Hydraulics 3

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Hydraulics 3: Topics

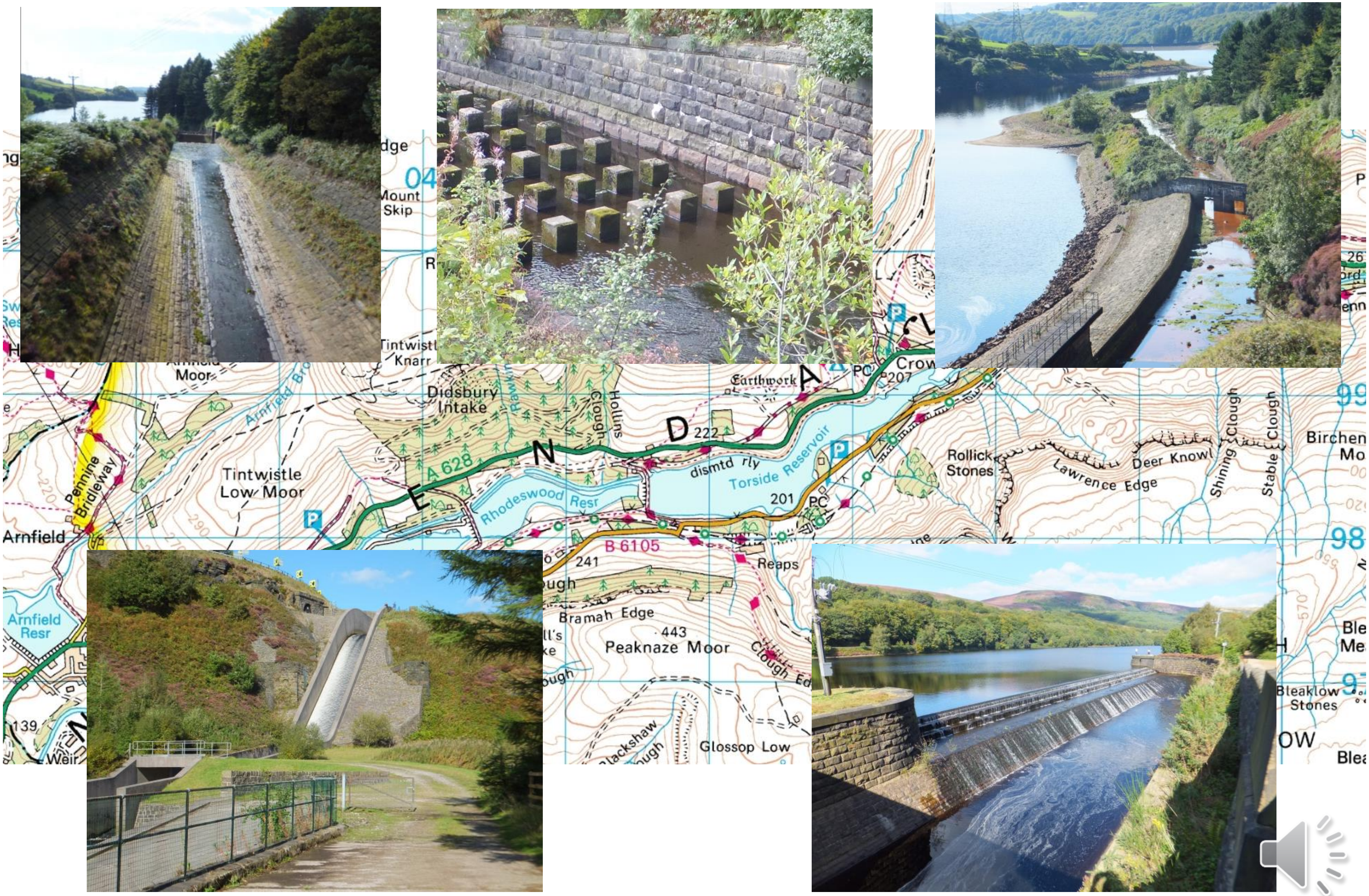
A. Open-channel flow

B. Waves

Hydraulics 3: Assessment

- **Coursework / labs** (20%)
  - 10% written coursework
  - 10% labs
- **Exam** (80%)
  - 4 compulsory questions
  - 2 questions from each of Sections A, B

A. Open-Channel Flow



A. Open-Channel Flow

1. Introduction

2. Rapidly-varied flow

3. Gradually-varied flow

4. Wave speed and analogy with compressible flow

Introduction

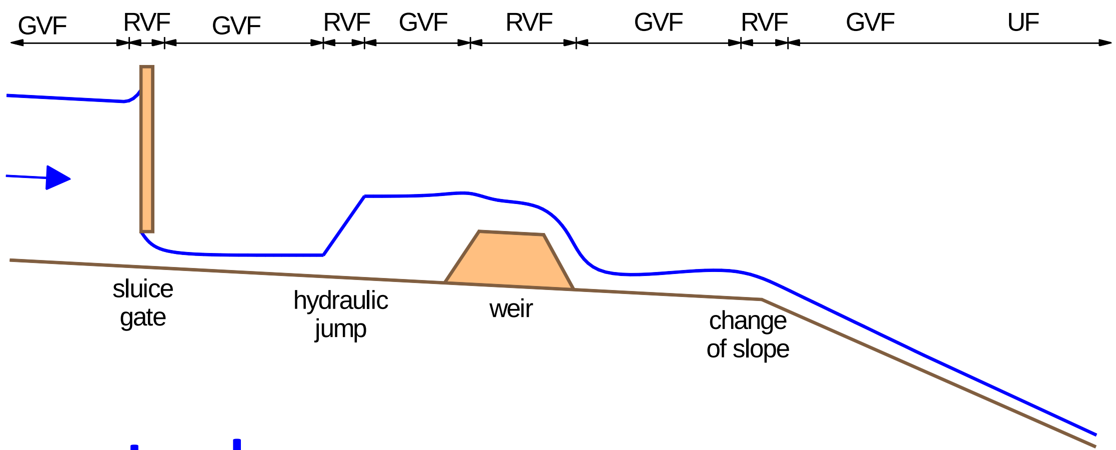
1. INTRODUCTION

- **1.1 Classification**
- 1.2 Normal flow
- 1.3 Flow energy: fluid head
- 1.4 Froude number

Characteristics of Open-Channel Flow

- Free surface ($p = 0$)
- Balance between gravity and friction
- Variable depth h

Classification of Open-Channel Flow



- **Steady** vs **unsteady**
- **Uniform flow** (steady uniform flow = **normal flow**)
  - requires a uniform channel!
  - the limiting behaviour, given sufficient distance
- **Rapidly-varied flow**
  - short distance; bed friction unimportant;
  - examples: hydraulic jump, weir, venturi, sluice, …
- **Gradually-varied flow**
  - long distance; depth adjustment following disturbance
  - result of imbalance between bed friction and component of weight

Introduction

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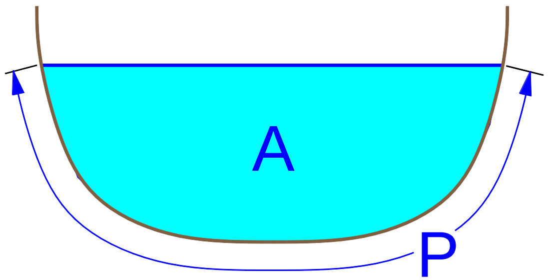
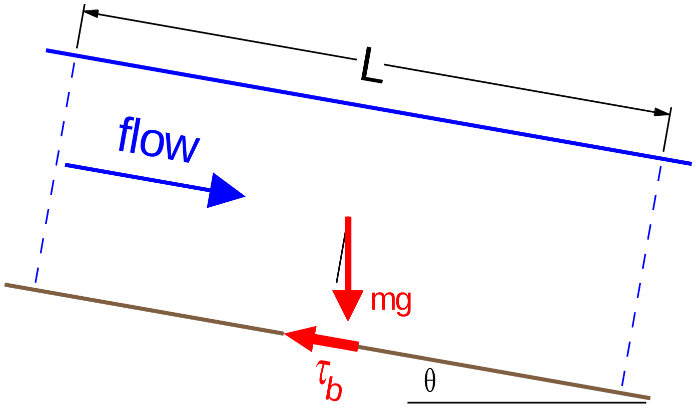
Normal Flow: Balance of Forces





$$\tau_b \times (PL) = (\rho AL) \times g \sin\theta$$

stress × wetted area = weight × sin θ

$$\tau_b = \rho \frac{A}{P} g \sin\theta$$

$$\tau_b = \rho g R_h S$$

hydraulic radius: $R_h \equiv \frac{A}{P}$

$$\tau_b = c_f \left(\tfrac{1}{2}\rho V^2\right)$$

definition of **skin-friction coefficient** c_f

$$c_f \left(\tfrac{1}{2}\rho V^2\right) = \rho g R_h S$$

$$V = \sqrt{\frac{2g}{c_f} R_h S}$$

Normal Flow: Friction Laws

$$V = \sqrt{\frac{2g}{c_f} R_h S}$$

Darcy $\lambda = 4c_f$ $D_h = 4R_h$ ➡ $V = \sqrt{2g\frac{D_h}{\lambda} S}$

Chézy $C = \sqrt{2g/c_f}$ ➡ $V = C\sqrt{R_h S}$

Manning $\sqrt{2g/c_f} = \frac{1}{n} R_h^{1/6}$ ➡ $V = \frac{1}{n} R_h^{2/3} S^{1/2}$

Typical Values of Manning's *n*

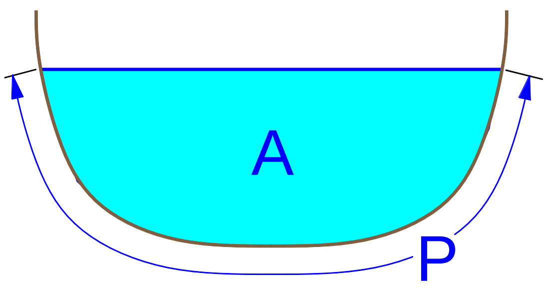
		n ($m^{-1/3}$ s)
Artificial lined channels	Glass	0.01
	Brass	0.011
	Steel, smooth	0.012
	painted	0.014
	riveted	0.015
	Cast iron	0.013
	Concrete, finished	0.012
	unfinished	0.014
	Planed wood	0.012
	Clay tile	0.014
	Brickwork	0.015
	Asphalt	0.016
	Corrugated metal	0.022
	Rubble masonry	0.025
Excavated earth channels	Clean	0.022
	Gravelly	0.025
	Weedy	0.03
	Stony, cobbles	0.035
Natural channels	Clean and straight	0.03
	Sluggish, deep pools	0.04
	Major rivers	0.035
Floodplains	Pasture, farmland	0.035
	Light brush	0.05
	Heavy brush	0.075
	Trees	0.15

Normal Flow: Calculation Formulae

Discharge: $Q = VA$

Manning's equation: $V = \frac{1}{n} R_h^{2/3} S^{1/2}$

Hydraulic radius: $R_h = \frac{A}{P}$



A and P depend on the channel geometry and the water depth h

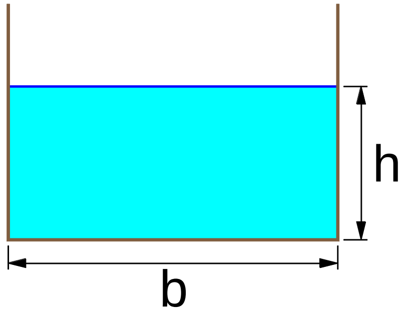
$$Q = f(h)$$

The more common problem is to find depth h, given discharge Q

Hydraulic Radius For Particular Shapes

Rectangular:

$$R_h = \frac{bh}{b+2h} = \frac{h}{1+2h/b}$$

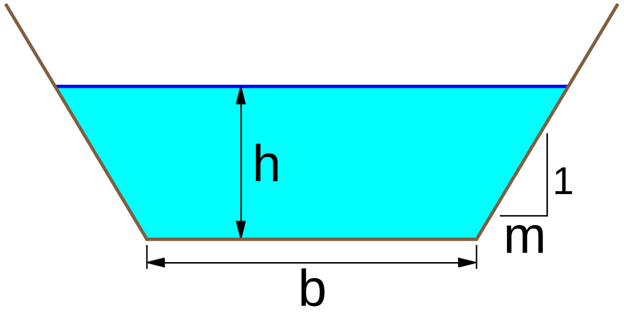


Wide:

$$R_h = h$$

Trapezoidal:

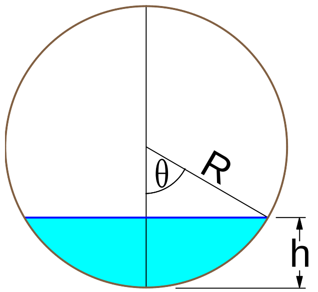
$$R_h = \frac{h(b+mh)}{b+2h\sqrt{1+m^2}}$$



Circular:

$$R_h = \frac{2(\frac{1}{2}R^2\theta - \frac{1}{2}R\sin\theta\, R\cos\theta)}{2R\theta}$$

$$= \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)$$

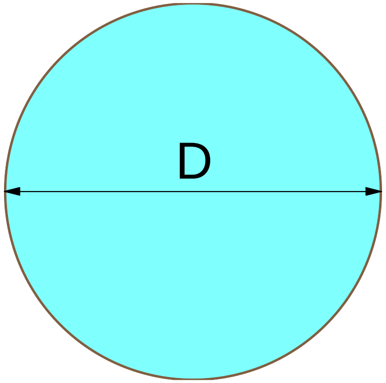


Two Special Cases

Wide channel:

$$R_h = h$$

Full circular pipe:



$$R_h = R/2$$

$$D = 4R_h$$

Example

The discharge in a channel with bottom width 3 m is 12 $m^3 s^{-1}$. If Manning's n is 0.013 $m^{-1/3}$ s and the streamwise slope is 1 in 200, find the normal depth if:

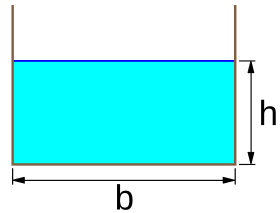
(a) the channel has vertical sides (i.e. rectangular channel);

(b) the channel is trapezoidal with side slopes 2H:1V.

The discharge in a channel with bottom width 3 m is 12 m^3 s^{-1}. If Manning's n is 0.013 m$^{-1/3}$ s and the streamwise slope is 1 in 200, find the normal depth if:

(a) the channel has vertical sides (i.e. rectangular channel);

$$b = 3 \text{ m}$$
$$Q = 12 \text{ m}^3 \text{ s}^{-1}$$
$$n = 0.013 \text{ m}^{-1/3} \text{ s}$$
$$S = 0.005$$



$$Q = VA \qquad V = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad A = bh \qquad R_h \equiv \frac{A}{P} = \frac{bh}{b + 2h} = \frac{h}{1 + 2\,h/b}$$

$$P = b + 2h$$

$$Q = \frac{1}{n}\left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} bh$$

$$Q = \frac{b\sqrt{S}}{n} \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\frac{nQ}{b\sqrt{S}}(1 + 2h/b)^{2/3} = h^{5/3}$$

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

$$h = 0.8316\,(1 + 2h/3)^{2/5}$$

0.8316

0.9921

1.0188

⋮

1.024 m

ANS

$$\boldsymbol{h_n = 1.02 \text{ m}}$$

The discharge in a channel with bottom width 3 m is 12 m^3 s^{-1}. If Manning's n is 0.013 m$^{-1/3}$ s and the streamwise slope is 1 in 200, find the normal depth if:

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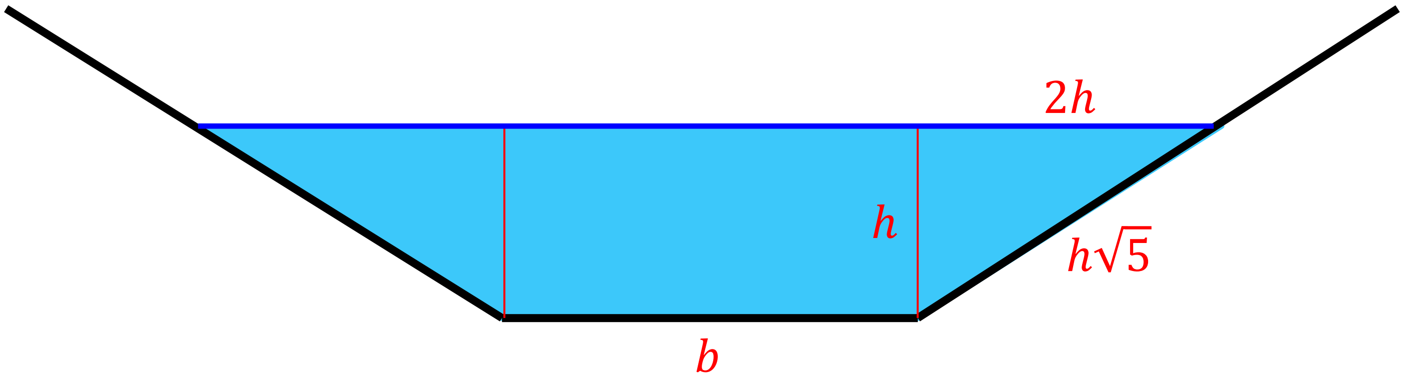

$$A = bh + 2 \times \frac{1}{2} h(2h) \quad = bh + 2h^2 \quad = bh(1 + 2h/b)$$

$$P = b + 2h\sqrt{5} \quad = b(1 + 2\sqrt{5}h/b)$$

$$R_h \equiv \frac{A}{P} \quad = \frac{bh(1 + 2h/b)}{b(1 + 2\sqrt{5}h/b)} \quad = h\left(\frac{1 + 2h/b}{1 + 2\sqrt{5}h/b}\right)$$

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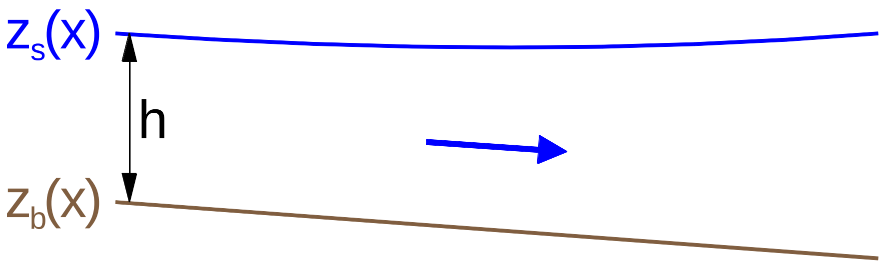
Fluid Head

Total pressure: $p + \rho g z + \frac{1}{2}\rho V^2$

Total head (H): $\frac{p}{\rho g} + z + \frac{V^2}{2g}$



If **hydrostatic**: $p + \rho g z$ is constant **along a vertical line**

$$\frac{p}{\rho g} + z = \left(\frac{p}{\rho g} + z\right)_s = z_s$$

Total head in (**gradually-varied**) open-channel flow: $H = z_s + \frac{V^2}{2g}$

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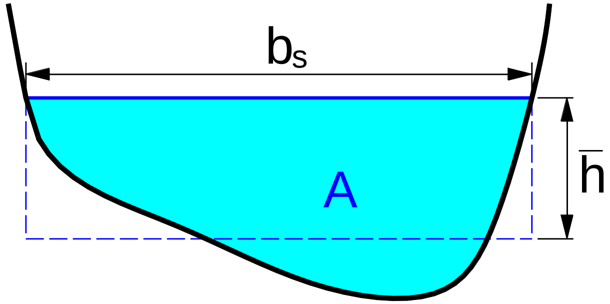
Froude Number in Open-Channel Flow

$$\mathrm{Fr} \equiv \frac{V}{\sqrt{g\bar{h}}}$$

Wide or rectangular channel: $\bar{h} = \text{depth}$

Non-rectangular channel: $\bar{h} = \text{mean depth} = \dfrac{A}{b_s}$



$\mathrm{Fr} < 1$: **subcritical** (tranquil)

$\mathrm{Fr} > 1$: **supercritical** (rapid)

$\mathrm{Fr} = 1$: **critical**

Interpretations of Froude Number

$$\mathrm{Fr} \equiv \frac{V}{\sqrt{gh}}$$

- (Square root of) ratio of inertial to gravitational forces
- Ratio of water velocity V to long-wave speed $\sqrt{gh}$
- **Critical depth** $(\mathrm{Fr} = 1) \Leftrightarrow$ **minimum specific energy**
- Separates:
  - deep, slow, subcritical flow $(\mathrm{Fr} < 1)$
  - shallow, fast, supercritical flow $(\mathrm{Fr} > 1)$
- Occurs at a **control point** in **critical-flow devices** such as broad-crested weirs and venturi flumes.

Example

The discharge in a rectangular channel of width 6 m with Manning's n = 0.012 m$^{-1/3}$ s is 24 m^3 s^{-1}. If the streamwise slope is 1 in 200 find:

(a) the normal depth;

(b) the Froude number at the normal depth;

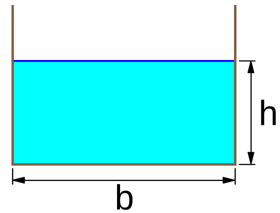
(c) the critical depth.

State whether the normal flow is subcritical or supercritical.

The discharge in a rectangular channel of width 6 m with Manning's n = 0.012 m$^{-1/3}$ s is 24 m^3 s^{-1}. If the streamwise slope is 1 in 200 find:

(a) the normal depth;

$$b = 6 \text{ m}$$
$$n = 0.012 \text{ m}^{-1/3} \text{ s}$$
$$Q = 24 \text{ m}^3 \text{ s}^{-1}$$
$$S = 0.005$$



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$$P = b + 2h$$

$$Q = \frac{1}{n}\left(\frac{h}{1+2h/b}\right)^{2/3} S^{1/2} bh$$

$$Q = \frac{b\sqrt{S}}{n} \frac{h^{5/3}}{(1+2h/b)^{2/3}}$$

$$\frac{nQ}{b\sqrt{S}}(1+2h/b)^{2/3} = h^{5/3}$$

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1+2h/b)^{2/5}$$

$$h = 0.7926\,(1 + h/3)^{2/5}$$

$$\boldsymbol{h_n = 0.8783 \text{ m}}$$

The discharge in a rectangular channel of width 6 m with Manning's n = 0.012 m$^{-1/3}$ s is 24 m^3 s^{-1}. If the streamwise slope is 1 in 200 find:

(a) the normal depth;

(b) the Froude number at the normal depth;

$$h_n = 0.8783 \text{ m}$$

$$\text{Fr} \equiv \frac{V}{\sqrt{gh}} \qquad V = \frac{Q}{A} \qquad V_n = \frac{Q}{bh_n} = 4.554 \text{ m s}^{-1}$$

$$\text{Fr}_n = \mathbf{1.551}$$

The discharge in a rectangular channel of width 6 m with Manning's n = 0.012 $m^{-1/3}$ s is 24 $m^3 s^{-1}$. If the streamwise slope is 1 in 200 find:

(a) the normal depth;

(b) the Froude number at the normal depth;

(c) the critical depth.

State whether the normal flow is subcritical or supercritical.

$h_n = 0.8783 \text{ m}$ $\quad\quad$ $\text{Fr}_n = 1.551$

$$\text{Fr}^2 = 1$$

$$\frac{V^2}{gh} = 1$$

$$V = \frac{Q}{bh} = \frac{q}{h} \quad \text{where } q = \frac{Q}{b} = 4 \text{ m}^2 \text{ s}^{-1}$$

$$\frac{q^2}{gh^3} = 1$$

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \mathbf{1.177 \text{ m}}$$

$\text{Fr}_n = 1.551 > 1$ **normal flow is supercritical**

$h_c = 1.177 \text{ m} \quad (\text{Fr} = 1)$

$h_n = 0.8783 \text{ m} \quad (\text{Fr} > 1)$

$h \downarrow \quad V \uparrow \quad \text{Fr} \uparrow$

$h_n < h_c$ **normal flow is supercritical**