EXAMPLES (SEDIMENT TRANSPORT) AUTUMN 2023

Q1.
Using Cheng's formula estimate the settling velocity of a sand particle of diameter 1 mm in:
(a) air;
(b) water.

Q2.
Find the critical Shields parameter τ^*_{crit} and critical absolute stress τ_{crit} for a sand particle of diameter 1 mm in water.

Q3.
Assuming a friction coefficient c_f = 0.005, estimate the velocities V at which:
(a) incipient motion;
(b) incipient suspended load;
occur in water for sand particles of density 2650 kg m^{-3} and diameter 1 mm.

Q4.
A sluice gate is lowered into a wide channel carrying a discharge of 0.9 m^3 s^{-1} per metre width. The bed of the channel is coarse gravel with particle diameter 60 mm and density 2650 kg m^{-3}. The critical Shields parameter is 0.056 and the bed friction coefficient is 0.01. The particles of gravel have settling velocity 1.1 m s^{-1}. Initially the bed of the channel under the sluice is horizontal and the depth of flow just upstream of the gate is 2.5 m.
(a) Show that the bed is stationary upstream of the gate.
(b) Determine the initial water depth just downstream of the gate. Show that the bed is mobile here.
(c) Assuming that the downstream water level is set by the gate and the discharge remains constant, find the final depth of scour and the final depths of flow upstream and downstream of the gate.
(d) Establish whether suspended load is likely to make any contribution to the initial scour.

Q5.
A long rectangular channel contracts smoothly from 5 m width to 3 m width for a region near its midpoint. The bed is composed of gravel with a median grain size of 10 mm and density 2650 kg m^{-3}. The friction coefficient c_f is 0.01 and the critical Shields parameter is 0.05. If the flow rate is 5 m^3 s^{-1} and the upstream depth is 1 m:
(a) show that the bed is stationary in the 5 m width and mobile in the 3 m width by assuming that the bed is initially flat;
(b) determine the depth of the scour hole which results just after the contraction.

Q6.

A river of width 12 m and slope 0.003, carrying a maximum discharge of 200 $m^3 s^{-1}$, is to be stabilised by using an armour layer of stones of density 2650 kg m^{-3}. Assuming a critical Shields parameter of 0.056, estimate the minimum size of stone that should be used and the corresponding river depth at maximum discharge.

Q7.

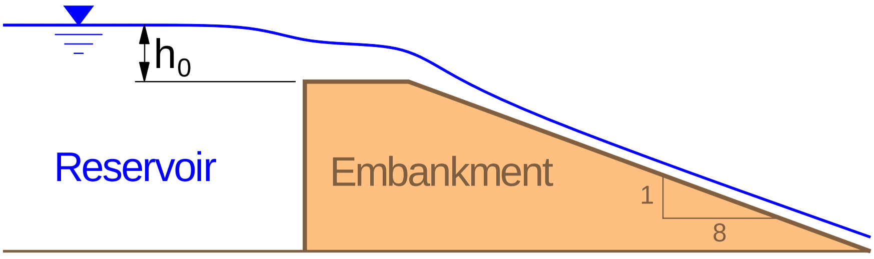
An embankment forms the boundary to a reservoir. After periods of heavy rain water may spill over the top of the embankment.

(a) If the freeboard (h_0 in the figure below) is 150 mm, estimate the overtopping flow rate (per metre width of embankment) and the depth of flow over the crest of the embankment, stating any assumptions made.

(b) The downstream slope of the embankment is 1 in 8 and its surface has a Manning's roughness coefficient $n = 0.013 \text{ m}^{-1/3}$ s. On the assumption that the flow rapidly approaches normal flow find the depth, velocity and bed shear stress on this slope.

(c) The downstream slope of the embankment is covered with fine gravel (diameter 2 mm and density 2650 kg m^{-3}). Use Soulsby's correlation to determine whether this will be eroded by the overtopping flow.

(d) Suggest how the downstream slope may be protected from erosion in occasional overtopping events.



Q8.

A wide channel of slope 1:800 has a gravel bed with $d_{50} = 3$ mm. The discharge is 4 $m^3 s^{-1}$ per metre width. The density of the gravel is 2650 kg m^{-3}.

(a) Estimate Manning's n using Strickler's formula.

(b) Find the depth of flow; (assume normal flow).

(c) Find the bed shear stress.

(d) Show that the bed is mobile and calculate the bed-load flux (per metre width) using (i) Meyer-Peter and Müller; (ii) Van Rijn models.

(e) Determine whether suspended load occurs.

Q9.
A wide channel of slope 1:800 has a fine sandy bed with $d_{50} = 0.5$ mm. The discharge is 5 m^3 s^{-1} per metre width. The specific gravity of the bed material is 2.65.

(a) Estimate Manning's n using Strickler's formula.
(b) Find the depth of flow; (assume normal flow).
(c) Find the bed shear stress.
(d) Show that the bed is mobile and calculate the bed-load flux (per metre width) using: (i) Meyer-Peter and Müller; (ii) Nielsen models.
(e) Find the particle settling velocity and show that suspended load will occur.
(f) Estimate the suspended-load flux (per metre width), explaining your method and stating any assumptions made.

Q10. (Exam 2015)
A long, wide channel of slope 1:1500 has a fine gravel bed, with particle diameter $d = 2.5$ mm and carries a flow of 3.5 m^3 s^{-1} per metre width. The specific gravity of the bed material is 2.65. Take the kinematic viscosity of water as 1.0×10^{-6} m^2 s^{-1}.

(a) Estimate Manning's n.
(b) Find the depth of flow and average velocity.
(c) Find the bed shear stress and show that the bed is mobile.
(d) Calculate the bed-load flux (per m width) using the Meyer-Peter and Müller model.
(e) Find the settling velocity and determine whether suspended load is likely to occur.
(f) Explain how you would calculate suspended-load flux if it did occur.

Data.
Strickler's equation:

$$n = \frac{d^{1/6}}{21.1} \qquad \text{(for } d \text{ in m, } n \text{ in m}^{-1/3}\text{ s)}$$

Soulsby correlation for critical Shields stress:

$$\tau^*_{\text{crit}} = \frac{0.30}{1 + 1.2d^*} + 0.055\,[1 - \exp(-0.020d^*)]$$

Meyer-Peter and Müller formula:

$$q^* = 8(\tau^* - \tau^*_{\text{crit}})^{3/2}$$

Cheng's formula:

$$w_s^* = [(25 + 1.2d^{*2})^{1/2} - 5]^{3/2}$$

Non-dimensional groups:

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} \qquad \tau^* = \frac{\tau_b}{(\rho_s - \rho)gd} \qquad q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}$$

$$w_s^* = \frac{w_s d}{\nu} \qquad s = \frac{\rho_s}{\rho}$$

where d is particle diameter, ρ and ρ_s are densities of water and sediment respectively, ν is kinematic viscosity, g is the acceleration due to gravity, τ_b is bed shear stress, q_b is bed-load flux per unit width, w_s is settling velocity.

Q11.

For small spherical particles the drag coefficient is given by:

$$c_D = \frac{24}{\text{Re}} \qquad \text{where} \qquad \text{Re} \equiv \frac{w_s d}{\nu}$$

Derive the following expression for settling velocity:

$$w_s = \frac{1}{18}\frac{(s-1)gd^2}{\nu}$$

Q12.

Show that, for the velocity profile

$$U(z) = \frac{u_\tau}{\kappa}\ln(33\frac{z}{k_s})$$

the velocity averaged over a channel depth h is

$$V = \frac{u_\tau}{\kappa}\ln(\frac{12h}{k_s})$$

Q13.

The mean-velocity profile in a rough-walled turbulent boundary layer is

$$U(z) = \frac{u_\tau}{\kappa}\ln(33\frac{z}{k_s})$$

where U is the mean velocity at distance z from the boundary, u_τ is the friction velocity, κ (= 0.41) is von Kármán’s constant and k_s is the roughness height.

(a) Define the friction velocity in terms of the bed shear stress.

(b) Assuming a linear shear-stress variation show that the eddy-viscosity profile is parabolic over the water depth.

(c) Show how the assumption of Fick’s law produces the following equation for sediment concentration C:

$$u_\tau z(1 - z/h)\frac{\mathrm{d}C}{\mathrm{d}z} + w_s C = 0$$

where w_s is settling velocity.

(d) Sketch the vertical profiles of velocity, eddy-viscosity and concentration.

(e) Describe the physical processes involved in bed-load sediment transport and explain how suspended load is calculated.

Q14.
Show that the solution of

$$-K\frac{\mathrm{d}C}{\mathrm{d}z} = w_s C \qquad \text{where} \qquad K = \kappa u_\tau z(1 - z/h)$$

has the form

$$\frac{C}{C_{\text{ref}}} = \left(\frac{h/z - 1}{h/z_{\text{ref}} - 1}\right)^{\frac{w_s}{\kappa u_\tau}}$$

Q15. (Exam 2014)

(a) Explain the physical basis for the following equation for the suspended-load concentration profile in a sediment-carrying flow:

$$-K\frac{\mathrm{d}C}{\mathrm{d}z} = w_s C \qquad (*)$$

Here, C is the sediment concentration (volume of sediment per volume of water), K is the eddy diffusivity, z is the vertical distance above the bed and w_s is the sediment settling velocity.

(b) The vertical profile of K is

$$K = \kappa u_\tau z(1 - z/h)$$

where κ is a universal constant with value 0.41, k_s is the roughness length, u_τ is the friction velocity and h is the channel depth. Use this, together with Equation (*) to derive the Rouse profile for concentration:

$$\frac{C}{C_{\text{ref}}} = \left(\frac{h/z - 1}{h/z_{\text{ref}} - 1}\right)^{\frac{w_s}{\kappa u_\tau}}$$

(c) The vertical profile of mean velocity U in a rough-walled turbulent flow is:

$$U(z) = \frac{u_\tau}{\kappa}\ln(33\frac{z}{k_s})$$

Use numerical integration by the trapezium rule with 3 intervals (4 ordinates) to calculate the suspended-load sediment flux in a channel using the following data:

channel width: $b = 5$ m;
flow depth: $h = 1.5$ m;
friction velocity: $u_\tau = 0.2$ m s^{-1};
settling velocity: $w_s = 0.03$ m s^{-1};
roughness length: $k_s = 0.001$ m;
reference concentration: $C_{\text{ref}} = 0.65$;
reference height: $z_{\text{ref}} = 0.001$ m.

Practical Exercises

E1

Undertake a review of the journal literature to expand the table of bed-load sediment transport models; (there are, literally, hundreds!). For the dimensionally-consistent ones try to rewrite the expression for the bed flux in the form:

$$q^* = f(\tau^*, d^*)$$

where

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}} \qquad \tau^* = \frac{\tau_b}{\rho(s-1)gd} \qquad d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

There are many journals available online from the University library's web pages. Good starting points are:

ASCE Journal of Hydraulic Engineering
Coastal Engineering
IAHR Journal of Hydraulic Research

from which you can branch out following their references.

E2

Construct spreadsheets or (better) computer programs to calculate:

(i) settling velocity w_s using Cheng's formula;
(ii) the critical Shields parameter for threshold of motion;
(iii) bed-load sediment flux according to the formulae in Section 3;
(iv) the Rouse profile for suspended load;
(v) suspended sediment flux by numerical integration.