Open-Channel Flow

4. Wave Speed and Analogy With Compressible Flow

Wave Speed and Analogy With Compressible Flow

4. WAVE SPEED AND ANALOGY WITH COMPRESSIBLE FLOW

4.1 Long-wave speed on shallow water

4.2 Zone of influence

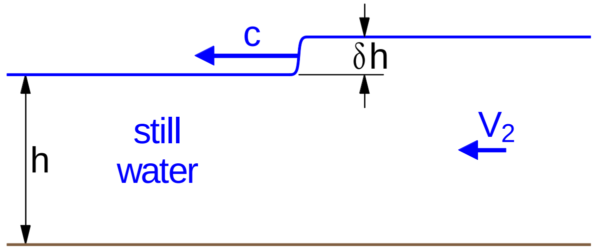
4.3 Analogy with compressible flow

Surge Wave

Tidal bore

Tsunami

Long-Wave Speed on Shallow Water

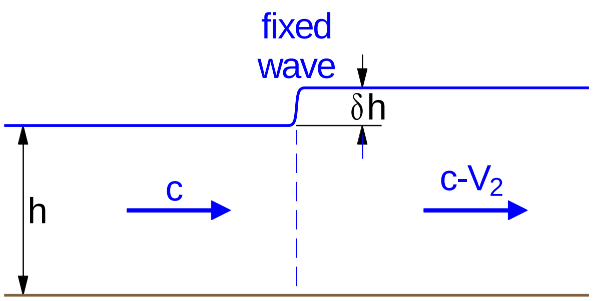


Continuity:

$$ch = (c - V_2)(h + \delta h)$$

$$ch = ch + c\delta h - V_2(h + \delta h)$$

$$V_2 = \frac{\delta h}{h + \delta h} c$$



Momentum:

$$(\bar{p}A)_L - (\bar{p}A)_R = \rho Q(V_R - V_L)$$

$$\tfrac{1}{2}\rho g[h^2 - (h + \delta h)^2] = \rho ch[(c - V_2) - c]$$

$$\tfrac{1}{2}g(-2h\delta h - \delta h^2) = -chV_2$$

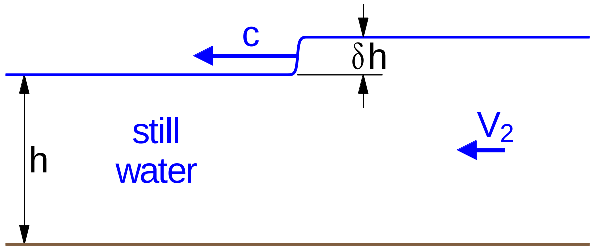
$$V_2 = \frac{g\delta h}{c}\left(1 + \frac{1}{2}\frac{\delta h}{h}\right)$$

$$\frac{\delta h}{h + \delta h} c = \frac{g\delta h}{c}\left(1 + \frac{1}{2}\frac{\delta h}{h}\right)$$

$$c^2 = gh\left(1 + \frac{1}{2}\frac{\delta h}{h}\right)\left(1 + \frac{\delta h}{h}\right)$$

Long-Wave Speed on Shallow Water

$$c^2 = gh\left(1 + \frac{1}{2}\frac{\delta h}{h}\right)\left(1 + \frac{\delta h}{h}\right)$$



- The bigger the amplitude (δh) the faster the wave (c).
- Small-amplitude limit ($\delta h/h \rightarrow 0$): $c = \sqrt{gh}$

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} = \frac{\text{water speed}}{\text{wave speed}}$$

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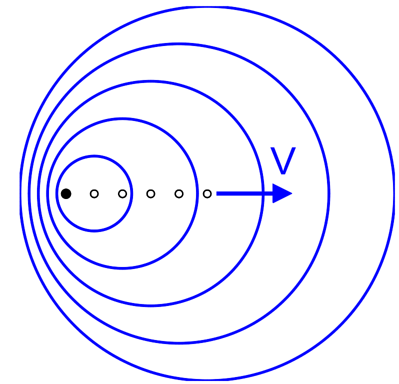
Zone of Influence

Subcritical ($\mathbf{Fr} < 1$)

If $V < c$, waves can travel upstream.

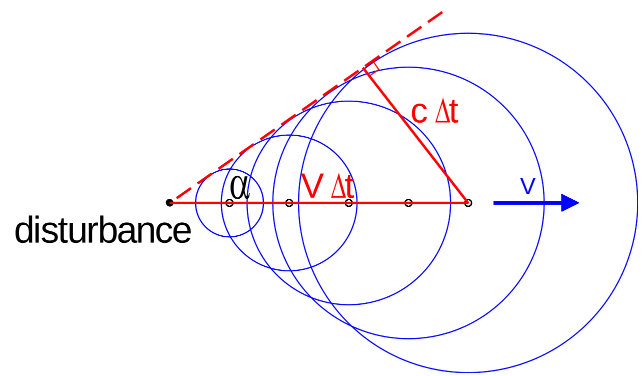
Downstream conditions **can** influence the flow upstream.



Supercritical ($\mathbf{Fr} > 1$)

If $V > c$, waves are swept downstream.

Downstream disturbances **can not** influence behaviour upstream.



Zone of influence:

$$\sin\alpha = \frac{c\Delta t}{V\Delta t} = \frac{c}{V} = \frac{\sqrt{gh}}{V}$$

$$\sin\alpha = \frac{1}{\mathrm{Fr}}$$

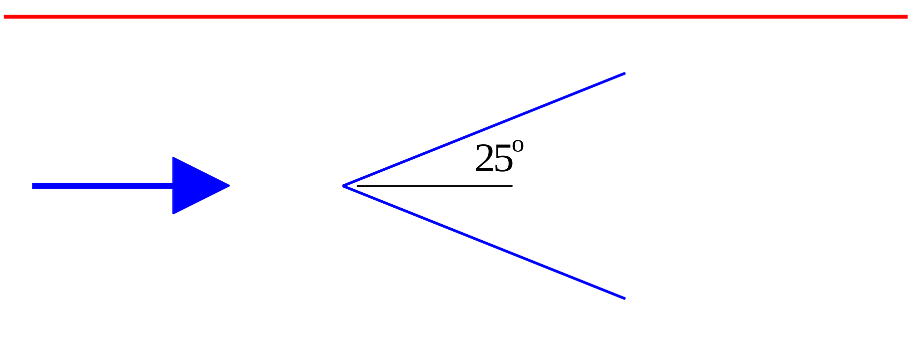
Example

A pencil point piercing the surface of a wide rectangular channel flow creates a wedgelike 25° half-angle wave, as in the figure. If the channel has a Manning's n of 0.014 $m^{-1/3}$ s and the depth is 350 mm, determine:

(a) the Froude number;

(b) the critical depth;

(c) the critical slope.

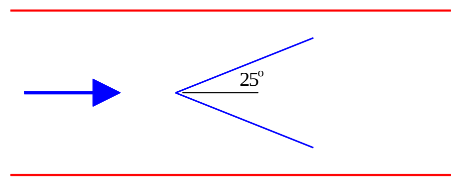


A pencil point piercing the surface of a wide rectangular channel flow creates a wedgelike 25° half-angle wave, as in the figure right. If the channel has a Manning's n of 0.014 $\mathrm{m^{-1/3}\,s}$ and the depth is 350 mm, determine:

(a) the Froude number;

(b) the critical depth;

(c) the critical slope.



$$\alpha = 25^\circ$$

$$n = 0.014\ \mathrm{m^{-1/3}\,s}$$

$$h = 0.35\ \mathrm{m}$$

$$\sin\alpha = \frac{1}{\mathrm{Fr}} \qquad \mathrm{Fr} = \frac{1}{\sin 25^\circ} = \mathbf{2.366}$$

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad q = Vh \qquad \mathrm{Fr} \equiv \frac{V}{\sqrt{gh}}$$

$$V = \mathrm{Fr}\sqrt{gh} = 4.384\ \mathrm{m\,s^{-1}}$$

$$q = Vh = 1.534\ \mathrm{m^2\,s^{-1}}$$

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \mathbf{0.6213\ m}$$

Want S such that $h_n = h_c (= 0.6213\ \mathrm{m})$

$$q = V_n h_n \qquad V_n = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad R_h = h \quad (\text{"wide"})$$

$$q = \frac{1}{n} h_n^{5/3} S^{1/2} \qquad S = \left(\frac{nq}{h_n^{5/3}}\right)^2 = \mathbf{2.254 \times 10^{-3}}$$

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Analogy With Compressible Flow

Ratio of flow velocity to wave speed

Froude number: $\mathrm{Fr} = \dfrac{V}{c}$ $\quad c = \sqrt{gh}$

Mach number: $\mathrm{Ma} = \dfrac{V}{c}$ $\quad c = \sqrt{\gamma p/\rho} \quad = \sqrt{\gamma RT}$

Zone of Influence

Wedge-shaped region of surface waves

Mach cone

Discontinuity

Hydraulic jump

Normal shock

Choked flow

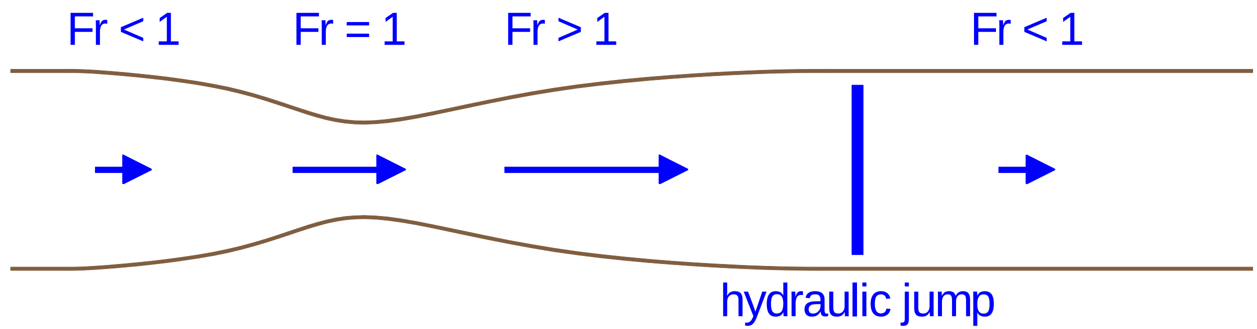
Venturi flume

Transonic nozzle

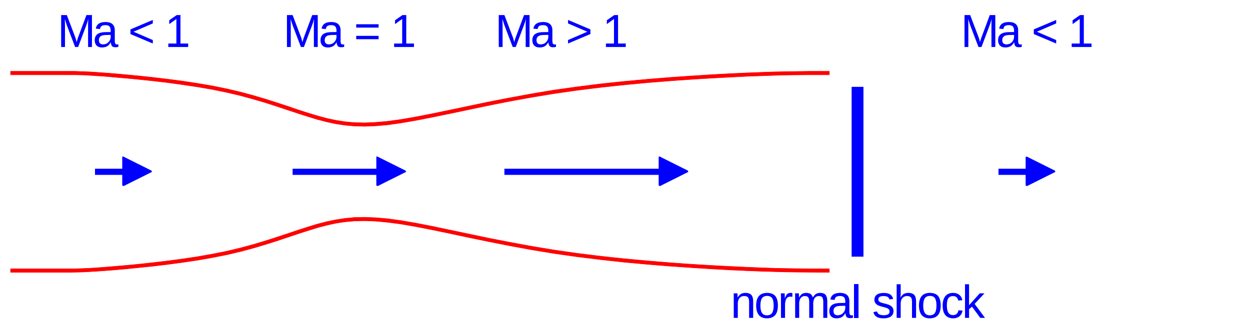
Mach Cone

Choked Flow

Venturi flume



Transonic nozzle



Choked Flow