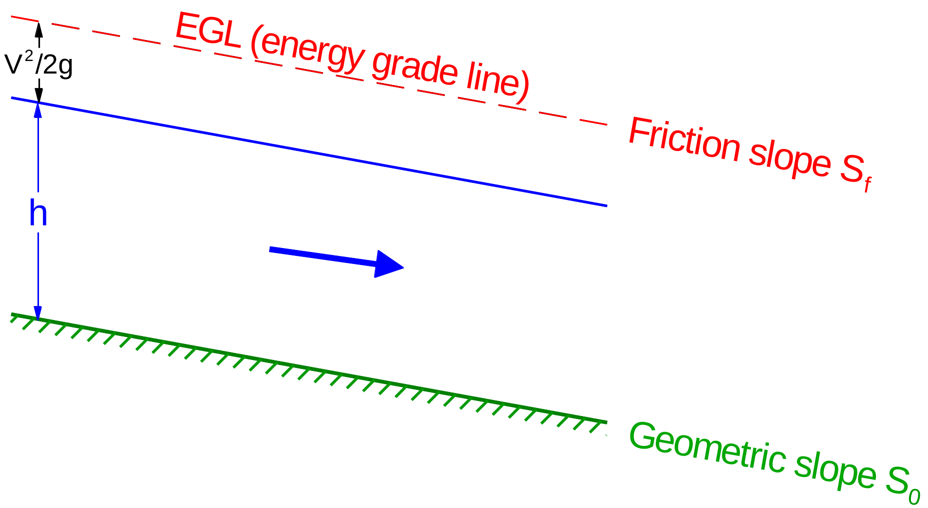
Open-Channel Flow

3. Gradually-Varied Flow

Gradually-Varied Flow

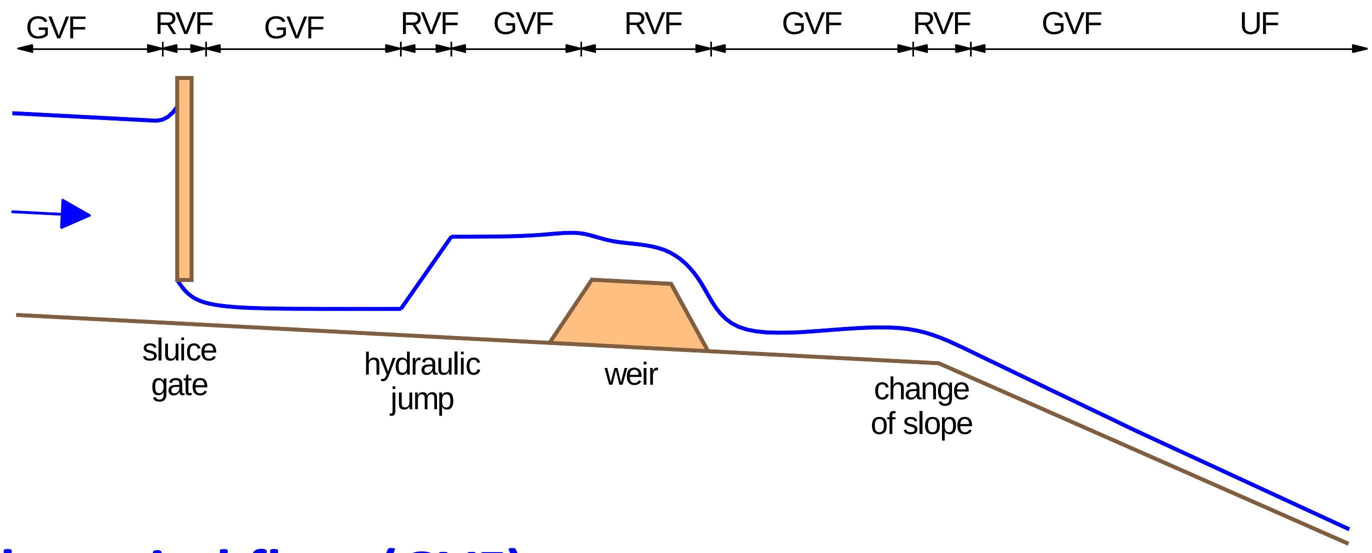
Normal Flow



Normal flow:

- Downslope component of weight balances bed friction
- Uniform depth and velocity
- Bed slope or **geometric slope** (S_0) is the same as the slope of the total head line or **friction slope** (S_f)
- “Preferred” depth, to which flow tends given sufficient fetch

Gradually-Varied Flow



Gradually-varied flow (GVF):

- Downslope component of weight does not balance bed friction
- Geometric slope (S_0) is different to friction slope (S_f)
- Depth h changes with distance

The **gradually-varied-flow equation** gives the change of depth with distance

Gradually-Varied-Flow Equation

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

Assumptions:

- Small slopes
- Quasi-1d
- Hydrostatic pressure

Depends on:

- Difference between geometric and friction slopes ($S_0 - S_f$)
- Sub- or supercritical flow (Fr)

Gradually-Varied Flow

3. GRADUALLY-VARIED FLOW

3.1 Normal flow vs gradually-varied flow

3.2 Derivation of the gradually-varied-flow equation

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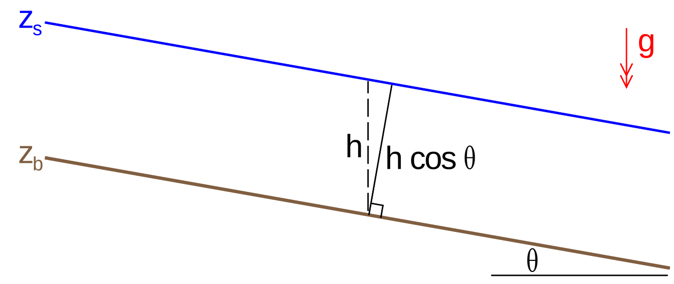
Derivation of the GVF Equation (1)



Total head: $H = z_s + \frac{V^2}{2g} \quad = z_b + h + \frac{V^2}{2g}$

$$H = z_b + E$$

$$\frac{\mathrm{d}H}{\mathrm{d}x} = \frac{\mathrm{d}z_b}{\mathrm{d}x} + \frac{\mathrm{d}E}{\mathrm{d}x}$$

Define: $\frac{\mathrm{d}H}{\mathrm{d}x} = -S_f$ **friction slope**

$\frac{\mathrm{d}z_b}{\mathrm{d}x} = -S_0$ **geometric slope**

GVF equation (specific-energy form): $\frac{\mathrm{d}E}{\mathrm{d}x} = S_0 - S_f$

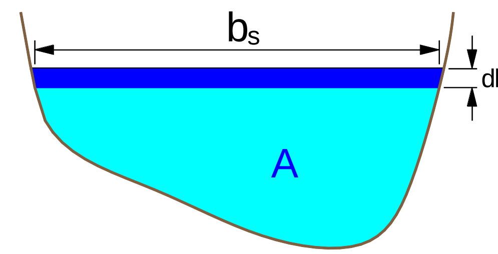
Derivation of the GVF Equation (2)

Specific energy: $E = h + \frac{V^2}{2g}$ ← $V = \frac{Q}{A}$

$$\frac{\mathrm{d}E}{\mathrm{d}x} = S_0 - S_f$$

$$E = h + \frac{Q^2}{2gA^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{\mathrm{d}h}{\mathrm{d}x} - \frac{Q^2}{gA^3}\frac{\mathrm{d}A}{\mathrm{d}x}$$ ← $\mathrm{d}A = b_s\,\mathrm{d}h$



$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{\mathrm{d}h}{\mathrm{d}x}\left(1 - \frac{Q^2 b_s}{gA^3}\right)$$ ← $V = \frac{Q}{A}$ $\quad \bar{h} = \frac{A}{b_s}$

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{\mathrm{d}h}{\mathrm{d}x}\left(1 - \frac{V^2}{g\bar{h}}\right)$$

$$S_0 - S_f = \frac{\mathrm{d}h}{\mathrm{d}x}\left(1 - \mathrm{Fr}^2\right)$$

GVF equation (depth form): $$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

Gradually-Varied Flow

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Finding the Friction Slope, S_f

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

Quasi-uniform-flow assumption:

rate of energy loss is the same as uniform flow of the same depth.

$$V = \frac{1}{n} R_h^{2/3} S_f^{1/2}$$

$$S_f = \frac{n^2 V^2}{R_h^{4/3}} = \frac{n^2 Q^2}{R_h^{4/3} A^2} = \text{function of depth } h$$

Greater depth $\Rightarrow$ lower velocity $\Rightarrow$ smaller S_f

Smaller depth $\Rightarrow$ higher velocity $\Rightarrow$ greater S_f

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Slope Classification

Critical depth h_c: depth at which $\mathrm{Fr} = 1$

Normal depth h_n: depth of uniform flow ($S_f = S_0$)

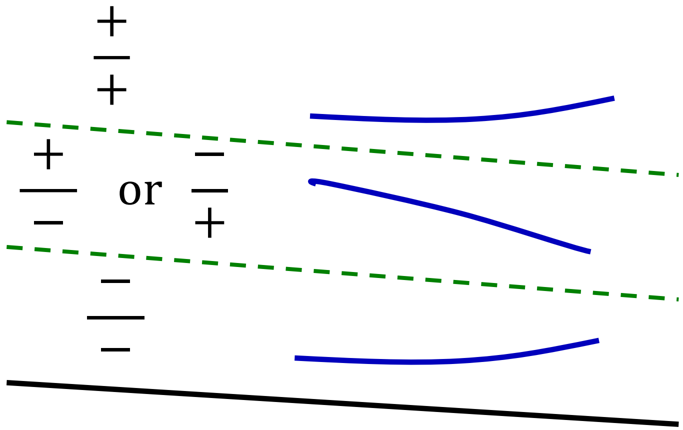
e.g. wide channel: $h_c = (q^2/g)^{1/3}$ $h_n = (nq/\sqrt{S_0})^{3/5}$

(For a given discharge) a slope is:

- **steep**, if the **normal flow is supercritical**
  (i.e. the normal depth is less than the critical depth)
- **mild**, if the **normal flow is subcritical**
  (i.e. the normal depth is greater than the critical depth)

Increasing or Decreasing Depth

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$



$S_0 - S_f > 0$ if and only if h is greater than normal depth

$1 - \mathrm{Fr}^2 > 0$ if and only if h is greater than critical depth

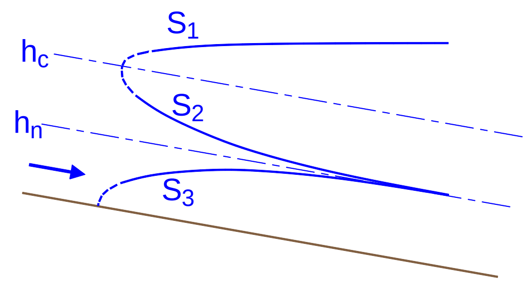
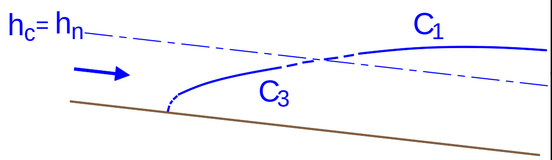
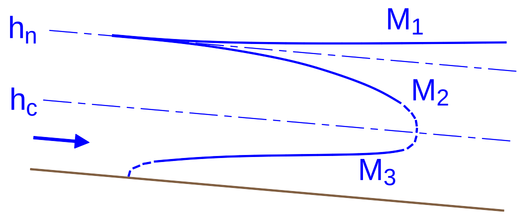
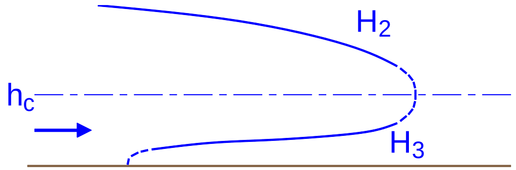
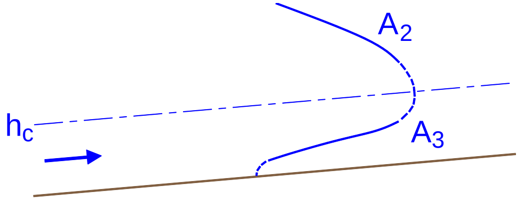
$\frac{\mathrm{d}h}{\mathrm{d}x} < 0$ **depth decreasing** ...

... if and only if h **lies between normal and critical depths**.

Water-Profile Classification

2 characters (e.g. S1, M3 etc.):

- **S**, C, **M**, H, A (Steep, Critical, Mild, Horizontal, Adverse)
- 1, 2, 3 (where h lies with respect to h_c and h_n)

Type	Symbol	Definition	Sketches	Examples
STEEP (normal flow supercritical)	S1	$h > h_c > h_n$		Hydraulic jump upstream with obstruction or reservoir controlling water level downstream.
	S2	$h_c > h > h_n$		Change to steeper slope.
	S3	$h_c > h_n > h$		Change to less steep slope.
CRITICAL (undesirable; undular unsteady flow)	C1	$h > h_c = h_n$		
	C3	$h_c = h_n > h$		
MILD (normal flow subcritical)	M1	$h > h_n > h_c$		Obstruction or reservoir controlling water level downstream.
	M2	$h_n > h > h_c$		Approach to free overfall.
	M3	$h_n > h_c > h$		Hydraulic jump downstream; change from steep to mild slope or downstream of sluice .
HORIZONTAL (limiting mild slope; $h_n \to \infty$)	H2	$h > h_c$		Approach to free overfall.
	H3	$h_c > h$		Hydraulic jump downstream; change from steep to horizontal or downstream of sluice.
ADVERSE (upslope)	A2	$h > h_c$		
	A3	$h_c > h$		

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Control Points

Definition: locations at which there is a known relationship between depth and flow rate (**stage-discharge relationship**)

Examples:

- Critical flow points: weir, venturi, free overfall, ...
- Sluices
- Entry/exit from reservoir
- Hydraulic jump

A control point often yields a boundary condition from which to start a GVF calculation

General Principles

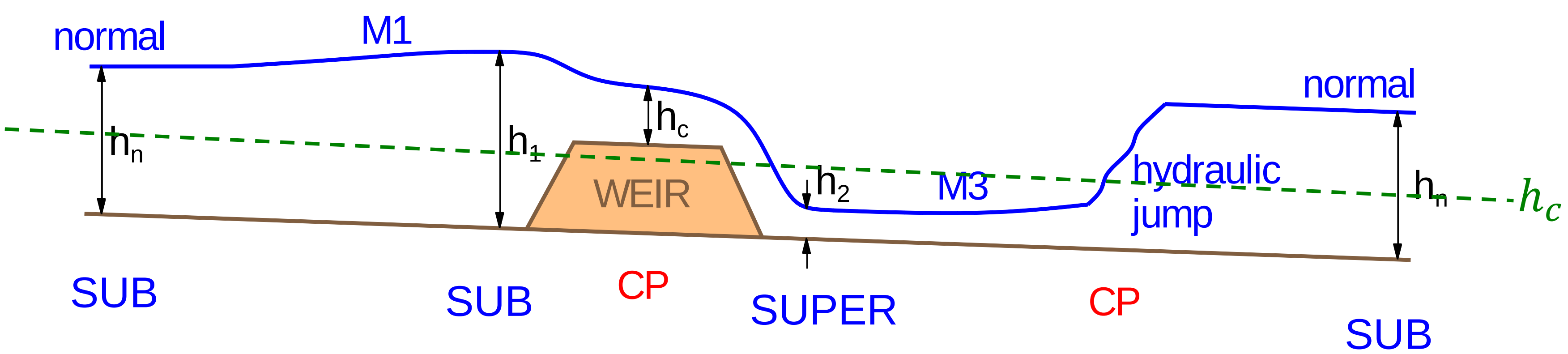
- **Supercritical** $\Rightarrow$ controlled by **upstream** conditions.
  Subcritical $\Rightarrow$ controlled by **downstream** conditions.

- Given a long-enough fetch the flow will try to revert to normal flow.

- A **hydraulic jump** occurs between regions of supercritical and subcritical gradually-varied flow **at the point where the jump condition for the sequent depths is correct**.

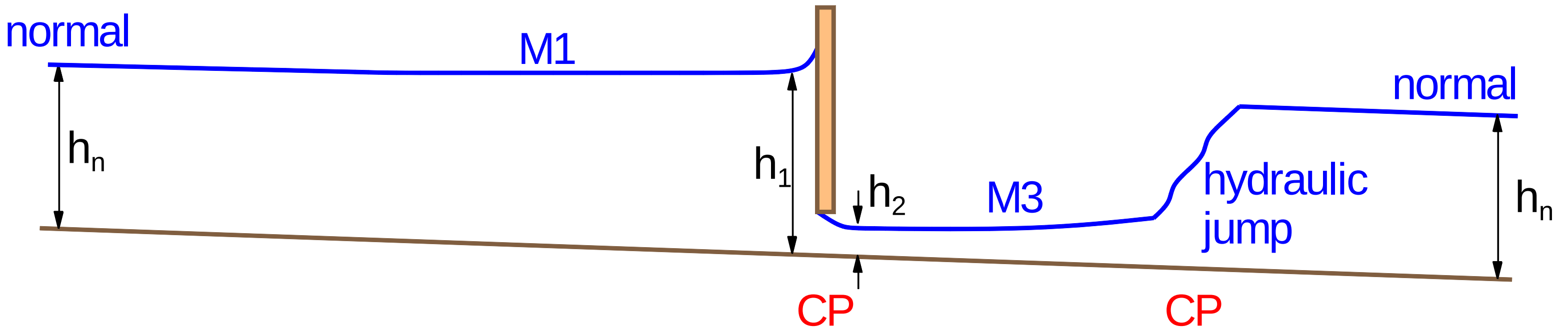
- Where the slope is mild (i.e. the normal flow is subcritical), and any downstream control is far away, a hydraulic jump can be assumed to **jump directly to the normal depth**.

Qualitative Examples: Weir (Mild Slope)

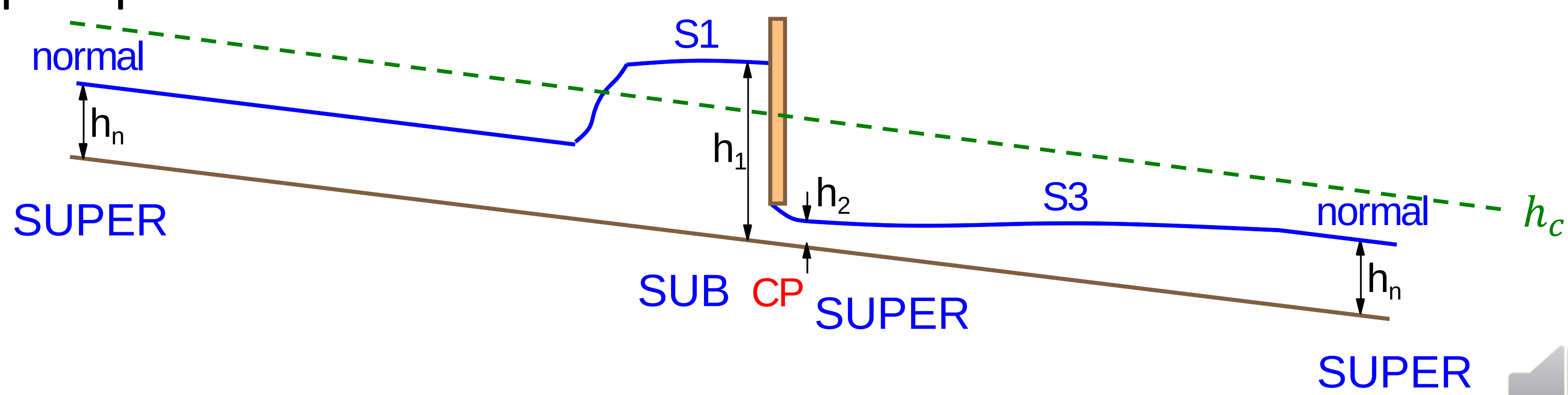


Qualitative Examples: Sluice

Mild slope

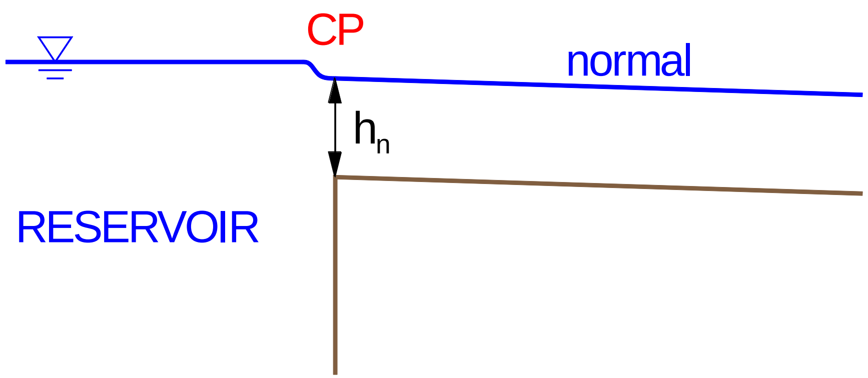


Steep slope

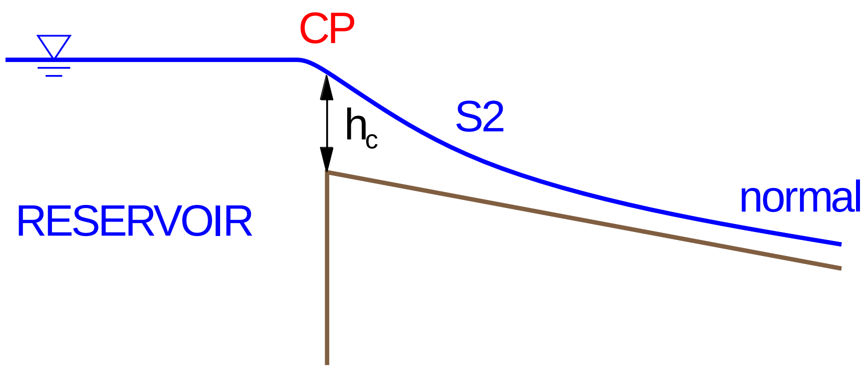


Qualitative Examples: Flow From Reservoir

Mild slope

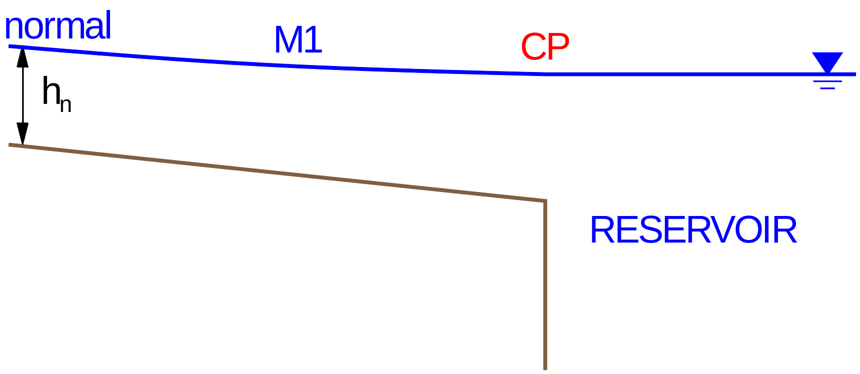


Steep slope

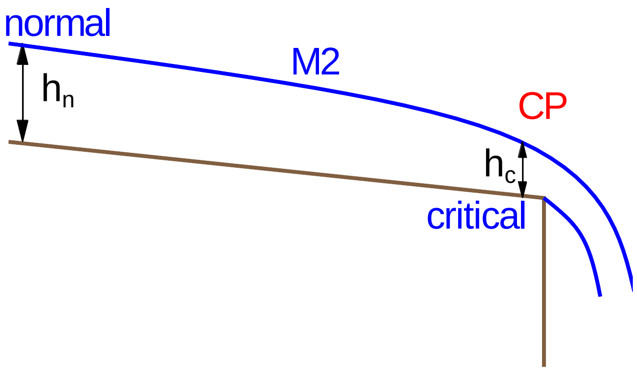


Qualitative Examples

Flow into reservoir (mild slope)



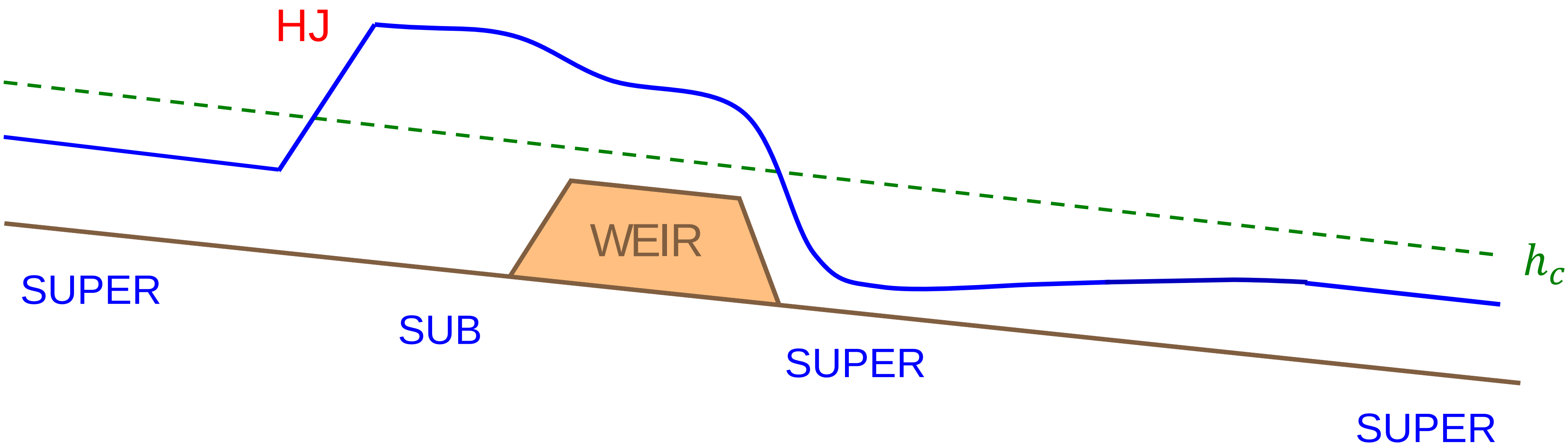
Free overfall (mild slope)



Qualitative Example: Exercise

Sketch the water profile for:

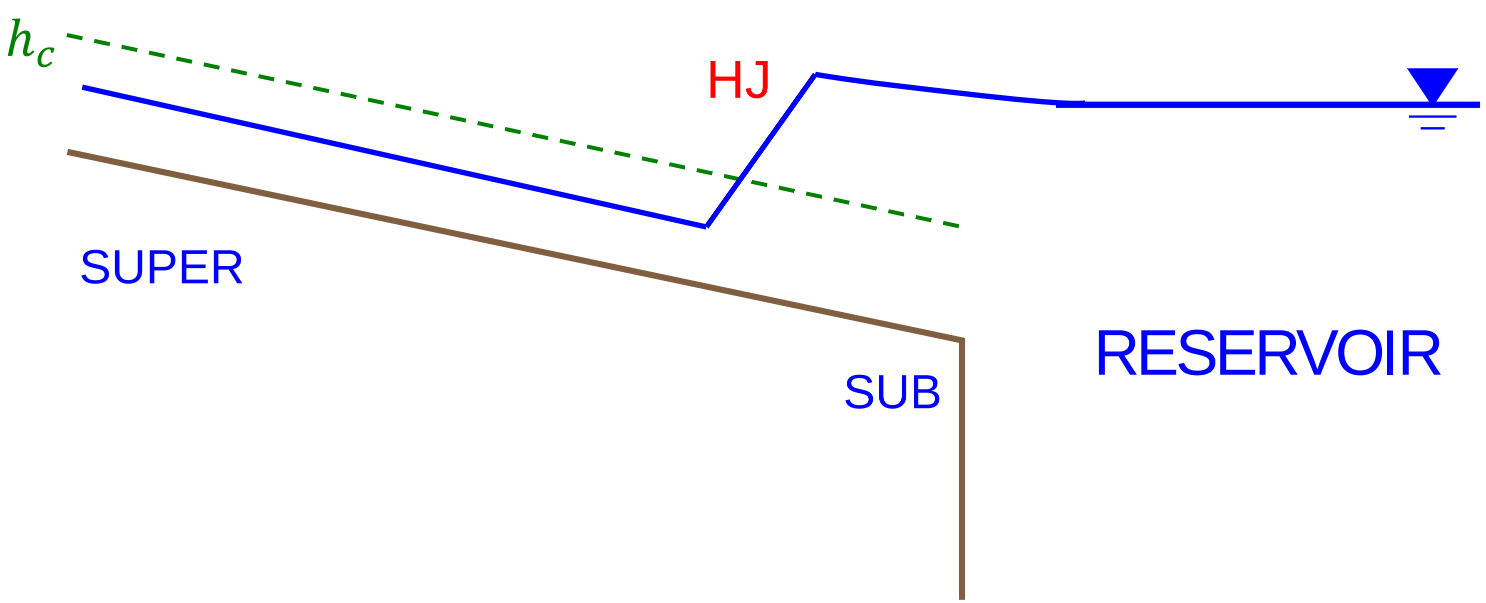
- Flow over weir (steep slope)



Qualitative Example: Exercise

Sketch the water profile for:

- Flow into reservoir (from a steep slope)



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The GVF Equation

Three forms:

Total head:

$$\frac{\mathrm{d}H}{\mathrm{d}x} = -S_f$$

Specific energy:

$$\frac{\mathrm{d}E}{\mathrm{d}x} = S_0 - S_f$$

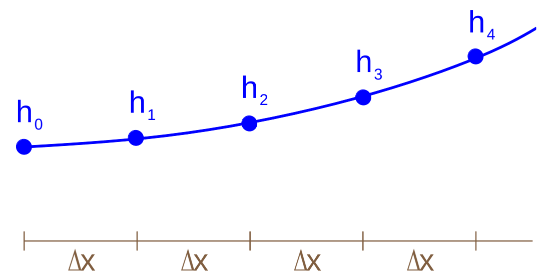
Depth:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

Solving the GVF Equation

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

Impossible to solve analytically (in most circumstances)



Find depths $h_1, h_2, h_3, \ldots$ at discrete points $x_1, x_2, x_3, \ldots$

$\frac{\mathrm{d}h}{\mathrm{d}x}$ approximated by $\frac{\Delta h}{\Delta x}$ where

$$\Delta h = h_{i+1} - h_i$$

$$\Delta x = x_{i+1} - x_i$$

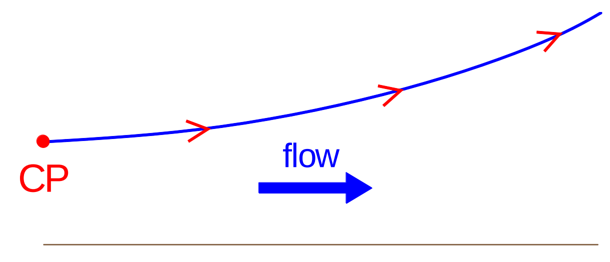
Starting Point and Direction

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

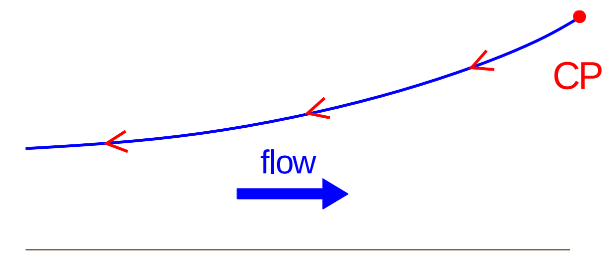
Start at a **control point**.

Proceed:

- **forward** in x if **supercritical** (upstream control);



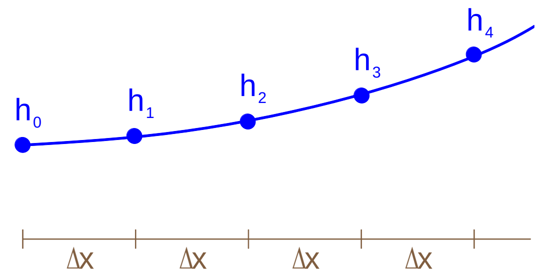
- **backward** in x if **subcritical** (downstream control).



Types of Method

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

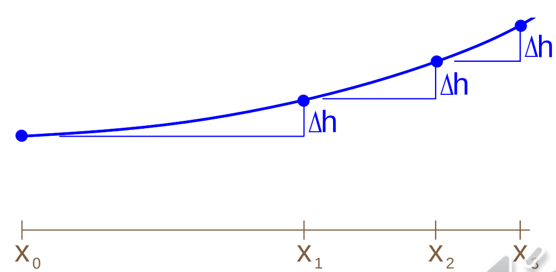
1. Standard-step methods



Solve for depth h_i at specified distance intervals Δx

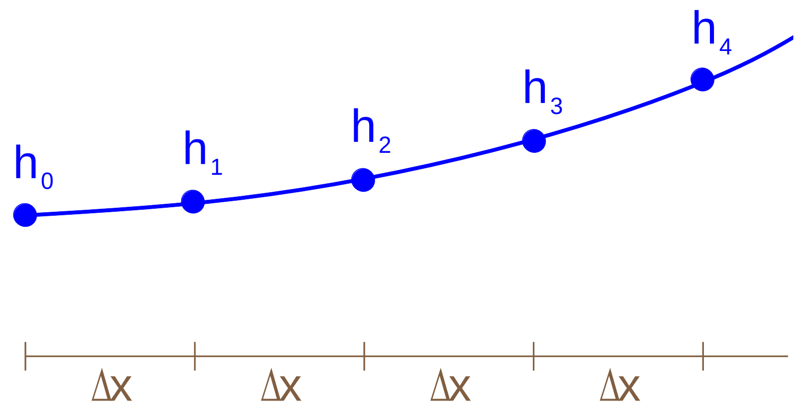
2. Direct-step methods

Solve for distance x_i at specified height intervals Δh



$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f}$$

Standard-Step Method: Total Head



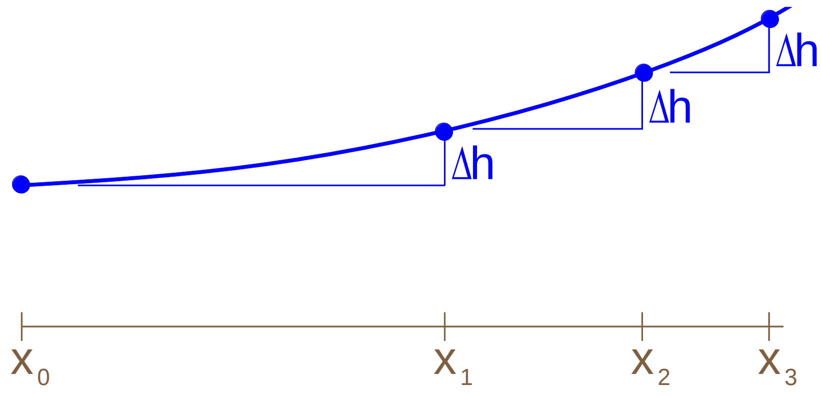
$$\frac{\mathrm{d}H}{\mathrm{d}x} = -S_f$$

$$H = z_b + h + \frac{V^2}{2g}$$

$$\frac{H_{i+1} - H_i}{\Delta x} = -\left(\frac{S_{f,i} + S_{f,i+1}}{2}\right)$$

Adjust depth h_{i+1} (iteratively) at each step until LHS = RHS.

Direct-Step Method: Specific Energy



$$\frac{dE}{dx} = S_0 - S_f$$

$$E = h + \frac{V^2}{2g}$$

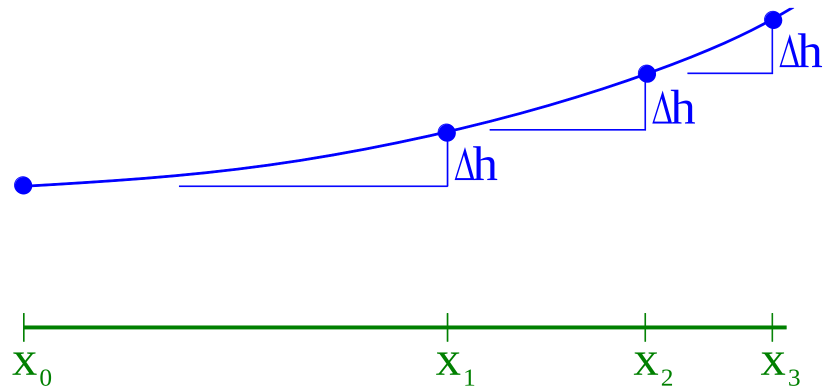
$$\frac{dx}{dE} = \frac{1}{S_0 - S_f}$$

$$\Delta x = \frac{\Delta E}{(S_0 - S_f)_{av}}$$

$$\Delta E = E_{i+1} - E_i$$

$$E = E(h)$$

Direct-Step Method: Depth



$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f}$$

Write S_f and Fr^2 as functions of h

$$\frac{\Delta x}{\Delta h} \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{av}}$$

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{av}} \Delta h$$

Example

A long, wide channel has a slope of 1:2747 with a Manning's n of 0.015 $m^{-1/3}$ s. It carries a discharge of 2.5 m^3 s^{-1} per metre width, and there is a free overfall at the downstream end. An undershot sluice is placed a certain distance upstream of the free overfall which determines the nature of the flow between sluice and overfall. The depth just downstream of the sluice is 0.5 m.

(a) Determine the critical depth and normal depth.

(b) Sketch, with explanation, the two possible gradually-varied flows between sluice and overfall.

(c) Calculate the particular distance between sluice and overfall which determines the boundary between these two flows. Use one step in the gradually-varied-flow equation.

A long, wide channel has a slope of 1:2747 with a Manning's n of 0.015 m$^{-1/3}$ s. It carries a discharge of 2.5 m^3 s^{-1} per metre width, and there is a free overfall at the downstream end. An undershot sluice is placed a certain distance upstream of the free overfall which determines the nature of the flow between sluice and overfall. The depth just downstream of the sluice is 0.5 m.

(a) Determine the critical depth and normal depth.

$$S_0 = 1/2747$$

$$n = 0.015\ \text{m}^{-1/3}\ \text{s}$$

$$q = 2.5\ \text{m}^2\ \text{s}^{-1}$$

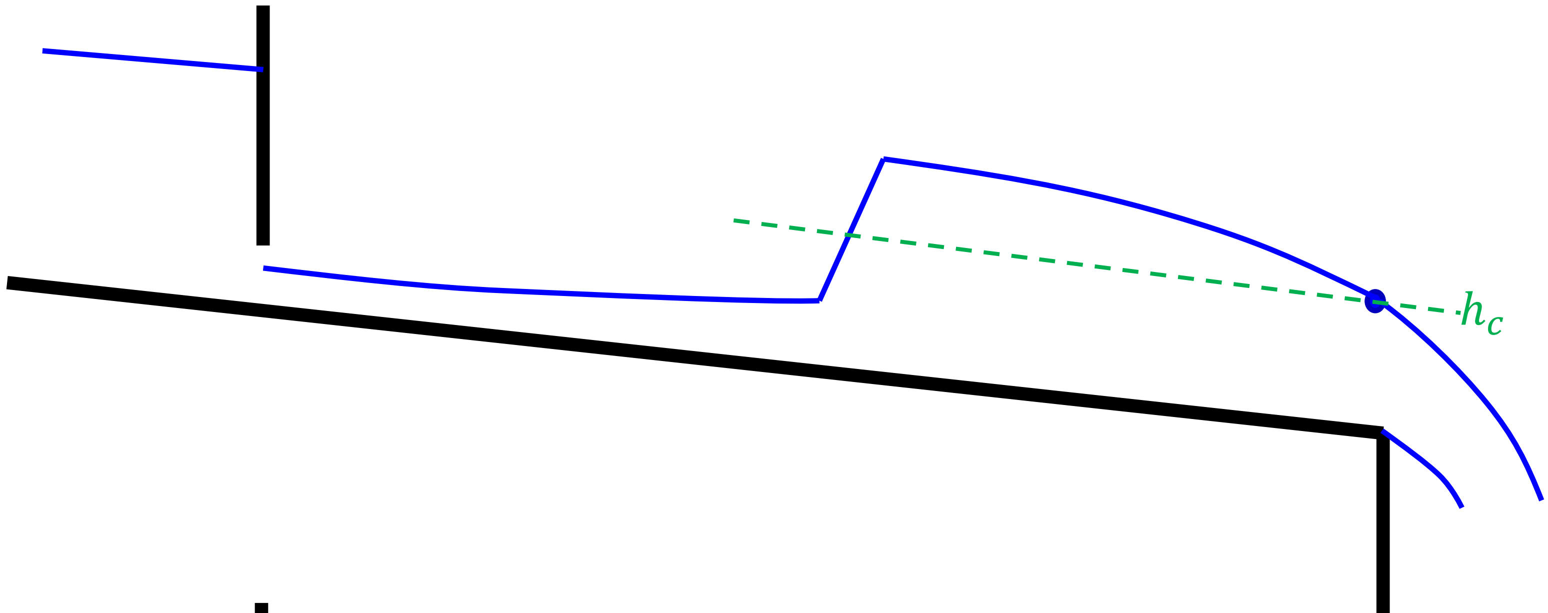
Critical: $h_c = \left(\dfrac{q^2}{g}\right)^{1/3} = \mathbf{0.8605\ m}$

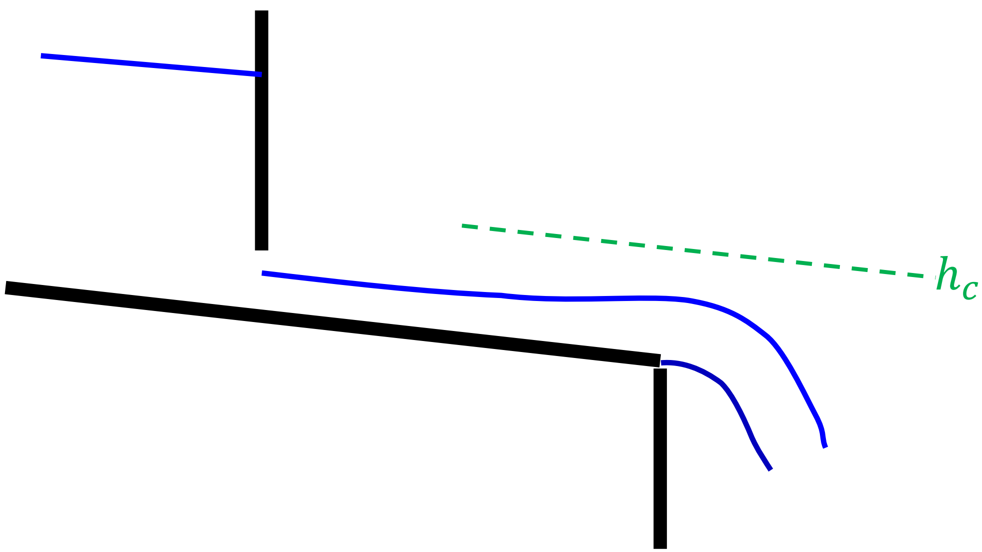
Normal: $q = Vh \qquad V = \dfrac{1}{n} R_h^{2/3} S_0^{1/2} \qquad R_h = h \quad (\text{"wide"})$

$$q = \frac{1}{n} h^{5/3} \sqrt{S_0} \qquad (*)$$

$$h_n = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} = \mathbf{1.500\ m}$$

(b) Sketch, with explanation, the two possible gradually-varied flows between sluice and overfall.





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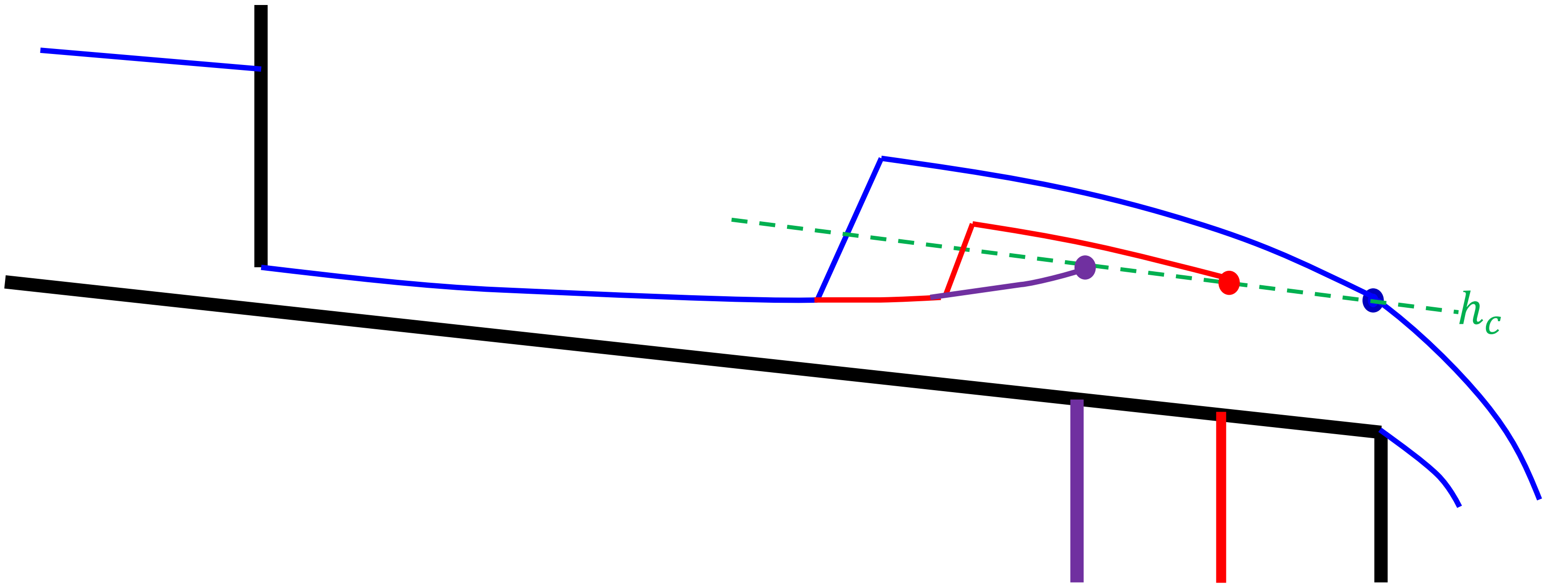
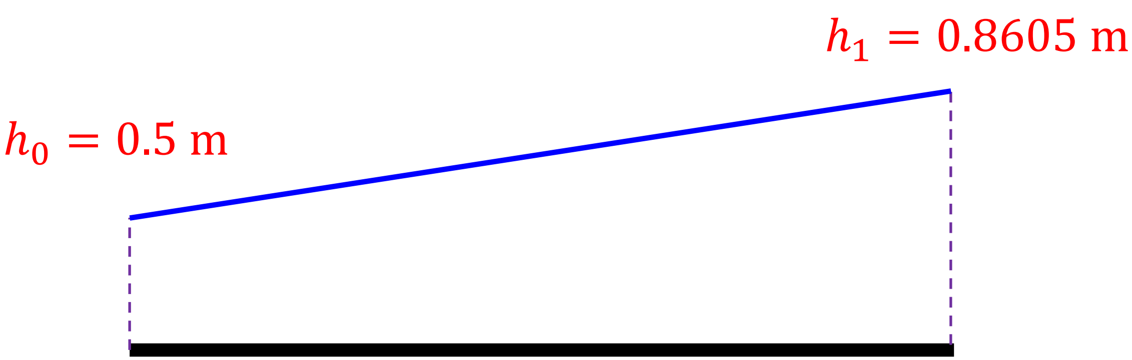




$$\Delta h = \frac{0.8605 - 0.5}{1}$$

$$= 0.3605 \text{ m}$$

(c) Calculate the particular distance between sluice and overfall which determines the boundary between these two flows. Use one step in the gradually-varied-flow equation.

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f}$$

$$\mathrm{Fr}^2 = \frac{V^2}{gh} = \frac{q^2}{gh^3} = \frac{0.6371}{h^3}$$

$$S_0 = 3.640 \times 10^{-4}$$

$$(*) \quad q = \frac{1}{n} h^{5/3} \sqrt{S_f}$$

$$S_f = \left(\frac{nq}{h^{5/3}}\right)^2 = \frac{14.06 \times 10^{-4}}{h^{10/3}}$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.6371}{h^3}}{\left(3.640 - \dfrac{14.06}{h^{10/3}}\right) \times 10^{-4}}$$

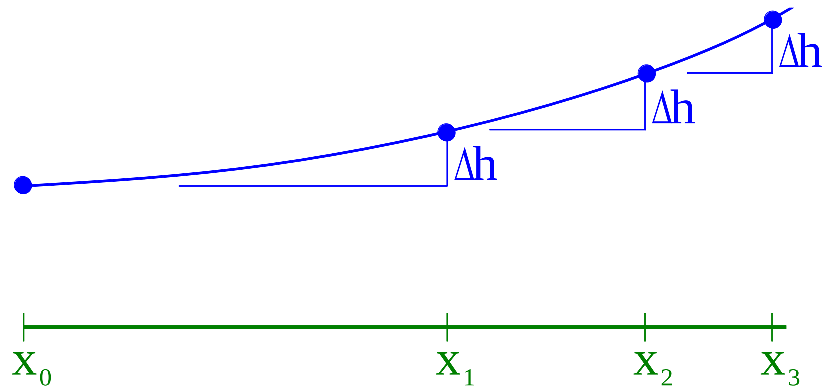
$$\frac{\Delta x}{\Delta h} \approx \frac{\mathrm{d}x}{\mathrm{d}h}$$

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

$$\Delta h = 0.3605$$

i	h_i	x_i	h_{mid}	$\left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}}$	Δx
0	0.5	0			
			0.6803	217.1	78.26
1	0.8605	**78.26**			

Direct-Step Method: Depth



$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

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Write S_f and Fr^2 as functions of h

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$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{av}} \Delta h$$

Example

A long rectangular channel of width $2.5\ \mathrm{m}$, slope 0.004 and Manning's roughness coefficient $n = 0.022\ \mathrm{m}^{-1/3}\mathrm{s}$ carries water at $4\ \mathrm{m}^3\ \mathrm{s}^{-1}$. Temporary works narrow the channel at one location to $1.1\ \mathrm{m}$ for a short distance.

(a) Find the normal depth in the main channel and show that the slope is hydraulically mild.

(b) Show that a hydraulic transition takes place at the narrow point and find the depth just downstream of the narrowed section, confirming that supercritical flow is possible here.

(c) Use two steps in the gradually-varied-flow equation to estimate the distance from the end of the narrow section to the downstream hydraulic jump.

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(a) Find the normal depth in the main channel and show that the slope is hydraulically mild.

$b = 2.5\ \mathrm{m}$ (main channel)

$S_0 = 0.004$

$n = 0.022\ \mathrm{m}^{-1/3}\ \mathrm{s}$

$Q = 4\ \mathrm{m}^3\ \mathrm{s}^{-1}$

$$Q = VA \qquad V = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad A = bh \qquad P = b + 2h \qquad R_h = \frac{bh}{b+2h} = \frac{h}{1 + 2h/b}$$

$$Q = \frac{1}{n}\frac{bh^{5/3}}{(1+2h/b)^{2/3}} S^{1/2} \qquad (*)$$

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

$$h = 0.7036(1 + 0.8h)^{2/5}$$

$$h_n = \mathbf{0.8690\ m}$$

$$V_n = \frac{Q}{bh_n} = 1.841\ \mathrm{m\ s}^{-1} \qquad \mathrm{Fr}_n \equiv \frac{V_n}{\sqrt{gh_n}} = \mathbf{0.6305}$$

subcritical normal flow
$\Rightarrow$ mild slope

(b) Show that a hydraulic transition takes place at the narrow point and find the depth just downstream of the narrowed section, confirming that supercritical flow is possible here.

Critical conditions at the throat ($b_m = 1.1\ \text{m}$):

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \qquad q_m = \frac{Q}{b_m} \quad = 3.636\ \text{m}^2\ \text{s}^{-1} \qquad h_c = 1.105\ \text{m}$$

$$H_c = z_b + E_c \quad = 0 + \frac{3}{2}h_c \quad = 1.658\ \text{m}$$

Approach flow:

$$H_a = h_n + \frac{V_n^2}{2g} \quad = 1.042\ \text{m} \qquad H_a < H_c \quad \Rightarrow \text{hydraulic transition occurs}$$

$$H = H_c = 1.658\ \text{m} \quad \text{throughout}$$

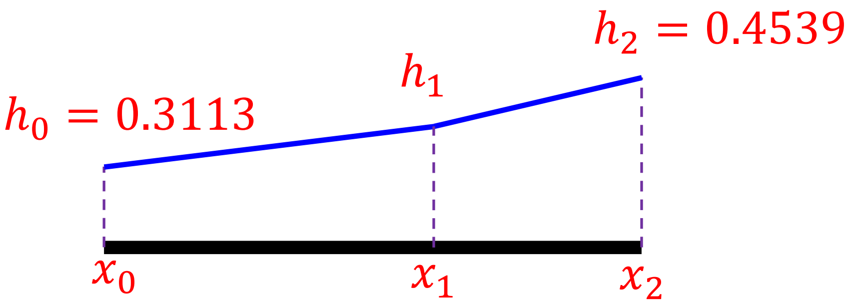
Main channel width ($b = 2.5\ \text{m}$):

$$H = h + \frac{Q^2}{2gb^2h^2} \qquad 1.658 = h + \frac{0.1305}{h^2} \qquad h = \sqrt{\frac{0.1305}{1.658 - h}} \qquad h_2 = \mathbf{0.3113\ m}$$

Mild slope, so any supercritical GVF would have to increase in depth with distance.

$$h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8\text{Fr}_n^2}\right) \quad = 0.4539\ \text{m} \qquad h_2 < h_J \quad \Rightarrow$$ **supercritical GVF occurs**

(c) Use two steps in the gradually-varied-flow equation to estimate the distance from the end of the narrow section to the downstream hydraulic jump.



$$\Delta h = \frac{0.4539 - 0.3113}{2} = 0.0713 \text{ m}$$

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f}$$

$$\mathrm{Fr}^2 = \frac{V^2}{gh} = \frac{Q^2}{gb^2h^3} = \frac{0.2610}{h^3}$$

$$S_0 = 0.004$$

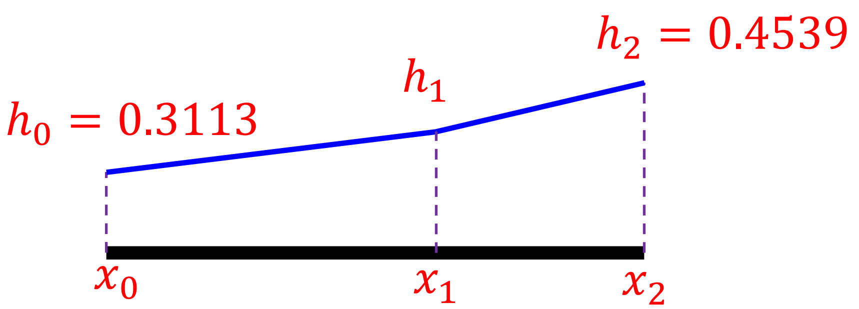
$$(*) \quad Q = \frac{1}{n}\frac{bh^{5/3}}{(1 + 2h/b)^{2/3}} S_f^{1/2}$$

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 (1 + 2h/b)^{4/3} = 1.239 \times 10^{-3} \frac{(1 + 0.8h)^{4/3}}{h^{10/3}}$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.2610}{h^3}}{0.004 - 1.239 \times 10^{-3} \dfrac{(1 + 0.8h)^{4/3}}{h^{10/3}}}$$

(c) Use two steps in the gradually-varied-flow equation to estimate the distance from the end of the narrow section to the downstream hydraulic jump.

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.2610}{h^3}}{0.004 - 1.239 \times 10^{-3} \dfrac{(1+0.8h)^{4/3}}{h^{10/3}}}$$



$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

$$\Delta h = 0.0713$$

i	h_i	x_i	h_{mid}	$\left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}}$	Δx
0	0.3113	0			
			0.34695	96.27	6.864
1	0.3826	6.864			
			0.41825	87.70	6.253
2	0.4539	**13.12**			

Example

A long rectangular channel of width 2.2 m, streamwise slope 1:100 and Chézy coefficient 80 $m^{1/2}\ s^{-1}$ carries a discharge of 4.5 $m^3\ s^{-1}$.

(a) Find the normal depth and critical depth and show that the slope is steep at this discharge.

(b) An undershot sluice gate causes a hydraulic transition in this flow. The depth of parallel flow downstream of the gate is 0.35 m. Find the depth immediately upstream of the gate and sketch the flow.

(c) Using 2 steps in the gradually-varied-flow equation, find the distance between the gate and the hydraulic jump.

A long rectangular channel of width 2.2 m, streamwise slope 1:100 and Chézy coefficient 80 $m^{1/2}\,s^{-1}$ carries a discharge of 4.5 $m^3\,s^{-1}$.

(a) Find the normal depth and critical depth and show that the slope is steep at this discharge.

$b = 2.2\ \mathrm{m} \qquad S_0 = 0.01 \qquad C = 80\ \mathrm{m^{1/2}\,s^{-1}} \qquad Q = 4.5\ \mathrm{m^3\,s^{-1}}$

Normal depth:

$$Q = VA \qquad V = CR_h^{1/2}S_0^{1/2} \qquad A = bh \qquad R_h = \frac{bh}{b+2h} = \frac{h}{1+2h/b}$$

$$P = b + 2h$$

$$Q = C\left(\frac{h}{1+2h/b}\right)^{1/2} S_0^{1/2} bh$$

$$\frac{Q}{Cb\sqrt{S_0}} = \frac{h^{3/2}}{(1+2h/b)^{1/2}} \qquad (*)$$

$$h = \left(\frac{Q}{Cb\sqrt{S_0}}\right)^{2/3}(1+2h/b)^{1/3} \qquad h = 0.4028(1+0.9091h)^{1/3} \qquad h_n = \mathbf{0.4518\ m}$$

Critical depth:

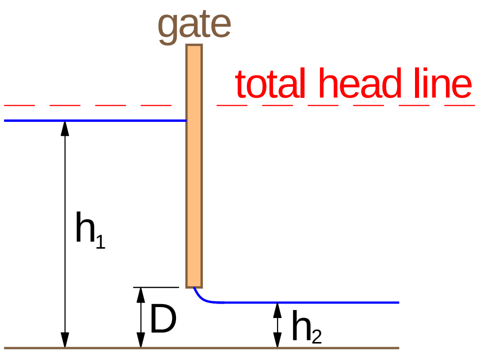
$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad q = \frac{Q}{b} = 2.045\ \mathrm{m^2\,s^{-1}} \qquad h_c = \mathbf{0.7526\ m} \qquad h_n < h_c$$

steep

(b) An undershot sluice gate causes a hydraulic transition in this flow. The depth of parallel flow downstream of the gate is 0.35 m. Find the depth immediately upstream of the gate and sketch the flow.

$Q = 4.5\ \mathrm{m^3\ s^{-1}}$ $b = 2.2\ \mathrm{m}$

$h_2 = 0.35\ \mathrm{m}$



$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$

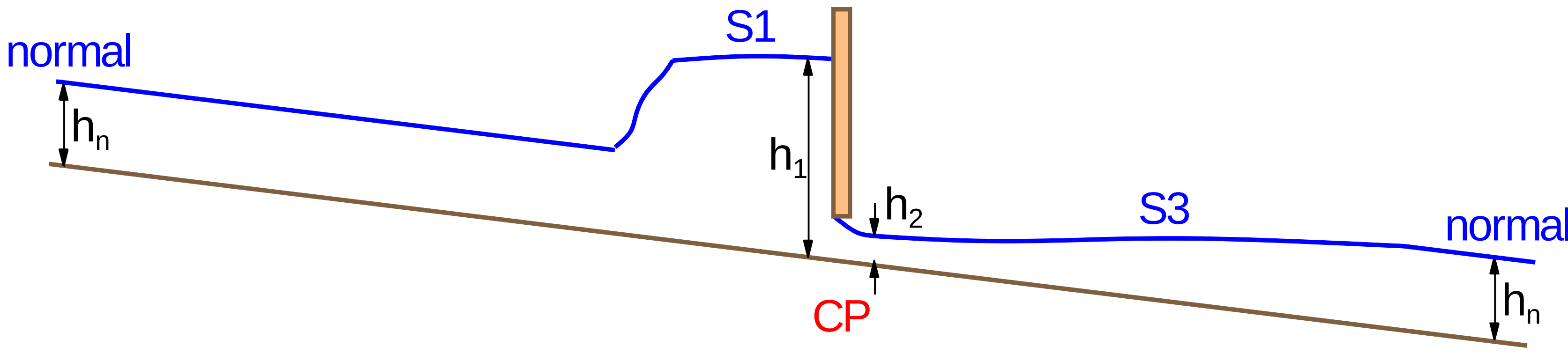
$$h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$$

$$h_1 + \frac{0.2132}{h_1^2} = 2.090$$

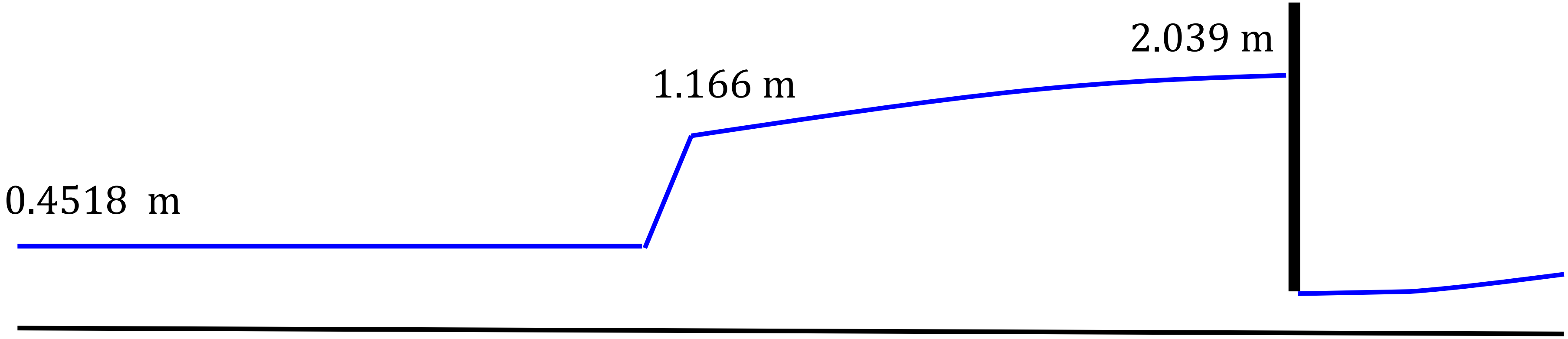
$$h_1 = 2.090 - \frac{0.2132}{h_1^2}$$

$$h_1 = \mathbf{2.039\ m}$$

... sketch the flow.



(c) Using 2 steps in the gradually-varied-flow equation, find the distance between the gate and the hydraulic jump.



$$h_n = 0.4518\ \mathrm{m}$$

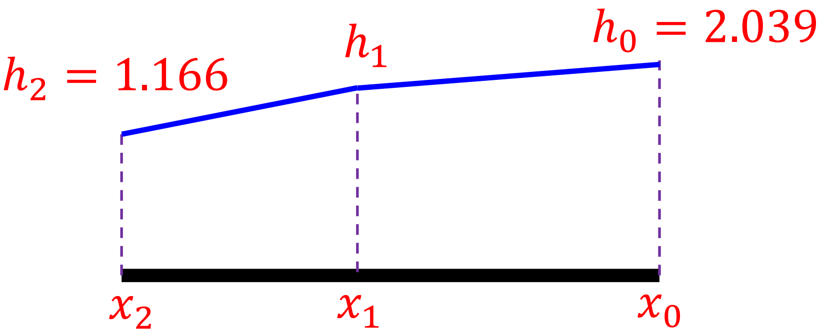
$$V_n = \frac{Q}{bh_n} = 4.527\ \mathrm{m\ s^{-1}}$$

$$\mathrm{Fr}_n = \frac{V_n}{\sqrt{gh_n}} = 2.150$$

$$h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_n^2}\right)$$

$$= 1.166\ \mathrm{m}$$

(d) Use 2 steps in the gradually-varied-flow equation to determine how far upstream of the sluice a hydraulic jump will occur.



$$\Delta h = \frac{1.166 - 2.039}{2} = -0.4365 \text{ m}$$

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f}$$

$$\mathrm{Fr}^2 = \frac{V^2}{gh} = \frac{Q^2}{gb^2h^3} = \frac{0.4263}{h^3}$$

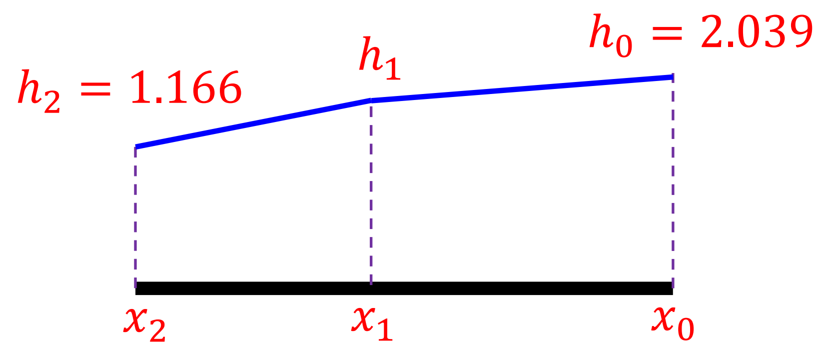
$$S_0 = 0.01$$

(*) $$\frac{Q}{Cb\sqrt{S_f}} = \frac{h^{3/2}}{(1 + 2h/b)^{1/2}}$$

$$S_f = \left(\frac{Q}{Cb}\right)^2 \frac{(1 + 2h/b)}{h^3} = 6.537 \times 10^{-4} \frac{(1 + 0.9091h)}{h^3}$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.4263}{h^3}}{0.01 - 6.537 \times 10^{-4} \dfrac{1 + 0.9091h}{h^3}}$$

(d) Use 2 steps in the gradually-varied-flow equation to determine how far upstream of the sluice a hydraulic jump will occur.



$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.4263}{h^3}}{0.01 - 6.537 \times 10^{-4}\dfrac{1 + 0.9091h}{h^3}}$$

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

$$\Delta h = -0.4365$$

i	h_i	x_i	h_{mid}	$\left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}}$	Δx
0	2.039	0			
			1.821	95.69	- 41.77
1	1.6025	- 41.77			
			1.384	88.87	- 38.79
2	1.166	**- 80.56**			