place;
- the head throughout is critical: $H = H_c = 2.258$ m

(c) In a region of the channel the bed is raised by a height of 0.8 m over a length sufficient for the flow to be parallel to the bed over this length. Determine the depths upstream, downstream and over the raised bed, ignoring frictional losses. Sketch the key features of the flow, indicating *all* hydraulic transitions caused by the bed rise.



Over the weir:

$$h = h_c = \mathbf{0.9717\ m}$$

Just up or downstream (h_1 and h_2):

$$H = z_s + \frac{V^2}{2g} \qquad z_s = h \qquad V = \frac{q}{h}$$

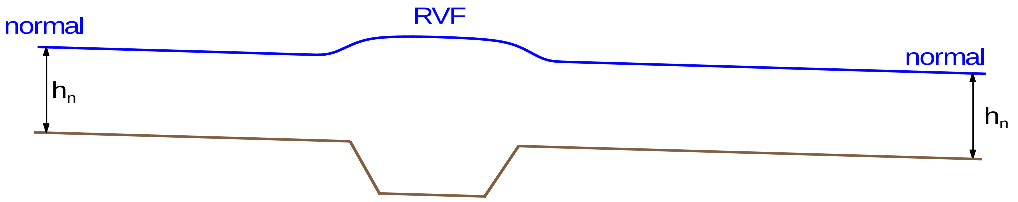
$$H = h + \frac{q^2}{2gh^2}$$

$$2.258 = h + \frac{0.4587}{h^2}$$

Deep (h_1): $h = 2.258 - \frac{0.4587}{h^2}$ $\quad \boldsymbol{h_1 = 2.160\ \mathrm{m}}$

Shallow (h_2): $h = \sqrt{\frac{0.4587}{2.258 - h}}$ $\quad \boldsymbol{h_2 = 0.5127\ \mathrm{m}}$

(d) In the same channel, the bed is lowered by 0.8 m from its original level. Determine the depths upstream, downstream and over the lowered bed, ignoring frictional losses. Sketch the flow.



No flow transition: $H = H_a = 1.536 \text{ m}$

Upstream/downstream: $h_1 = h_2 = h_n \quad = \mathbf{1.236 \text{ m}}$

In the depressed-bed region: $H = z_s + \dfrac{V^2}{2g} \qquad z_s = -0.8 + h \qquad V = \dfrac{q}{h}$

$$1.536 = -0.8 + h + \frac{q^2}{2gh^2}$$

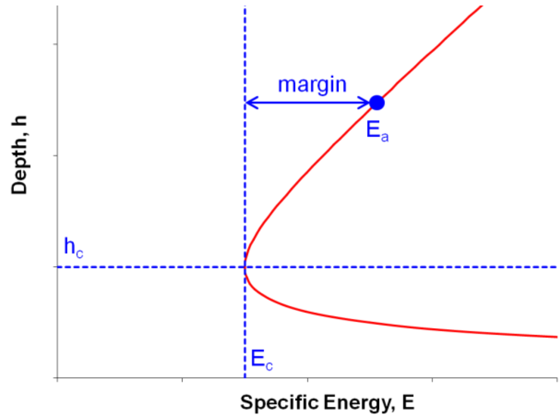
$$2.336 = h + \frac{0.4587}{h^2}$$

Subcritical (deep): $h = 2.336 - \dfrac{0.4587}{h^2}$ $\qquad \boldsymbol{h = 2.245 \text{ m}}$

Broad-Crested Weir: Test For Critical

First find, for the given discharge q:

- **approach-flow** conditions (often normal): h_a and E_a
- **weir** critical conditions (h_c and E_c)



Then either:

Method 1

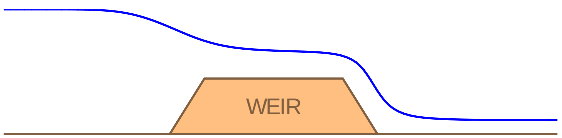
- Calculate specific energy following rise, $E_a - z_{\text{weir}}$, **assuming not critical**.
- If this is less than E_c then the flow must actually be critical over the weir.

Method 2 (my preference)

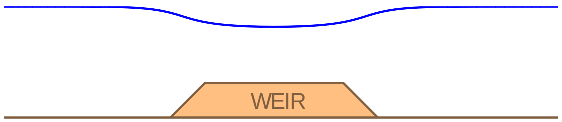
- Calculate total head over weir **assuming critical**; i.e. $H_c = z_{\text{weir}} + E_c$.
  (This is the **minimum energy needed to get over the weir at this flow rate**.)
- If this exceeds the available head H_a $(= E_a)$ then critical conditions occur.
  (The depth just upstream must increase to supply the necessary head.)

Broad-Crested Weir: Total Head

- If the flow **does** go critical then:
  - the total head throughout is critical;
  - the flow goes smoothly from sub- to supercritical.



- If the flow **does not** go critical then:
  - the total head throughout is that from upstream;
  - there is simply a localised dip in the free surface.



- In both cases the total head through the device is:
  - constant
  - equal to the larger of the critical or the approach-flow head

Broad-Crested Weir: Downstream Conditions

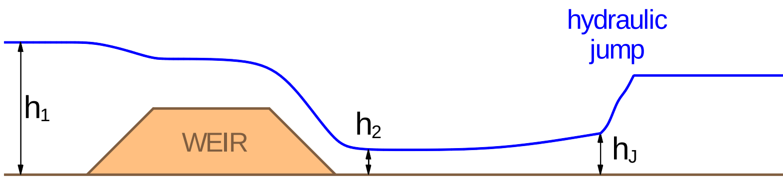
On a mild slope (i.e. where the normal flow is subcritical), the flow must go through a hydraulic jump back to subcritical flow.

Depths either side of the hydraulic jump are connected by the sequent-depth formula.

On a mild slope, any supercritical GVF increases in depth (**see later**).

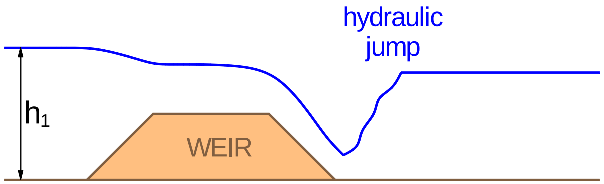
Case $h_2 < h_J$

Region of supercritical GVF between weir and jump.



Case $h_2 > h_J$

No region of supercritical GVF between weir and jump. Hydraulic jump occurs immediately.



Example

A long channel of rectangular cross-section with width 3.5 m and streamwise slope 1 in 800 carries a discharge of 15 $m^3\ s^{-1}$. Manning's n may be taken as 0.016 $m^{-1/3}$ s. A broad-crested weir of height 0.7 m is constructed at the centre of the channel.

Determine:

(a) the depth far upstream of the weir;

(b) the depth just upstream of the weir;

(c) whether or not a region of supercritical gradually-varied flow exists downstream of the weir.

A long channel of rectangular cross-section with width 3.5 m and streamwise slope 1 in 800 carries a discharge of 15 m^3 s^{-1}. Manning's n may be taken as 0.016 m$^{-1/3}$ s. A broad-crested weir of height 0.7 m is constructed at the centre of the channel. Determine:

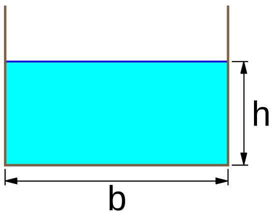
(a) the depth far upstream of the weir;

$b = 3.5 \text{ m}$

$S = 0.00125$

$Q = 15 \text{ m}^3 \text{ s}^{-1}$

$n = 0.016 \text{ m}^{-1/3} \text{ s}$



$Q = VA$ $\qquad V = \frac{1}{n} R_h^{2/3} S^{1/2}$ $\qquad R_h = \frac{bh}{b + 2h} = \frac{h}{1 + 2h/b}$

$$Q = \frac{1}{n}\left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} bh$$

$$Q = \frac{b\sqrt{S}}{n} \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\frac{nQ}{b\sqrt{S}}(1 + 2h/b)^{2/3} = h^{5/3}$$

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

$$h = 1.488\,(1 + 0.5714h)^{2/5}$$

$$\mathbf{h_n = 2.023\ m}$$

(b) the depth just upstream of the weir;

Available head in the approach flow:

$$H_a = 0 + E_a = h_n + \frac{V_n^2}{2g} \qquad V = \frac{Q}{bh}$$

$$= h_n + \frac{Q^2}{2gb^2h_n^2} \qquad h_n = 2.023 \text{ m} \qquad Q = 15 \text{ m}^3 \text{ s}^{-1} \qquad b = 3.5 \text{ m}$$

$$= \mathbf{2.252\ m}$$

Minimum head required (critical conditions):

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = 1.233 \text{ m}$$

$$E_c = \frac{3}{2}h_c = 1.850 \text{ m} \qquad z_{\text{weir}} = 0.7 \text{ m}$$

$$H_c = z_b + E_c = \mathbf{2.550\ m}$$

Available head (H_a) is less than the minimum required (H_c). Hence:

- the water depth must increase ("back up"), to raise the head immediately upstream;
- a hydraulic transition (subcritical to supercritical) must take place;
- the head throughout is critical: $H = H_c = 2.550 \text{ m}$

(b) the depth just upstream of the weir;

Total head near the weir:

$$H = 2.550\ \text{m}$$

Just upstream (or downstream):

$$H = z_s + \frac{V^2}{2g} \qquad z_s = h \qquad V = \frac{Q}{bh}$$

$$H = h + \frac{Q^2}{2gb^2h^2}$$

$$2.550 = h + \frac{0.9362}{h^2}$$

Upstream (deep):

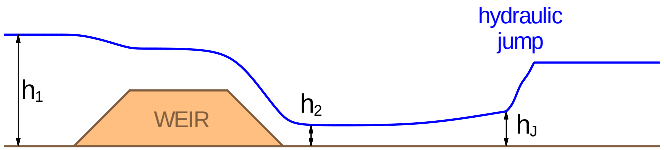
$$h = 2.550 - \frac{0.9362}{h^2}$$

$$\boldsymbol{h_1 = 2.385\ \text{m}}$$

(c) whether or not a region of supercritical gradually-varied flow exists downstream of the weir.

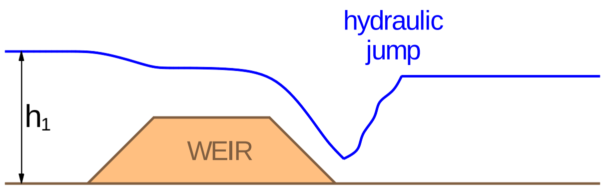
Total head near the weir:

$$2.550 = h + \frac{0.9362}{h^2}$$



Downstream of the weir (<u>if</u> supercritical flow is reached):

$$h = \sqrt{\frac{0.9362}{2.550 - h}} \qquad h_2 = \mathbf{0.7141\ m}$$



Hydraulic jump:

Downstream: $h_n = 2.023\ \text{m}$ $\quad V_n = \frac{Q}{bh_n} = 2.118\ \text{m s}^{-1}$ $\quad \text{Fr}_n = \frac{V_n}{\sqrt{gh_n}} = 0.4754$

Upstream: $h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8\text{Fr}_n^2}\right) = \mathbf{0.6835\ m}$

$h_2 > h_J$, so there is no room for supercritical flow between weir and hydraulic jump

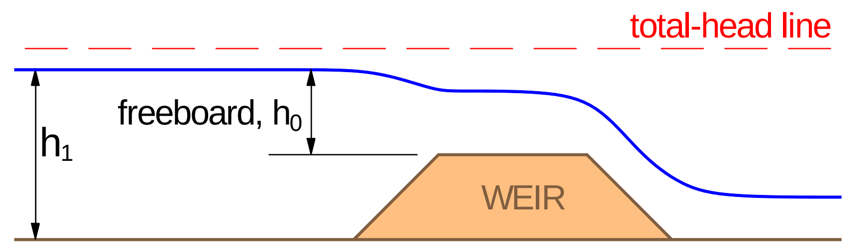
The hydraulic jump occurs **immediately, at the downstream end of the weir**

Measurement of Discharge

Head over weir = head upstream

$$\frac{3}{2}h_c = h_0 + \frac{V_1^2}{2g}$$

$$\frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} = h_0 + \frac{q^2}{2gh_1^2}$$



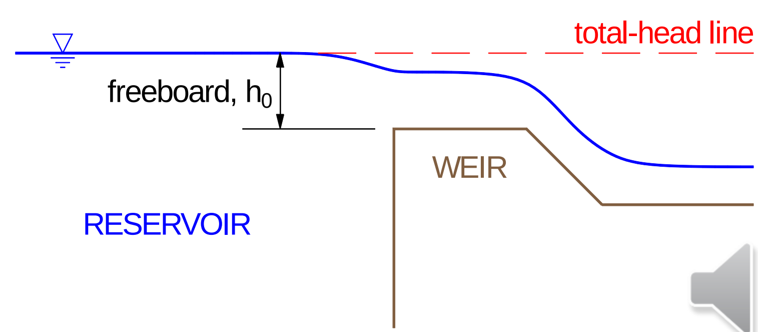
Discharge per unit width: $q = (2/3)^{3/2}\sqrt{g}\ (h_0 + \frac{q^2}{2gh_1^2})^{3/2}$

Ideal total discharge: $Q = (2/3)^{3/2}\sqrt{g}b\ (h_0 + \frac{Q^2}{2gb^2h_1^2})^{3/2}$

Actual total discharge: $Q = c_d Q_{\text{ideal}}$

If discharging from still water, $V_1 = 0$:

$$\frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} = h_0$$



Example

A reservoir discharge is controlled by a weir of width 8 m and discharge coefficient 0.9.

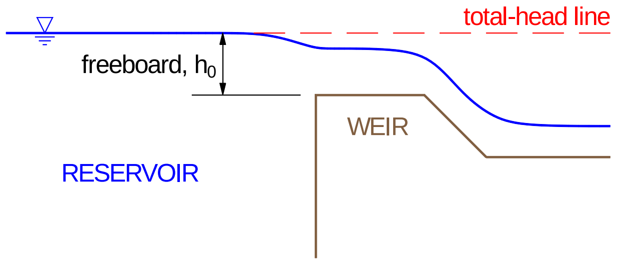
(a) Calculate the flow rate over the weir when the freeboard is 0.65 m.

(b) Assuming negligible inflow and a constant plan area for the reservoir of 1.5 km^2, calculate the time in hours to reduce the level of the reservoir by 0.4 m.

A reservoir discharge is controlled by a weir of width 8 m and discharge coefficient 0.9.

(a) Calculate the flow rate over the weir when the freeboard is 0.65 m.

(b) Assuming negligible inflow and a constant plan area of 1.5 km^2 for the reservoir, calculate the time in hours taken to reduce the level of the reservoir by 0.4 m.



$b = 8$ m

$c_d = 0.9$

Head in reservoir = head over weir

Relative to the top of the weir:

$$h_0 = \frac{3}{2} h_c$$

$$h_0 = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3}$$

$$\left(\frac{2}{3} h_0\right)^3 = \frac{q^2}{g}$$

$$q = \sqrt{(2/3)^3 g h_0^3} = 1.705 h_0^{3/2}$$

$$Q = c_d Q_{\text{ideal}} = c_d q b$$

$$Q = 12.28 h_0^{3/2}$$

A reservoir discharge is controlled by a weir of width 8 m and discharge coefficient 0.9.

(a) Calculate the flow rate over the weir when the freeboard is 0.65 m.

(b) Assuming negligible inflow and a constant plan area of 1.5 km^2 for the reservoir, calculate the time in hours taken to reduce the level of the reservoir by 0.4 m.

$$Q = 12.28h^{3/2}$$

(a) When $h = 0.65$ m, $Q = \mathbf{6.435\ m^3\ s^{-1}}$

(b) Generally:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\text{volume}) = Q_{\text{in}} - Q_{\text{out}}$$

$$A_{ws}\frac{\mathrm{d}h}{\mathrm{d}t} = 0 - 12.28h^{3/2}$$

$$A_{ws} = 1.5 \times 10^6\ \mathrm{m}^2$$

$$1.5 \times 10^6 \frac{\mathrm{d}h}{h^{3/2}} = -12.28\ \mathrm{d}t$$

$$\frac{-1.5 \times 10^6}{12.28}\int_{0.65}^{0.25} h^{-3/2}\,\mathrm{d}h = \int_0^T \mathrm{d}t$$

$$-122100\left[\frac{h^{-1/2}}{(-1/2)}\right]_{0.65}^{0.25} = T$$

$$244200\left(\frac{1}{\sqrt{0.25}} - \frac{1}{\sqrt{0.65}}\right) = T$$

$T = 185500$ s = **51.5 hours**

Maximum Discharge (Per Unit Width) at Given Energy

$$E = h + \frac{V^2}{2g} \quad \longleftarrow \quad V = \frac{q}{h}$$ (rectangular channel)

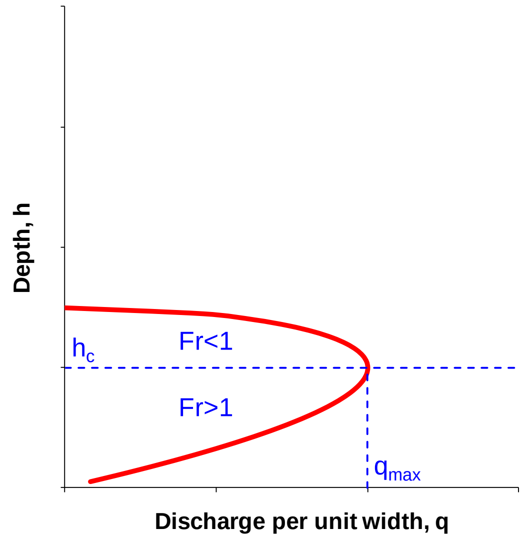
$$E = h + \frac{q^2}{2gh^2} \quad \Rightarrow \quad q^2 = 2gh^2(E-h) \quad = 2g(Eh^2 - h^3)$$

$$\frac{\mathrm{d}}{\mathrm{d}h}(q^2) = 2g(2Eh - 3h^2)$$

$$\frac{\mathrm{d}(q^2)}{\mathrm{d}h} = 0 \quad \Rightarrow \quad E = \frac{3}{2}h$$

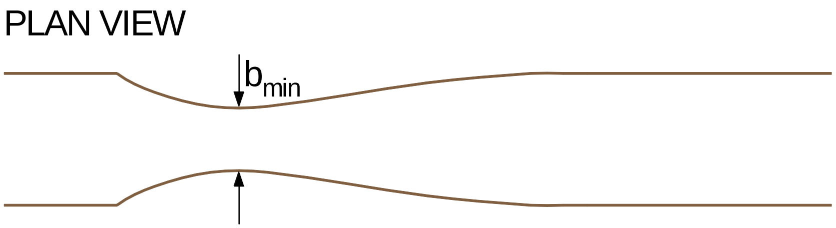
$$\Rightarrow \quad q^2 = gh^3$$

$$\mathrm{Fr}^2 = \frac{V^2}{gh} \quad = \frac{q^2}{gh^3} \quad = 1$$



- For a given specific energy there is a **maximum discharge per unit width**, occurring at the critical depth where $\mathrm{Fr} = 1$.

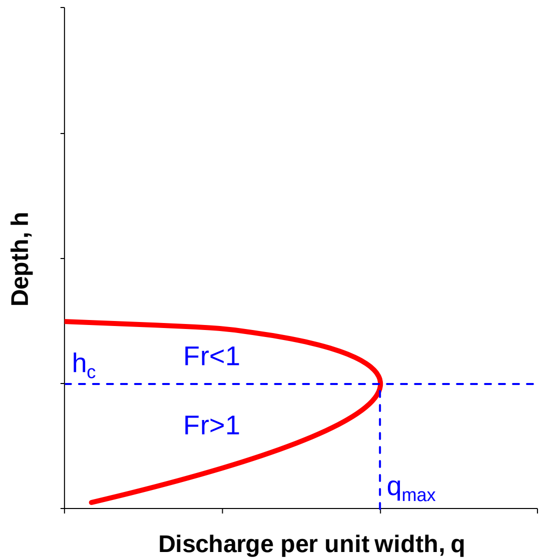
Venturi Flume



A **venturi** is any narrowing of a channel.

If a channel narrows then the discharge per unit width, $q = Q/b$, increases.

BUT, this cannot exceed the maximum discharge per unit width, $q_{\max}$, at this specific energy.

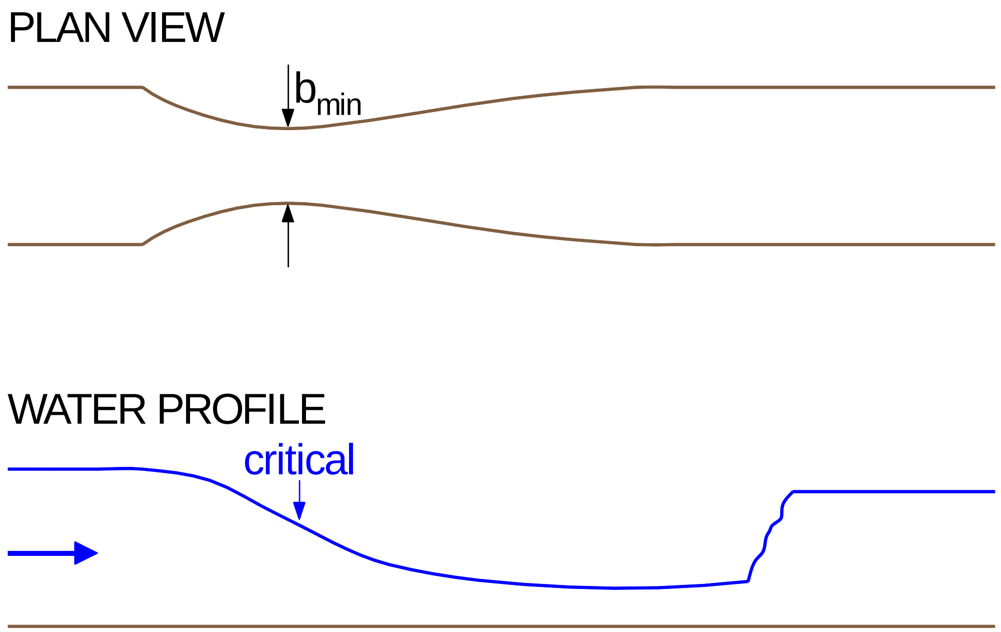


The maximum discharge occurs at a flow depth such that

$$E = \frac{3}{2}h \qquad \mathrm{Fr} = 1$$

If the discharge per unit width does exceed this then the flow is **choked** and backs up, the upstream depth increasing so as to increase the specific energy. Critical conditions are maintained at the venturi throat.

Venturi Flume: Water Profile



Venturi Flume: Critical Flow

<u>If</u> critical conditions occur:

- There is smooth acceleration from sub- to supercritical flow through the throat.
- At the venturi throat: $h_c = \left(\frac{q_m^2}{g}\right)^{1/3}$ $\qquad q_m = \frac{Q}{b_{\min}}$

$$E_c = \frac{3}{2} h_c$$

- Total head throughout the device is fixed by that at the throat:

$$H = H_c \quad = z_b + E_c$$

where z_b is the bed level (often 0).

- Depths anywhere in the flume are the solutions of

$$H = z_s + \frac{V^2}{2g} \qquad \text{or} \qquad E = h + \frac{Q^2}{2gb^2h^2}$$

Venturi Flume: Determining Criticality

Compare:

- head in approach flow, H_a
- critical head at the throat, $H_c = (z_b + E_c)_{\text{throat}}$

If $H_a < H_c$, critical conditions occur, the flow backs up and a flow transition occurs.

If $H_a > H_c$, the flow just dips, then returns to original depth.

As for the broad-crested weir …
the total head through the device is constant and equal to the larger of the critical head and approach-flow head.

Example

A venturi flume is placed near the middle of a long rectangular channel with Manning's n = 0.012 m$^{-1/3}$ s. The channel has a width of 5 m, a discharge of 12.5 m^3 s^{-1} and a slope of 1:2500.

(a) Determine the critical depth and the normal depth in the main channel.

(b) Determine the venturi flume width which will just make the flow critical at the contraction.

(c) If the contraction width is 2 m find the depths just upstream, downstream and at the throat of the venturi flume (neglecting friction in this short section).

(d) Sketch the surface profile.

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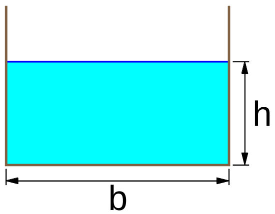
(a) Determine the critical depth and the normal depth in the main channel.

$$n = 0.012\ \mathrm{m}^{-1/3}\ \mathrm{s}$$
$$b = 5\ \mathrm{m} \quad \text{(main channel)}$$
$$Q = 12.5\ \mathrm{m}^3\ \mathrm{s}^{-1}$$
$$S = 0.0004$$



Critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad q = \frac{Q}{b} \quad = 2.5\ \mathrm{m}^2\ \mathrm{s}^{-1}$$

$$\boldsymbol{h_c = 0.8605\ \mathrm{m}}$$

Normal depth:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

$$h = 1.275(1 + 0.4h)^{2/5}$$

$$\boldsymbol{h_n = 1.546\ \mathrm{m}}$$

(b) Determine the venturi flume width which will just make the flow critical at the contraction.

Just critical if: $H_a(\text{approach flow}) = H_c(\text{contraction})$

Approach flow :

$$h_a = 1.546 \text{ m} \qquad V_a = \frac{Q}{bh_a} = 1.617 \text{ m s}^{-1}$$

$$H_a = 0 + E_a \quad = h_a + \frac{V_a^2}{2g} \quad = 1.679 \text{ m}$$

Critical head at the contraction:

$$H_c = \frac{3}{2}\left(\frac{q_m^2}{g}\right)^{1/3} = \frac{3}{2}\left(\frac{Q^2}{gb_m^2}\right)^{1/3} = \frac{3.774}{b_m^{2/3}}$$

$$1.679 = \frac{3.774}{b_m^{2/3}}$$

$$\boldsymbol{b_m = 3.370 \text{ m}}$$

(c) If the contraction width is 2 m find the depths just upstream, downstream and at the throat of the venturi flume (neglecting friction in this short section)

$2\ \text{m} < 3.370\ \text{m}$ There **is** a hydraulic transition

Critical depth at the throat:

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \qquad q_m = \frac{Q}{b_m} = 6.25\ \text{m}^2\ \text{s}^{-1} \qquad b_m = 2\ \text{m}$$

$$h_c = 1.585\ \text{m}$$

Total head:

$$H = H_c(\text{contraction}) = 0 + \frac{3}{2}h_c = 2.378\ \text{m}$$

Just upstream and downstream in main channel ($b = 5$ m):

$$H = z_s + \frac{V^2}{2g} \qquad z_s = h \qquad V = \frac{Q}{bh}$$

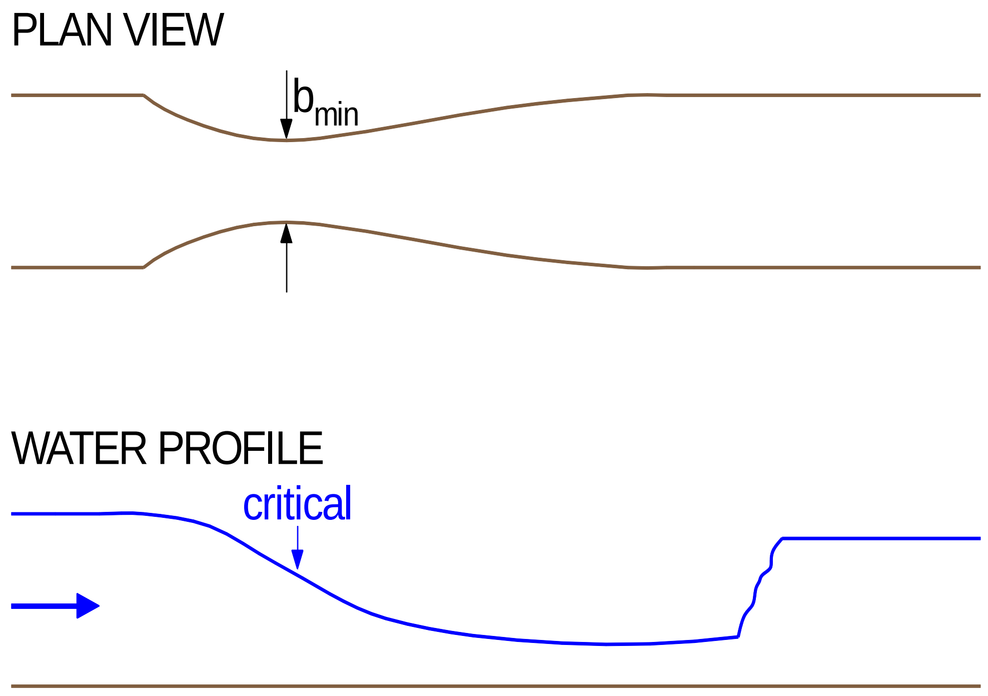
$$H = h + \frac{Q^2}{2gb^2h^2}$$

$$2.378 = h + \frac{0.3186}{h^2}$$

Upstream (deep): $h = 2.378 - \dfrac{0.3186}{h^2}$ $\boldsymbol{h_1 = 2.319\ \text{m}}$

Downstream (shallow): $h = \sqrt{\dfrac{0.3186}{2.378 - h}}$ $\boldsymbol{h_2 = 0.4015\ \text{m}}$

(d) Sketch the surface profile.

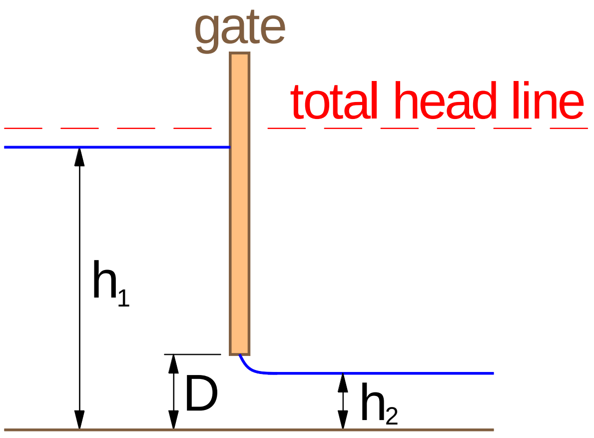


Sluice Gate

RVF: total head the same both sides

$$H_1 = H_2$$

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$



h_1 and h_2 are the subcritical and supercritical values in

$$\text{total head} \;=\; h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$$

h_2 is the depth at the vena contracta (≈ 0.6 × gate opening)

Gate opening plus either upstream head or upstream depth determine the discharge.

Example

The water depth upstream of a sluice gate is 0.8 m and the depth just downstream (at the vena contracta) is 0.2 m.

Calculate:

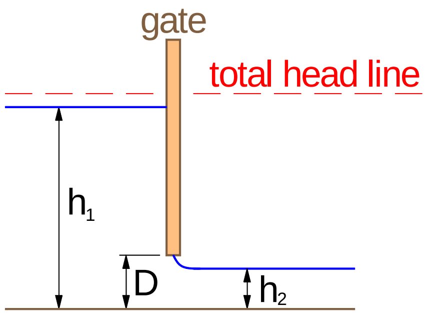
(a) the discharge per unit width;

(b) the Froude numbers upstream and downstream.

The water depth upstream of a sluice gate is 0.8 m and the depth just downstream (at the vena contracta) is 0.2 m. Calculate:

(a) the discharge per unit width;

(b) the Froude numbers upstream and downstream.



$$h_1 = 0.8 \text{ m}$$

$$h_2 = 0.2 \text{ m}$$

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g} \qquad z_s = h \qquad V = \frac{q}{h}$$

$$h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$$

$$0.8 + 0.0796q^2 = 0.2 + 1.2742q^2$$

$$0.6 = 1.1946q^2$$

$$\boldsymbol{q = 0.7087 \ \text{m}^2 \ \text{s}^{-1}}$$

$$\text{Fr} = \frac{V}{\sqrt{gh}} \qquad V = \frac{q}{h}$$

$$\text{Fr} = \frac{q}{\sqrt{gh^3}} \qquad \mathbf{Fr_1 = 0.3162} \qquad \mathbf{Fr_2 = 2.530}$$

Example

A sluice gate controls the flow in a channel of width 2 m. If the discharge is 0.5 $m^3\ s^{-1}$ and the upstream water depth is 1.5 m, calculate the downstream depth and velocity.

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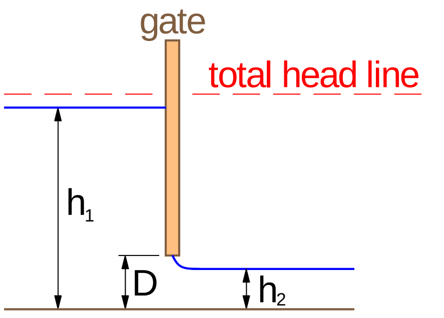


$b = 2$ m

$Q = 0.5$ m^3 s^{-1}

$h_1 = 1.5$ m

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g} \qquad z_s = h \qquad V = \frac{Q}{bh}$$

$$h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$$

$$1.501 = h_2 + \frac{0.003186}{h_2^2}$$

Downstream: shallow (supercritical) solution:

$$h_2 = \sqrt{\frac{0.003186}{1.501 - h_2}} \qquad \boldsymbol{h_2 = 0.04681\ \mathrm{m}}$$

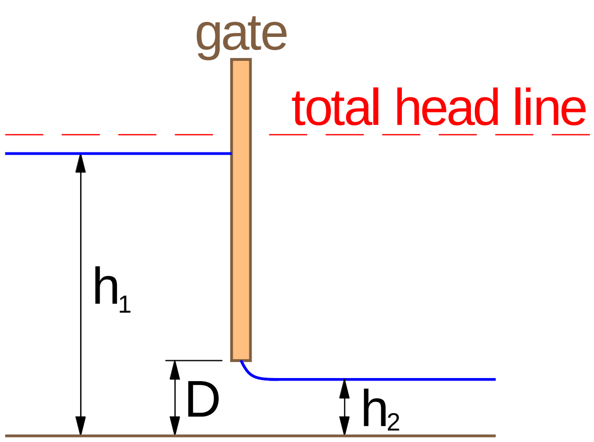
$$V_2 = \frac{Q}{bh_2} \qquad \boldsymbol{V_2 = 5.341\ \mathrm{m\ s^{-1}}}$$

Sluice Gate: Ideal Discharge

Constant head: $$z_{s1} + \frac{V^2}{2g} = z_{s2} + \frac{V^2}{2g}$$

$$h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$$

$$Q = bh_2\sqrt{\frac{2gh_1}{1 + h_2/h_1}}$$



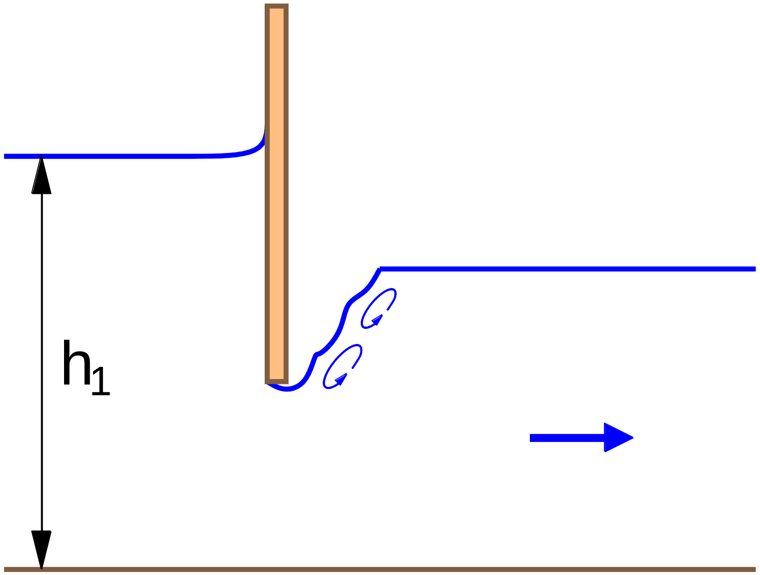
Ideal approximations:

- RVF (no losses)
- $h_2 = D$
- $h_2 \ll h_1$

$$Q_{\text{ideal}} = bD\sqrt{2gh_1}$$

Actual discharge: $$Q = c_d Q_{\text{ideal}}$$

Drowned Gate

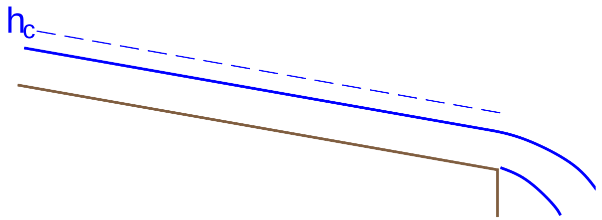


Gate opened too far or downstream control too close

Free Overfall

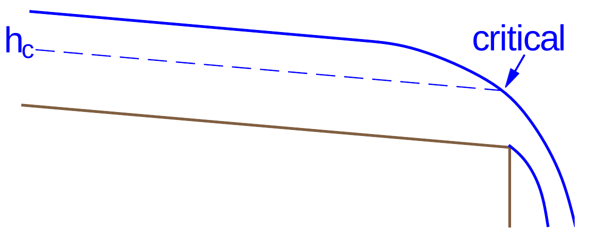
Supercritical $(\mathrm{Fr} > 1)$ approach flow:

- upstream control;
- supercritical flow continues over the overfall.



Subcritical $(\mathrm{Fr} < 1)$ approach flow:

- downstream control;
- flow passes through critical near the overfall.



Rapidly-Varied Flow

2. RAPIDLY-VARIED FLOW

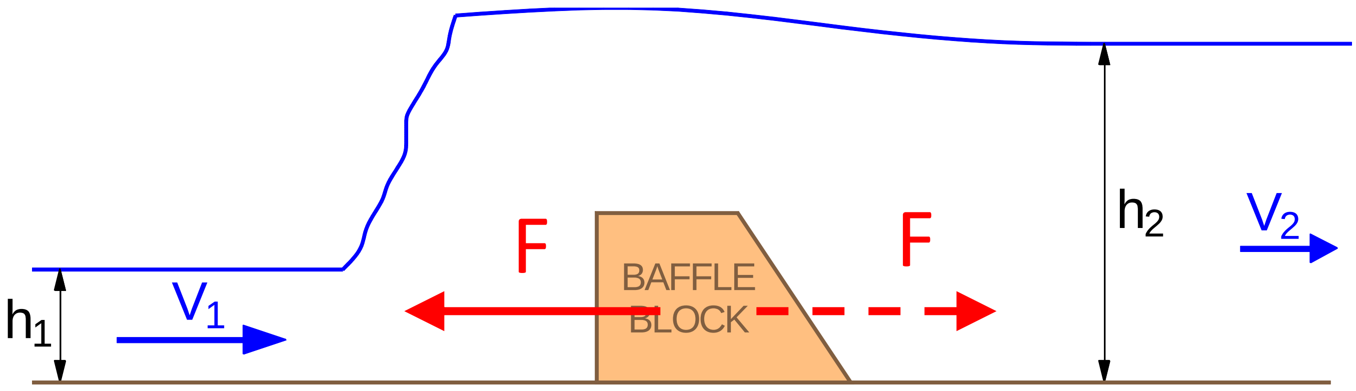
2.1 Hydraulic jump

2.2 Specific energy

2.3 Critical-flow devices

2.4 Forces on objects

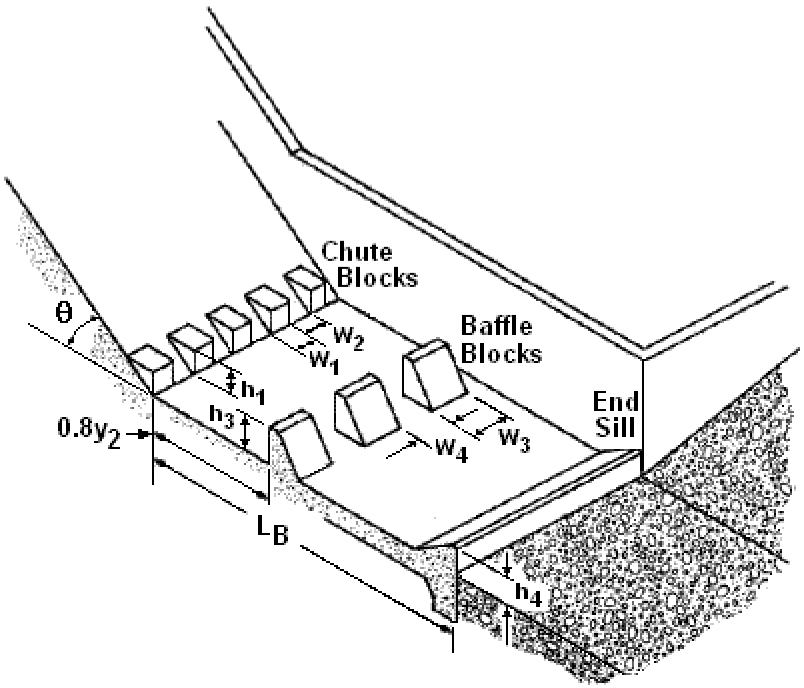
Forces On Objects



- Obstacles in the flow provide a reactive force
- Often they provoke a flow transition; e.g. hydraulic jump
- Analysis by momentum principle

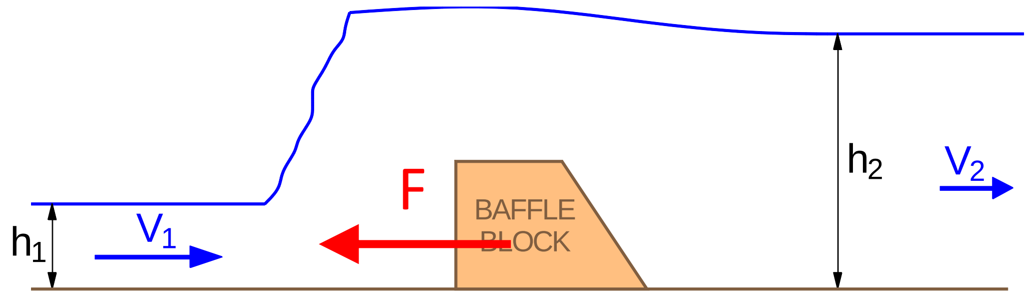
Baffle Blocks

Baffle blocks are widely used in stilling basins to dissipate fluid energy before discharging into rivers.



Baffle Blocks

Control-Volume Analysis



Force = rate of change of momentum

$$-F + \bar{p}_1 A_1 - \bar{p}_2 A_2 = \rho Q (V_2 - V_1)$$

$$\bar{p} = \rho g (\tfrac{1}{2} h) \qquad A = bh$$

$$-F + \tfrac{1}{2}\rho g h_1^2 b - \tfrac{1}{2}\rho g h_2^2 b = \rho Q (V_2 - V_1)$$

$$F = (\rho Q V_1 + \tfrac{1}{2}\rho g h_1^2 b) - (\rho Q V_2 + \tfrac{1}{2}\rho g h_2^2 b)$$

$$F = (M_1 + F_{p1}) - (M_2 + F_{p2})$$

$$M = \rho Q V = \text{momentum flux}$$

$$F_p = \tfrac{1}{2}\rho g h^2 b = \text{pressure force}$$

Special case: hydraulic jump ($F = 0$)

Example

Water flows at 0.8 $m^3\ s^{-1}$ per metre width down a long, wide spillway of slope 1 in 30 onto a wide apron of slope 1 in 1000. Manning's roughness coefficient n = 0.014 $m^{-1/3}$ s on both slopes.

(a) Find the normal depths in both sections and show that normal flow is supercritical on the spillway and subcritical on the apron.

(b) Baffle blocks are placed a short distance downstream of the slope transition to provoke a hydraulic jump. Assuming that flow is normal on both the spillway and downstream of the hydraulic jump, calculate the force per metre width of channel that the blocks must impart.

(c) Find the head loss across the blocks.

Water flows at 0.8 m^3 s^{-1} per metre width down a long, wide spillway of slope 1 in 30 onto a wide apron of slope 1 in 1000. Manning's roughness coefficient n = 0.014 m$^{-1/3}$ s on both slopes.

(a) Find the normal depths in both sections and show that normal flow is supercritical on the spillway and subcritical on the apron.

$$q = 0.8\ \mathrm{m^2\ s^{-1}}$$

$$S_1 = 1/30 \qquad S_2 = 1/1000$$

$$n = 0.014\ \mathrm{m^{-1/3}\ s}$$

Normal depths:

$$q = Vh \qquad V = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad R_h = h \quad (\text{"wide"})$$

$$q = \frac{1}{n} h^{2/3} S^{1/2} h$$

$$q = \frac{h^{5/3}\sqrt{S}}{n}$$

$$h = \left(\frac{nq}{\sqrt{S}}\right)^{3/5} \qquad \boldsymbol{h_1 = 0.1874\ \mathrm{m}} \qquad \boldsymbol{h_2 = 0.5365\ \mathrm{m}}$$

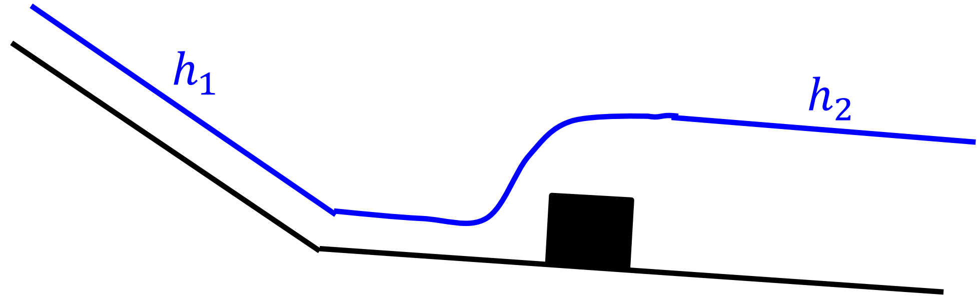
Critical depth:

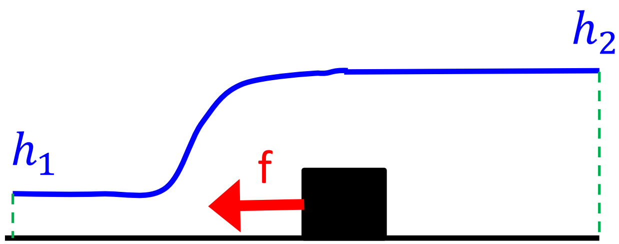
$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.4026\ \mathrm{m}$$

Spillway: $h_1 < h_c$ supercritical

Apron: $h_2 > h_c$ subcritical

(b) Baffle blocks are placed a short distance downstream of the slope transition to provoke a hydraulic jump. Assuming that flow is normal on both the spillway and downstream of the hydraulic jump, calculate the force per metre width of channel that the blocks must impart.





$$-f + \bar{p}_1 h_1 - \bar{p}_2 h_2 = \rho q (V_2 - V_1)$$

$$\bar{p} = \rho g (\tfrac{1}{2} h) \qquad V = \frac{q}{h}$$

$$-f + \tfrac{1}{2}\rho g h_1^2 - \tfrac{1}{2}\rho g h_2^2 = \rho q \left(\frac{q}{h_2} - \frac{q}{h_1}\right)$$

$$f = \tfrac{1}{2}\rho g (h_1^2 - h_2^2) + \rho q^2 \left(\frac{1}{h_1} - \frac{1}{h_2}\right)$$

$$q = 0.8 \text{ m}^2 \text{ s}^{-1} \qquad h_1 = 0.1874 \text{ m}$$

$$h_2 = 0.5365 \text{ m}$$

$$\boldsymbol{f = 982.7 \text{ N m}^{-1}}$$

(c) Find the head loss across the blocks.

$$H = z_s + \frac{V^2}{2g} \qquad z_s = h \qquad V = \frac{q}{h}$$

$$H = h + \frac{q^2}{2gh^2} \qquad q = 0.8\ \mathrm{m^2\ s^{-1}} \qquad h_1 = 0.1874\ \mathrm{m}$$

$$h_2 = 0.5365\ \mathrm{m}$$

$$H_1 = 1.1162\ \mathrm{m}$$

$$H_2 = 0.6498\ \mathrm{m}$$

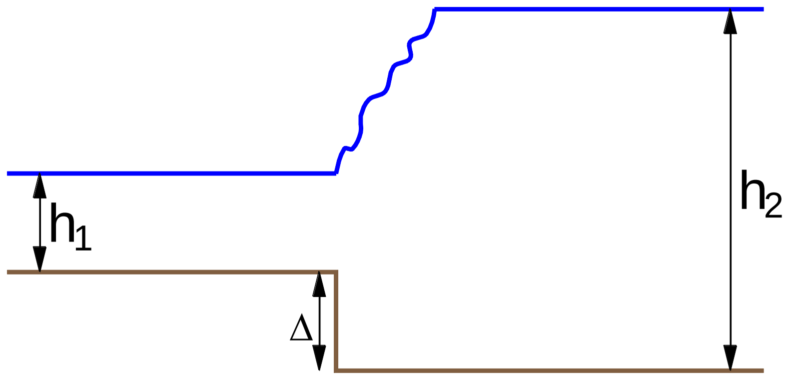
$$\text{Head loss} = H_1 - H_2 \quad = \mathbf{0.4664\ m}$$

Hydraulic Jumps in Expanding Channels

- Sudden expansion causes a rapid drop in velocity.
- May be sufficient to trigger a hydraulic jump
- Analysed by momentum principle:
  - control volume in expanded section only
  - assume hydrostatic pressure on walls

Example

A downward step of height 0.5 m causes a hydraulic jump in a wide channel when the depth and velocity of the flow upstream are 0.5 m and 10 m s^{-1}, respectively.

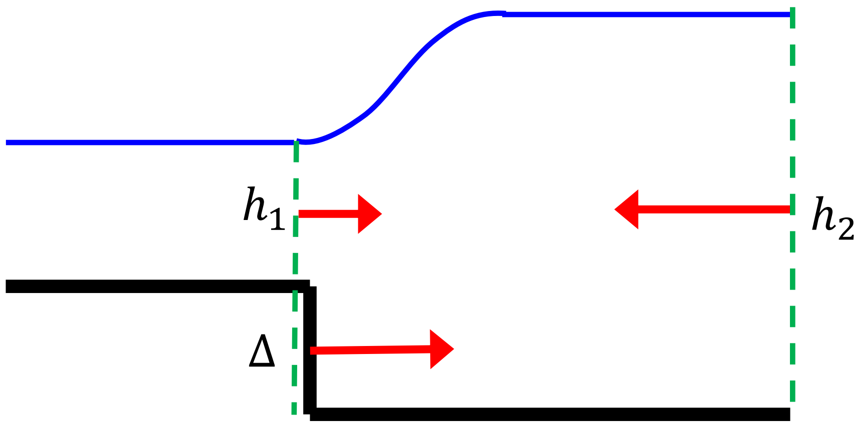


(a) Find the downstream depth.

(b) Find the head lost in the jump.

A downward step of height 0.5 m causes a hydraulic jump in a wide channel when the depth and velocity of the flow upstream are 0.5 m and 10 m s^{-1}, respectively.

(a) Find the downstream depth.



$$\Delta = 0.5 \text{ m}$$
$$h_1 = 0.5 \text{ m}$$
$$V_1 = 10 \text{ m s}^{-1}$$

$$q = V_1 h_1 = 5 \text{ m}^2\text{ s}^{-1}$$

$$\frac{1}{2}\rho g(h_1+\Delta)\times(h_1+\Delta) - \frac{1}{2}\rho g h_2 \times h_2 = \rho q(V_2 - V_1) \qquad V = \frac{q}{h}$$

$$(h_1+\Delta)^2 - h_2^2 = \frac{2q}{g}\left(\frac{q}{h_2} - \frac{q}{h_1}\right)$$

$$1 - h_2^2 = 1.019\left(\frac{5}{h_2} - 10\right)$$

$$11.19 = h_2^2 + \frac{5.095}{h_2}$$

Deep solution:

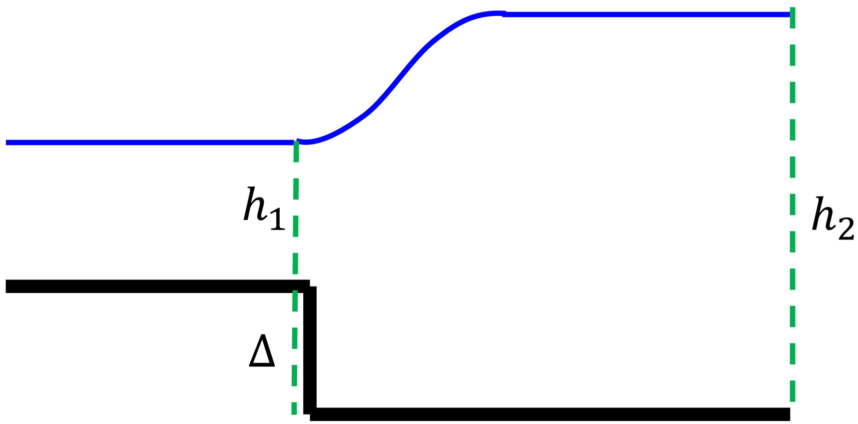
$$h_2 = \sqrt{11.19 - \frac{5.095}{h_2}} \qquad \boldsymbol{h_2 = 3.089 \text{ m}}$$

(b) Find the head lost in the jump.

$\Delta = 0.5\text{ m}$

$h_1 = 0.5\text{ m}$ $\quad V_1 = 10\text{ m s}^{-1}$

$h_2 = 3.089\text{ m}$ $\quad V_2 = \dfrac{q}{h_2} = 1.619\text{ m s}^{-1}$



$$H = z_s + \frac{V^2}{2g}$$

$$H_1 = 1.0 + \frac{10^2}{2g} = 6.097\text{ m}$$

$$H_2 = 3.089 + \frac{1.619^2}{2g} = 3.223\text{ m}$$

$$\text{Head loss} = H_1 - H_2 = \mathbf{2.874\ m}$$