Section 1.2

Example.
The discharge in a channel with bottom width 3 m is 12 m^3 s^{-1}. If Manning's n is 0.013 $m^{-1/3}$ s and the streamwise slope is 1 in 200, find the normal depth if:
(a) the channel has vertical sides (i.e. rectangular channel);
(b) the channel is trapezoidal with side slopes 2H:1V.

$b = 3$ m (base width)
$Q = 12\ \text{m}^3\ \text{s}^{-1}$
$n = 0.013\ \text{m}^{-1/3}\ \text{s}$
$S = 0.005$

(a)
Discharge:

$$Q = VA$$

where, in normal flow,

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b+2h} = \frac{h}{1+2h/b}$$

Hence,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1+2h/b)^{2/3}} S^{1/2}$$

Rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1+2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 0.8316\ (1+2h/3)^{2/5}$$

Iteration (from, e.g., $h = 0.8316$) gives

$$h_n = 1.024\ \text{m}$$

Answer: 1.02 m.

(b) Geometry: trapezoidal cross-section with base width b, surface width $b + 2 \times (2h)$ and two sloping side lengths $\sqrt{h^2 + (2h)^2} = h\sqrt{5}$.

Area and wetted perimeter:

$$A = \tfrac{1}{2}(b \quad + b + 4h)h \quad = h(b+2h) \quad = hb(1+2h/b)$$

$$P = b + 2h\sqrt{5}$$

Hydraulic radius:

$$R_h \equiv \frac{A}{P} \quad = \frac{h(b+2h)}{b+2h\sqrt{5}} \quad = h\left(\frac{1+2h/b}{1+2\sqrt{5}h/b}\right)$$

Discharge:

$$Q = VA \quad = \frac{1}{n} R_h^{2/3} S^{1/2} A$$

Hence,

$$Q = \frac{1}{n} h^{2/3} \left(\frac{1+2h/b}{1+2\sqrt{5}h/b}\right)^{2/3} S^{1/2} hb(1+2h/b)$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = h^{5/3} \frac{(1+2h/b)^{5/3}}{(1+2\sqrt{5}h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} \frac{(1+2\sqrt{5}h/b)^{2/5}}{1+2h/b}$$

Here, with lengths in metres,

$$h = 0.8316 \frac{(1+1.491h)^{2/5}}{1+2h/3}$$

Iteration (from, e.g., $h = 0.8316$) gives

$$h_n = 0.7487 \text{ m}$$

Answer: 0.749 m.

Section 1.4

Example.
The discharge in a rectangular channel of width 6 m with Manning's $n = 0.012 \text{ m}^{-1/3}$ s is 24 $\text{m}^3 \text{ s}^{-1}$. If the streamwise slope is 1 in 200 find:
(a) the normal depth;
(b) the Froude number at the normal depth;
(c) the critical depth.

State whether the normal flow is subcritical or supercritical.

$b = 6 \text{ m}$
$n = 0.012 \text{ m}^{-1/3} \text{ s}$
$Q = 24 \text{ m}^3 \text{ s}^{-1}$
$S = 0.005$

(a)
Discharge:

$$Q = VA$$

where, in normal flow,

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1 + 2h/b)^{2/3}} S^{1/2}$$

or, rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}} \right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 0.7926\ (1 + h/3)^{2/5}$$

Iteration (from, e.g., $h = 0.7926$) gives

$$h_n = 0.8783 \text{ m}$$

Answer: 0.878 m.

(b) At the normal depth, $h = 0.8783$ m:

$$V = \frac{Q}{A} = \frac{24}{6 \times 0.8783} = 4.554 \text{ m s}^{-1}$$

$$\text{Fr} \equiv \frac{V}{\sqrt{gh}} = \frac{4.554}{\sqrt{9.81 \times 0.8783}} = 1.551$$

Answer: 1.55.

(c) The critical depth is that depth (at the given flow rate) for which $\mathrm{Fr} = 1$. It is *not* normal flow, and does not depend on the slope S or the roughness n.

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}}$$

where

$$V = \frac{Q}{A} \qquad \text{(in general)}$$

or $$V = \frac{Q}{bh} = \frac{q}{h} \qquad \text{(for a rectangular channel; } q \text{ is the flow per unit width)}$$

Hence, for a rectangular channel,

$$\mathrm{Fr}^2 = \frac{(q/h)^2}{gh} = \frac{q^2}{gh^3}$$

For critical flow, $\mathrm{Fr} = 1$ and so

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Here, the flow per unit width is $q = 24/6 = 4\ \mathrm{m^2\ s^{-1}}$, so that

$$h_c = \left(\frac{4^2}{9.81}\right)^{1/3} = 1.177\ \mathrm{m}$$

Answer: 1.18 m.

The normal depth is supercritical because, when $h = h_n$, then $\mathrm{Fr} > 1$ (part (b)).

Alternatively (and often more conveniently), the normal depth here is supercritical because $h_n < h_c$; so speed V is larger, and depth h is smaller in normal flow than critical flow, so that $\mathrm{Fr} \equiv V/\sqrt{gh}$ must be greater than 1.

Section 2.2

Example.
A 3-m wide channel carries a total discharge of 12 $m^3 s^{-1}$. Calculate:
(a) the critical depth;
(b) the minimum specific energy;
(c) the alternate depths when $E = 4$ m.

$b = 3 \text{ m}$
$Q = 12 \text{ m}^3 \text{ s}^{-1}$

(a)
Discharge per unit width:

$$q = \frac{Q}{b} \quad = \frac{12}{3} \quad = 4 \text{ m}^2 \text{ s}^{-1}$$

Then, for a rectangular channel:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{4^2}{9.81}\right)^{1/3} \quad = 1.177 \text{ m}$$

Answer: 1.18 m.

(b) For a rectangular channel,

$$E_c = \frac{3}{2} h_c \quad = \frac{3}{2} \times 1.177 \quad = 1.766 \text{ m}$$

Answer: 1.77 m.

(c) As $E > E_c$, there are two possible depths for a given specific energy.

$$E \equiv h + \frac{V^2}{2g} \qquad \text{where} \qquad V = \frac{Q}{A} \quad = \frac{q}{h} \qquad \text{(for a rectangular channel)}$$

$$\Rightarrow \qquad E \equiv h + \frac{q^2}{2gh^2}$$

Substituting values in metre-second units:

$$4 \equiv h + \frac{0.8155}{h^2}$$

For the *subcritical* (slow, deep) solution, the first term, associated with potential energy, dominates, so rearrange as:

$$h = 4 - \frac{0.8155}{h^2}$$

Iteration (from, e.g., h =4) gives $h = 3.948$ m.

For the *supercritical* (fast, shallow) solution, the second term, associated with kinetic energy, dominates, so rearrange as:

$$h = \sqrt{\frac{0.8155}{4 - h}}$$

Iteration (from, e.g., $h = 0$) gives $h = 0.4814$ m.

Answer: 3.95 m and 0.481 m.

Section 2.3.1

Example. (Exam 2020)

(a) Define:

(i) specific energy

(ii) Froude number

for open-channel flow. What is special about these quantities in critical conditions?

A long, wide channel has a slope of 1:1000, a Manning's n of 0.015 m$^{-1/3}$ s and a discharge of 3 m^3 s^{-1} per metre width.

(b) Calculate the normal and critical depths.

(c) In a region of the channel the bed is raised by a height of 0.8 m over a length sufficient for the flow to be parallel to the bed over this length. Determine the depths upstream, downstream and over the raised bed, ignoring frictional losses. Sketch the key features of the flow, indicating *all* hydraulic transitions caused by the bed rise.

(d) In the same channel, the bed is lowered by 0.8 m from its original level. Determine the depths upstream, downstream and over the lowered bed, ignoring frictional losses. Sketch the flow.

(a)

(i) *Specific energy* is the head relative to the local bed of the channel:

$$E = h + \frac{V^2}{2g}$$

(ii) The Froude number is

$$\text{Fr} = \frac{V}{\sqrt{g\bar{h}}}$$

In *critical conditions* Fr = 1 and the specific energy is the minimum for that discharge.

(b)

$S = 0.001$

$n = 0.015 \text{ m}^{-1/3} \text{ s}$

$q = 3 \text{ m}^2 \text{ s}^{-1}$

<u>Normal depth:</u>

Discharge per unit width:

$$q = Vh, \quad \text{where} \quad V = \frac{1}{n} R_h^{2/3} S^{1/2} \quad \text{(Manning)}, \qquad R_h = h \quad \text{(“wide” channel)}$$

$$\Rightarrow \qquad q = \frac{1}{n} h^{2/3} S^{1/2} h$$

$$\Rightarrow \qquad q = \frac{h^{5/3}\sqrt{S}}{n}$$

$$\Rightarrow \qquad h_n = \left(\frac{nq}{\sqrt{S}}\right)^{3/5} \quad = \left(\frac{0.015 \times 3}{\sqrt{0.001}}\right)^{3/5} \quad = 1.236 \text{ m}$$

Critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{3^2}{9.81}\right)^{1/3} \quad = 0.9717 \text{ m}$$

Answer: normal depth = 1.24 m; critical depth = 0.972 m.

(c) To determine the type of behaviour over the raised bed, compare the total head under critical conditions (the minimum energy necessary to get over the weir at this flow rate) with that available in the approach flow.

Critical

$$h_c = 0.9717 \text{ m}$$

$$E_c = \frac{3}{2}h_c \quad = 1.458 \text{ m}$$

$$z_b = 0.8 \text{ m}$$

$$H_c = z_b + E_c \quad = 2.258 \text{ m}$$

Approach Flow

Because the channel is described as "long" it will have sufficient fetch to develop normal flow; hence the approach-flow head is that for the normal depth ($h = 1.236$ m):

$$H_a = E_a \quad = h_n + \frac{V_n^2}{2g}$$

$$= h_n + \frac{q^2}{2gh_n^2}$$

$$= 1.236 + \frac{3^2}{2 \times 9.81 \times 1.236^2}$$

$$= 1.536 \text{ m}$$

At the normal depth the available head (H_a) is less than the minimum required to get over the bed rise (H_c). Hence the water depth must increase upstream ("back up"), to raise the head immediately upstream. Thus:

- critical conditions *do* occur;
- the total head in the vicinity is the critical head ($H = H_c = 2.258$ m).

Over the raised bed there is a hydraulic transition, so the depth over this is critical: $h = h_c = 0.9717$ m.

Just up- or downstream,

$$H = E = h + \frac{V^2}{2g} \qquad \text{where} \qquad V = \frac{q}{h}$$

$$\Rightarrow \qquad H = h + \frac{q^2}{2gh^2}$$

$$\Rightarrow \qquad 2.258 = h + \frac{0.4587}{h^2}$$

Upstream, rearrange for the deep, subcritical, solution:

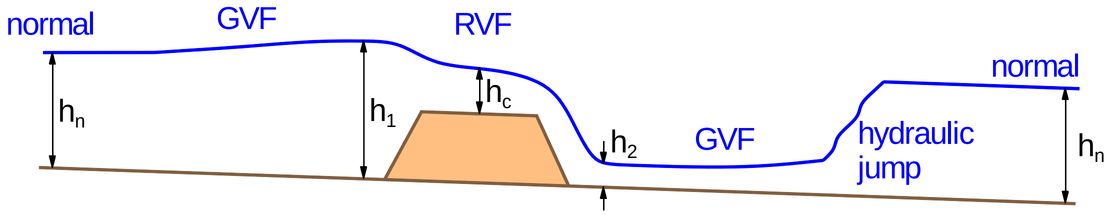
$$h = 2.258 - \frac{0.4587}{h^2}$$

Iteration (from, e.g., $h = 2.258$) gives $h = 2.160$ m.

Downstream, rearrange for the shallow, supercritical solution:

$$h = \sqrt{\frac{0.4587}{2.258 - h}}$$

Iteration (from, e.g., $h = 0$) gives $h = 0.5127$ m.



Since the preferred (i.e. normal) depth is subcritical, there must be a downstream hydraulic jump. (A quick calculation shows that the upstream depth for this jump is greater than h_2, so there is indeed a length of GVF between the area of bed rise and the jump.)

Answer: depths upstream, over, downstream of the raised bed: 2.16 m, 0.972 m, 0.513 m.

(d) The flow does not require additional energy to pass a depressed section; hence, the total head throughout is that supplied by the approach flow ($H = H_a = 1.536$ m) and the flow remains subcritical. The depths just upstream and downstream of the lowered section are those in the approach flow; i.e. normal depth.

As bed height z_b decreases, specific energy E must increase to maintain the same total head. In the lowered section:

$$H = z_b + E$$

$$\Rightarrow \qquad 1.536 = -0.8 + E$$

$$\Rightarrow \qquad E = 2.336 \text{ m}$$

Then

$$E = h + \frac{V^2}{2g} \qquad \text{where} \qquad V = \frac{q}{h}$$

$$\Rightarrow \qquad E = h + \frac{q^2}{2gh^2}$$

$$\Rightarrow \qquad 2.336 = h + \frac{0.4587}{h^2}$$

As we require the subcritical solution, rearrange as

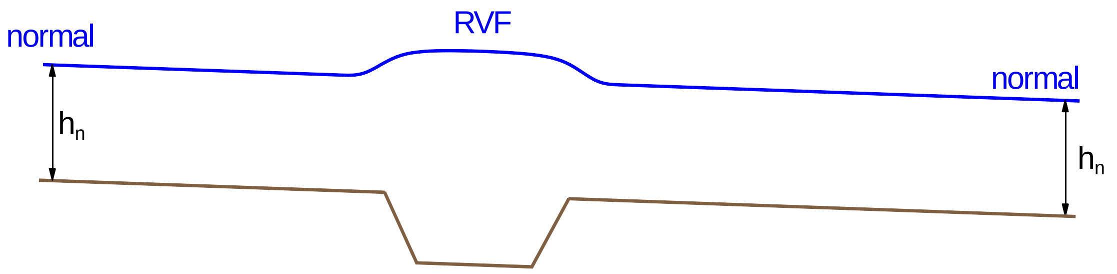
$$h = 2.336 - \frac{0.4587}{h^2}$$

Iteration (from, e.g., $h = 2.336$) gives $h = 2.245$ m.

(Note that this is the depth of the water column. The actual surface level here is

$$z_s = -0.8 + h \quad = 1.445 \text{ m}$$

so the overall water level also rises in this section.)



Answer: depths upstream, within, downstream of the lowered section: 1.24 m, 2.24 m, 1.24 m.

Section 2.3.1

Example.
A long channel of rectangular cross-section with width 3.5 m and streamwise slope 1 in 800 carries a discharge of 15 $m^3 s^{-1}$. Manning's n may be taken as 0.016 $m^{-1/3}$ s. A broad-crested weir of height 0.7 m is constructed at the centre of the channel. Determine:
(a) the depth far upstream of the weir;
(b) the depth just upstream of the weir;
(c) whether or not a region of supercritical gradually-varied flow exists downstream of the weir.

$b = 3.5$ m
$S = 0.00125$
$Q = 15$ m^3 s^{-1}
$n = 0.016$ m$^{-1/3}$ s
$z_{\text{weir}} = 0.7$ m

(a) The depth far upstream is normal since the channel is described as "long". For normal flow in a rectangular channel:

$$Q = VA$$

where:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad A = bh \qquad R_h = \frac{bh}{b+2h} = \frac{h}{1+2h/b}$$

Hence,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1+2h/b)^{2/3}} S^{1/2}$$

or, rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1+2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 1.488\,(1 + 0.5714h)^{2/5}$$

Iteration (from, e.g., $h = 1.488$) gives

$$h = 2.023 \text{ m}$$

Answer: 2.02 m.

(b) To establish depths near the weir we need to know the flow behaviour at the weir. Compare the energy in the approach flow with that under critical conditions.

<u>Approach flow</u>

$h = 2.023$ m (from part (a))

$$V = \frac{Q}{A} = \frac{Q}{bh} = \frac{15}{3.5 \times 2.023} = 2.118 \text{ m s}^{-1}$$

Specific energy in the approach flow:

$$E_a = h + \frac{V^2}{2g} = 2.023 + \frac{2.118^2}{2 \times 9.81} = 2.252 \text{ m}$$

Referring heads to the undisturbed bed near the weir:

$$H_a = E_a = 2.252 \text{ m}$$

<u>Critical conditions</u>

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad q = \frac{Q}{b} = \frac{15}{3.5} = 4.286 \text{ m}^2 \text{ s}^{-1}$$

$$\Rightarrow \qquad h_c = \left(\frac{4.286^2}{9.81}\right)^{1/3} = 1.233 \text{ m}$$

$$E_c = \tfrac{3}{2} h_c = \tfrac{3}{2} \times 1.233 = 1.850 \text{ m}$$

$$H_c = z_{\text{weir}} + E_c = 0.7 + 1.850 = 2.550 \text{ m}$$

Since the head required to flow over the weir ($H_c = 2.550$ m) exceeds that in the approach flow ($H_a = 2.252$ m), the depth just upstream of the weir must increase and the flow back up. The total head at any position in the vicinity of the weir is $H = H_c = 2.550$ m.

Just upstream and downstream of the weir (i.e. at undisturbed bed level):

$$H = E = h + \frac{V^2}{2g} \qquad V = \frac{Q}{bh}$$

$$\Rightarrow \qquad H = h + \frac{Q^2}{2gb^2h^2}$$

$$\Rightarrow \qquad 2.550 = h + \frac{0.9362}{h^2} \qquad (*)$$

The depth just upstream is the deep, subcritical solution. Hence, rearrange as

$$h = 2.550 - \frac{0.9362}{h^2}$$

Iteration (from, e.g., $h = 2.550$) gives

$$h = 2.385 \text{ m}$$

Answer: 2.39 m.

(c) Since the normal flow is subcritical, the flow must return to it via a hydraulic jump on the downstream side of the weir.

If the flow in the vicinity of the weir is unaffected by the hydraulic jump the flow goes smoothly supercritical on the downstream side, with total head $H = 2.550$ m (equation (*)). Rearranging to get an iterative formula for the supercritical solution:

$$h = \sqrt{\frac{0.9362}{2.550 - h}}$$

Iteration (from, e.g., $h = 0$) gives

$$h = 0.7141 \text{ m}$$

Denote by subscripts A and B respectively the conditions upstream and downstream of the hydraulic jump. On the downstream side conditions may be assumed normal, since the channel is "long" and hence there is sufficient fetch to develop the preferred depth:

$$h_B = 2.023 \text{ m}$$

$$V_B = 2.118 \text{ m s}^{-1} \qquad \text{(from part (b))}$$

$$\text{Fr}_B = \frac{V_B}{\sqrt{gh_B}} \quad = \frac{2.118}{\sqrt{9.81 \times 2.023}} \quad = 0.4754$$

Hence, from the hydraulic-jump relation for the sequent depths:

$$h_A = \frac{h_B}{2}(-1 + \sqrt{1 + 8\text{Fr}_B^2}) \quad = \frac{2.023}{2}(-1 + \sqrt{1 + 8 \times 0.4754^2}) \quad = 0.6835 \text{ m}$$

Any gradually-varied supercritical flow downstream of the weir would increase in depth until a hydraulic jump occurred (see the lectures on GVF). Since the depth downstream of the weir is already greater than any sequent depth upstream of the hydraulic jump, no such increasing-depth GVF is possible and the hydraulic jump must actually occur at (or just before) the downstream end of the weir.

Section 2.3.1

Example.
A reservoir discharge is controlled by a weir of width 8 m and discharge coefficient 0.9.
(a) Calculate the flow rate over the weir when the freeboard is 0.65 m.
(b) Assuming negligible inflow and a constant plan area of 1.5 km^2 for the reservoir, calculate the time in hours taken to reduce the level of the reservoir by 0.4 m.

(a) The weir controls the flow, so there must be critical flow over the weir and the freeboard corresponds to critical head:

$$H_c = \frac{3}{2}h_c \qquad = h_0$$

$$\Rightarrow \qquad \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} = h_0$$

$$\Rightarrow \qquad q = \sqrt{g\left(\frac{2}{3}h_0\right)^3}$$

For the full width $b = 8$ m, and with a discharge coefficient $c_d = 0.9$,

$$Q = c_d q b \qquad = c_d b\sqrt{g\left(\frac{2}{3}h_0\right)^3} \qquad = 12.28 h_0^{3/2}$$

When $h_0 = 0.65$ m,

$$Q = 6.435 \text{ m}^3\text{ s}^{-1}$$

Answer: 6.44 m^3 s^{-1}.

(b) By continuity:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = Q_{\text{in}} - Q_{\text{out}}$$

Here, $\mathrm{d}V = A_{ws}\,\mathrm{d}h$, $Q_{\text{in}} = 0$ and Q_{out} can be written as a function of h (dropping subscript 0) as above. Hence,

$$A_{ws}\frac{\mathrm{d}h}{\mathrm{d}t} = -12.28\,h^{3/2}$$

Writing $A_{ws} = 1.5\text{ km}^2 = 1.5\times10^6\text{ m}^2$ and separating variables:

$$-\frac{1.5\times10^6}{12.28}h^{-3/2}\mathrm{d}h = \mathrm{d}t$$

Integrating between $t = 0$ (where $h = 0.65$ m) and $t = T$ (where $h = 0.25$ m):

$$-122100\int_{0.65}^{0.25} h^{-3/2}\mathrm{d}h = \int_0^T \mathrm{d}t$$

$$\Rightarrow \qquad -122100\left[\frac{h^{-1/2}}{-1/2}\right]_{0.65}^{0.25} = T$$

$$\Rightarrow \qquad 244200\left(\frac{1}{\sqrt{0.25}} - \frac{1}{\sqrt{0.65}}\right) = T$$

$$\Rightarrow \qquad T = 185500 \text{ s} \quad = 51.53 \text{ hr}$$

Answer: 51.5 hr.

Section 2.3.2

Example.

A venturi flume is placed near the middle of a long rectangular channel with Manning's $n = 0.012\ \text{m}^{-1/3}$ s. The channel has a width of 5 m, a discharge of 12.5 m^3 s^{-1} and a slope of 1:2500.

(a) Determine the critical depth and the normal depth in the main channel.
(b) Determine the venturi flume width which will just make the flow critical at the contraction.
(c) If the contraction width is 2 m find the depths just upstream, downstream and at the throat of the venturi flume (neglecting friction in this short section).
(d) Sketch the surface profile.

$n = 0.012\ \text{m}^{-1/3}$ s
$b = 5$ m (main channel)
$Q = 12.5\ \text{m}^3\ \text{s}^{-1}$
$S = 4 \times 10^{-4}$

(a)
In the main channel,

$$q = \frac{Q}{b} = \frac{12.5}{5} = 2.5\ \text{m}^2\ \text{s}^{-1}$$

Critical Depth

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.5^2}{9.81}\right)^{1/3} = 0.8605\ \text{m}$$

Normal Depth

$$Q = VA$$

where:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1 + 2h/b)^{2/3}} S^{1/2}$$

or, rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 1.275\ (1 + 0.4h)^{2/5}$$

Iteration (from, e.g., $h = 1.275$) gives

$$h_n = 1.546\ \text{m}$$

Answer: critical depth = 0.860 m;   normal depth = 1.55 m.

(b) The flow will just go critical if the head in the throat (H_c) is exactly equal to that in the approach flow (H_a). Measure heads relative to the bed of the channel in the vicinity of the venturi.

Critical Head

$$H_c = E_c \quad = \frac{3}{2} h_c \quad = \frac{3}{2}\left(\frac{q_m^2}{g}\right)^{1/3}, \qquad \text{where} \qquad q_m = \frac{Q}{b_m}$$

(Note that the critical depth is different at the throat to that in the main channel, due to the narrower width.)

$$\Rightarrow \qquad H_c = \frac{3}{2}\left(\frac{Q^2}{g b_m^2}\right)^{1/3}$$

Approach Flow

The approach flow is normal, since the channel is "long". Hence,

$$h_a = 1.546\ \text{m}$$

$$V_a = \frac{Q}{b h_a} \quad = \frac{12.5}{5 \times 1.546} \quad = 1.617\ \text{m s}^{-1}$$

$$H_a = E_a \quad = h_a + \frac{V_a^2}{2g} \quad = 1.546 + \frac{1.617^2}{2 \times 9.81} \quad = 1.679\ \text{m}$$

For the flow just to go critical at the throat,

$$H_c = H_a$$

$$\Rightarrow \qquad \frac{3}{2}\left(\frac{Q^2}{g b_m^2}\right)^{1/3} = 1.679$$

$$\Rightarrow \qquad b_m = \frac{12.5}{\left(\frac{2}{3} \times 1.679\right)^{3/2} \sqrt{9.81}} \quad = 3.370\ \text{m}$$

Answer: 3.37 m.

(c) If the throat width is reduced further, then the flow will back up and undergo a critical transition at the throat.

At the throat,

$$q_m = \frac{Q}{b_m} \quad = \frac{12.5}{2} \quad = 6.25 \text{ m}^2 \text{ s}^{-1}$$

$$h_c = \frac{3}{2}\left(\frac{q_m^2}{g}\right)^{1/3} \quad = \left(\frac{6.25^2}{9.81}\right)^{1/3} \quad = 1.585 \text{ m}$$

$$E_c = \frac{3}{2}h_c \quad = \frac{3}{2} \times 1.585 \quad = 2.378 \text{ m}$$

$$H_c = z_b + E_c \quad = 0 + 2.378 \quad = 2.378 \text{ m}$$

The head throughout the venturi will be the critical head ($H = H_c = 2.378$ m).

Anywhere in the flume,

$$H = z_s + \frac{V^2}{2g}, \qquad \text{where} \qquad z_s = h \qquad V = \frac{Q}{bh}$$

$$\Rightarrow \qquad H = h + \frac{Q^2}{2gb^2h^2}$$

At the throat the depth will be the critical depth there; i.e. $h = h_c = 1.585$ m.

Just upstream and downstream, $b = 5$ m; hence,

$$2.378 = h + \frac{0.3186}{h^2}$$

Upstream

Rearrange for the deep, subcritical solution:

$$h = 2.378 - \frac{0.3186}{h^2}$$

Iteration (from, e.g., $h = 2.378$) gives $h = 2.319$ m.

Downstream

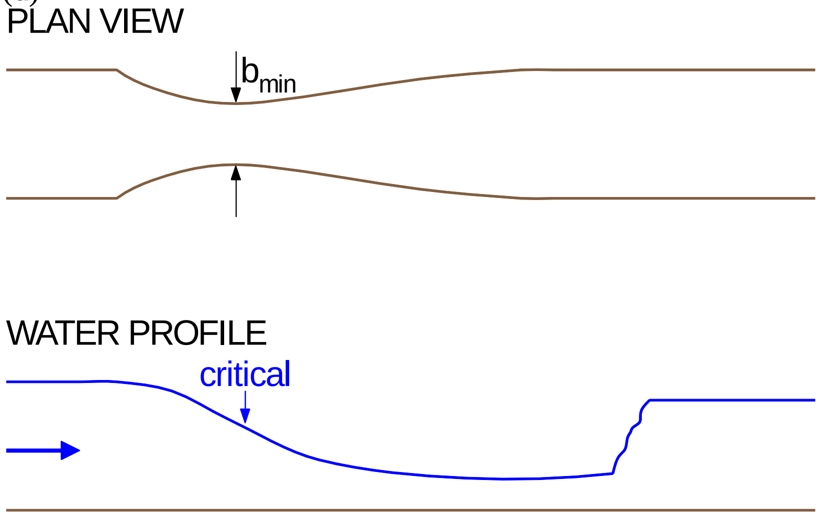
Rearrange for the shallow, supercritical solution:

$$h = \sqrt{\frac{0.3186}{2.378 - h}}$$

Iteration (from, e.g., $h = 0$) gives $h = 0.4015$ m.

Answer: depths upstream, in the throat, downstream = 2.32 m, 1.59 m, 0.401 m.

(d)



Section 2.3.3

Example.
The water depth upstream of a sluice gate is 0.8 m and the depth just downstream (at the vena contracta) is 0.2 m. Calculate:
(a) the discharge per unit width;
(b) the Froude numbers upstream and downstream.

$h_1 = 0.8 \text{ m}$
$h_2 = 0.2 \text{ m}$

(a) Assuming total head the same on either side of the gate:

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$

Substituting $z_s = h$ and $V = q/h$:

$$h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$$

From the given data, in metre-second units:

$$0.8 + 0.0796q^2 = 0.2 + 1.2742q^2$$

$$\Rightarrow \qquad 0.6 = 1.1946q^2$$

$$\Rightarrow \qquad q = 0.7087 \text{ m}^2 \text{ s}^{-1}$$

Answer: 0.709 $\text{m}^2 \text{ s}^{-1}$.

(b) Use, on each side of the gate,

$$V = \frac{q}{h}$$

$$\text{Fr} = \frac{V}{\sqrt{gh}}$$

to get

$$V_1 = 0.8859 \text{ m s}^{-1}$$

$$V_2 = 3.544 \text{ m s}^{-1}$$

and then

$$\text{Fr}_1 = 0.3162$$

$$\text{Fr}_2 = 2.530$$

Answer: Froude numbers upstream, downstream = 0.316, 2.53.

Section 2.3.3

Example.
A sluice gate controls the flow in a channel of width 2 m. If the discharge is 0.5 m^3 s^{-1} and the upstream water depth is 1.5 m, calculate the downstream depth and velocity.

$b = 2$ m
$Q = 0.5$ m^3 s^{-1}
$h_1 = 1.5$ m

Use upstream conditions to get total head. Then, assuming no losses, find the supercritical flow with the same head.

Total head (either side):

$$H = z_s + \frac{V^2}{2g} \qquad \text{where} \qquad z_s = h \qquad \text{and} \qquad V = \frac{Q}{bh}$$

$$\Rightarrow \qquad H = h + \frac{Q^2}{2gb^2h^2} \quad = h + \frac{3.186 \times 10^{-3}}{h^2}$$

The upstream depth $h_1 = 1.5$ m gives

$$H = 1.5 + \frac{3.186 \times 10^{-3}}{1.5^2} \quad = 1.501 \text{ m} \qquad \text{(dominated by } h_1\text{)}$$

Hence,

$$1.501 = h + \frac{3.186 \times 10^{-3}}{h^2}$$

Rearrange for the shallow, supercritical solution:

$$h = \sqrt{\frac{3.186 \times 10^{-3}}{1.501 - h}}$$

Iteration (from, e.g., $h = 0$) gives

$$h_2 = 0.04681 \text{ m}$$

$$V_2 = \frac{Q}{bh_2} \quad = \frac{0.5}{2 \times 0.04681} \quad = 5.341 \text{ m s}^{-1}$$

Answer: downstream depth = 0.0468 m;   velocity = 5.34 m s^{-1}.

Section 2.4

Example. (Exam 2018)
Water flows at 0.8 m^3 s^{-1} per metre width down a long, wide spillway of slope 1 in 30 onto a wide apron of slope 1 in 1000. Manning's roughness coefficient $n = 0.014$ m$^{-1/3}$ s on both slopes.

(a) Find the normal depths in both sections and show that normal flow is supercritical on the spillway and subcritical on the apron.

(b) Baffle blocks are placed a short distance downstream of the slope transition to provoke a hydraulic jump. Assuming that flow is normal on both the spillway and downstream of the hydraulic jump, calculate the force per metre width of channel that the blocks must impart.

(c) Find the head loss across the blocks.

$S_1 = 1/30$; $S_2 = 1/1000$
$q = 0.8$ m^2 s^{-1}
$n = 0.014$ m$^{-1/3}$ s

(a) Normal flow:

$$q = Vh, \qquad \text{where} \quad V = \frac{1}{n} R_h^{2/3} S^{1/2} \quad \text{(Manning)}, \quad R_h = h \quad \text{("wide" channel)}$$

$$\Rightarrow \qquad q = \frac{1}{n} h^{2/3} S^{1/2} h$$

$$\Rightarrow \qquad q = \frac{h^{5/3}\sqrt{S}}{n}$$

$$\Rightarrow \qquad h = \left(\frac{nq}{\sqrt{S}}\right)^{3/5}$$

For the two slopes this gives

$$h_1 = 0.1874 \text{ m}$$

$$h_2 = 0.5365 \text{ m}$$

Answer: depth on spillway = 0.187 m;   depth on apron = 0.536 m.

For subcritical/supercritical:
<u>Method 1</u> (use critical depth)
The critical depth is

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{0.8^2}{9.81}\right)^{1/3} \quad = 0.4026 \text{ m}$$

On the spillway, $h_1 < h_c$: this is shallower than critical flow (where $\mathrm{Fr} = 1$) and hence faster; both ensure $\mathrm{Fr}_1 > 1$, so supercritical.
On the apron, $h_2 > h_c$: this is deeper than critical flow and hence slower; both ensure $\mathrm{Fr}_2 < 1$, so subcritical.

Method 2 (find Froude numbers)

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{q}{\sqrt{gh^3}}$$

Applying this for both depths we find $\mathrm{Fr}_1 = 3.148$ (supercritical) and $\mathrm{Fr}_2 = 0.650$ (subcritical).

(b)
The corresponding velocities are deduced from $V = q/h$, whence:

$$V_1 = 4.269 \text{ m s}^{-1}$$

$$V_2 = 1.491 \text{ m s}^{-1}$$

Let f be the *magnitude* of the force per unit width exerted by the fluid on the blocks and, by reaction, the blocks on the fluid, which is clearly acts in the upstream direction.

On each side of the blocks the hydrostatic pressure force is given by

$$\text{average pressure} \times \text{area}$$

or $\frac{1}{2}\rho g h \times h$ (per unit width)

Hence, from the steady-state momentum principle:

force = rate of change of momentum (mass flux × change in velocity)

$$-f + \frac{1}{2}\rho g h_1^2 - \frac{1}{2}\rho g h_2^2 = \rho q(V_2 - V_1)$$

Hence,

$$\begin{aligned} f &= \frac{1}{2}\rho g(h_1^2 - h_2^2) + \rho q(V_1 - V_2) \\ &= -1240 + 2222 \\ &= 982 \text{ N m}^{-1} \end{aligned}$$

Answer: 982 N m^{-1}.

(c) Where hydrostatic, head in open-channel flow is given by

$$H = z_s + \frac{V^2}{2g}$$

Here, relative to the bed of the apron, $z_s = h$. Hence,

$$\begin{aligned}\text{head loss} &= h_1 - h_2 + \frac{V_1^2 - V_2^2}{2g} \\ &= -0.3491 + 0.8156 \\ &= 0.4665 \text{ m}\end{aligned}$$

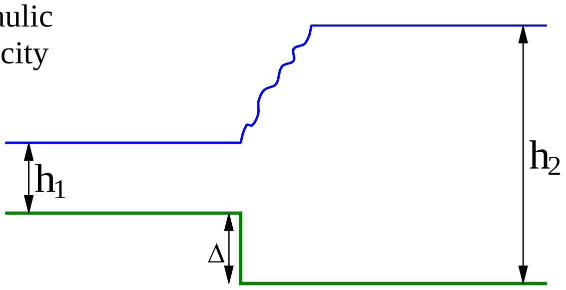
Answer: 0.467 m.

Section 2.4

Example.
A downward step of height 0.5 m causes a hydraulic jump in a wide channel when the depth and velocity of the flow upstream are 0.5 m and 10 m s^{-1}, respectively.

(a) Find the downstream depth.

(b) Find the head lost in the jump.



(a) The downstream depth can be deduced from the momentum principle if the reaction force from the step is known. The approximation is that this is the same as would occur if it were in equilibrium with a hydrostatic pressure distribution here.

Flow rate per unit width:

$$q = V_1 h_1 \quad = 10 \times 0.5 \quad = 5 \ \ \text{m}^2\ \text{s}^{-1}$$

Steady-state momentum principle

force = rate of change of momentum (mass flux × change in velocity)

Per unit width:

$$\frac{1}{2}\rho g(h_1+\Delta)^2 - \frac{1}{2}\rho g h_2^2 = \rho q (V_2 - V_1)$$

Since

$$V_2 = \frac{q}{h_2} = \frac{5}{h_2}$$

this gives, in metre-second units,

$$4905(1-h_2^2) = 5000(\frac{5}{h_2} - 10)$$

$$\Rightarrow \qquad 1 - h_2^2 = 1.019(\frac{5}{h_2} - 10)$$

$$\Rightarrow \qquad 11.19 = h_2^2 + \frac{5.095}{h_2}$$

Rearrange for the deep, subcritical solution:

$$h_2 = \sqrt{11.19 - \frac{5.095}{h_2}}$$

Iterating (from, e.g., $h_2 = \sqrt{11.19}$) gives $h_2 = 3.089$ m.

Answer: 3.09 m.

(b) Head either side is given by:

$$H = z_s + \frac{V^2}{2g}$$

The datum is not important as it is only the difference in head that is required. For convenience, measure z relative to the bed of the expanded part. Then,

$$z_{s1} = 1 \text{ m (note: water surface level, not depth)}, V_1 = 10 \text{ m s}^{-1} \quad \Rightarrow \quad H_1 = 6.097 \text{ m}$$

$$z_{s2} = 3.089 \text{ m}, V_2 = q/h_2 \quad = 1.619 \text{ m s}^{-1} \quad \Rightarrow \quad H_2 = 3.223 \text{ m}$$

Hence,

$$\text{head lost} = 6.097 - 3.223 \quad = 2.874 \text{ m}$$

Answer: 2.87 m.

Section 3.6.2

Example.

A long, wide channel has a slope of 1:2747 with a Manning's n of 0.015 m$^{-1/3}$ s. It carries a discharge of 2.5 m^3 s^{-1} per metre width, and there is a free overfall at the downstream end. An undershot sluice is placed a certain distance upstream of the free overfall which determines the nature of the flow between sluice and overfall. The depth just downstream of the sluice is 0.5 m.

(a) Determine the critical depth and normal depth.

(b) Sketch, with explanation, the two possible gradually-varied flows between sluice and overfall.

(c) Calculate the particular distance between sluice and overfall which determines the boundary between these two flows. Use one step in the gradually-varied-flow equation.

$S_0 = 1/2747 = 3.640 \times 10^{-4}$
$n = 0.015 \text{ m}^{-1/3} \text{ s}$
$q = 2.5 \text{ m}^2 \text{ s}^{-1}$

(a)
Critical Depth

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{2.5^2}{9.81}\right)^{1/3} \quad = 0.8605 \text{ m}$$

Normal Depth

$$q = Vh, \quad \text{where} \quad V = \frac{1}{n} R_h^{2/3} S_0^{1/2}, \quad \text{(Manning)}, \quad R_h = h \quad \text{(“wide” channel)}$$

$$\Rightarrow \quad q = \frac{1}{n} h^{2/3} S_0^{1/2} h$$

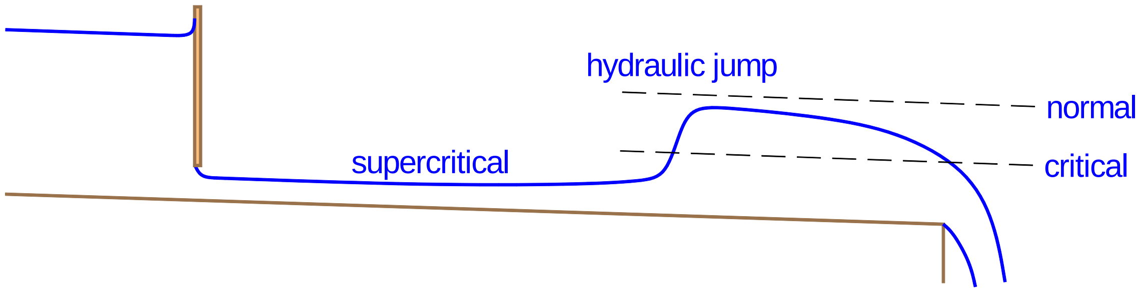
$$\Rightarrow \quad q = \frac{h^{5/3}\sqrt{S_0}}{n}$$

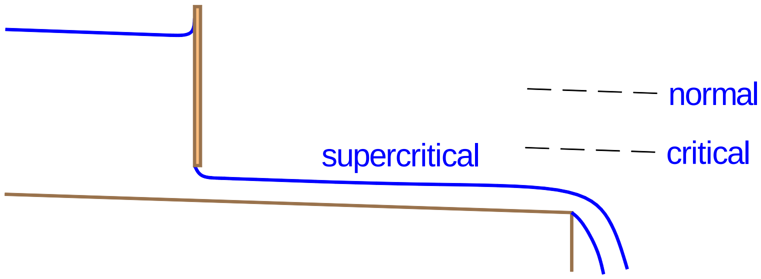
$$\Rightarrow \quad h = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} \quad = \left(\frac{0.015 \times 2.5}{\sqrt{1/2747}}\right)^{3/5} \quad = 1.500 \text{ m}$$

Answer: critical depth = 0.860 m; normal depth = 1.50 m.

(b) The depth just downstream of the sluice is supercritical (0.5 m $< h_c$). However, the preferred depth is subcritical ($h_n > h_c$). Hence, if the channel is long enough then there will be a downstream hydraulic jump, with the flow depth then decreasing to pass through critical again near the overfall.

If the channel is too short, however, the region of supercritical flow from the sluice will extend to the overfall.





(c) As the channel shortens, the depth change across the hydraulic jump diminishes. The boundary between the two possible flow behaviours occurs when the supercritical GVF just reaches critical depth at the overfall (i.e. the limiting depth change across the hydraulic jump is zero).

As the flow is supercritical, integrate the GVF equation forward from the downstream side of the sluice gate (where $h = 0.5$ m) to the overfall (where $h = h_c = 0.8605$ m). Use 1 step.

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)\Delta h$$

For the working, write the derivative as a function of h; (all lengths in metres).

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} = \frac{q}{\sqrt{gh^3}} \qquad \Rightarrow \qquad \mathrm{Fr}^2 = \frac{q^2}{gh^3} = \frac{0.6371}{h^3}$$

$$S_f = \left(\frac{nq}{h^{5/3}}\right)^2 = \frac{1.406 \times 10^{-3}}{h^{10/3}}$$

$$\Delta h = 0.8605 - 0.5 = 0.3605$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.6371}{h^3}}{3.640 \times 10^{-4} - \dfrac{1.406 \times 10^{-3}}{h^{10/3}}}, \qquad \Delta h = 0.3605$$

i	h_i	x_i	h_{mid}	$\left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}}$	Δx
0	0.5	0			
			0.6803	217.1	78.26
1	0.8605	**78.26**			

Answer: 78 m.

Example. (Exam 2022)
A long rectangular channel of width 2.5 m, slope 0.004 and Manning's roughness coefficient $n = 0.022$ m$^{-1/3}$s carries water at 4 m^3 s^{-1}. Temporary works narrow the channel at one location to 1.1 m for a short distance.

(a) Find the normal depth in the main channel and show that the slope is hydraulically mild.

(b) Show that a hydraulic transition takes place at the narrow point and find the depth just downstream of the narrowed section, confirming that supercritical flow is possible here.

(c) Use two steps in the gradually-varied-flow equation to estimate the distance from the end of the narrow section to the downstream hydraulic jump.

(a)
$b = 2.5$ m ($b_m = 1.1$ m in the narrow section)
$S_0 = 0.004$
$n = 0.022$ m$^{-1/3}$ s
$Q = 4$ m^3 s^{-1}

Discharge:

$$Q = VA$$

where, in normal flow:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1 + 2h/b)^{2/3}} S^{1/2}$$

or, rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5} \qquad (*)$$

When $S = S_0 = 0.004$, substitution of numerical values in (*) yields an iterative formula

$$h = 0.7036(1 + 0.8h)^{2/5}$$

Iteration (from, e.g., $h = 0.7036$) gives

$$h_n = 0.8690 \text{ m}$$

At the normal depth the velocity is $V_n = Q/(bh_n) = 1.841$ m s^{-1} and the Froude number is

$$\text{Fr}_n \equiv \frac{V_n}{\sqrt{gh_n}} = 0.6305$$

This is less than 1, i.e. subcritical. Hence, the slope is hydraulically mild.

Answer: 0.869 m.

(b) At the throat of the narrow section the critical depth is

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3}$$

where the flow rate per unit width $q_m = Q/b_m = 3.636 \text{ m}^2 \text{ s}^{-1}$. Hence

$$h_c = 1.105 \text{ m}$$

The critical head is

$$H_c = z_b + E_c \quad = 0 + \frac{3}{2}h_c \quad = 1.658 \text{ m}$$

The approach flow head is based on normal flow (since the channel is long):

$$H_a = h_n + \frac{V_n^2}{2g} \quad = 1.042 \text{ m}$$

This is less than the critical head (the minimum energy required to pass this flow through the constricted section). Hence, upstream of the narrow section the flow must back up, increasing in depth to provide the required energy, and so a hydraulic transition takes place. The total head in the vicinity of the narrowed section is

$$H = H_c \quad = 1.658 \text{ m}$$

Downstream of the constricted section we require supercritical flow at the same head:

$$H = h + \frac{Q^2}{2gb^2h^2}$$

i.e., in m-s units,

$$1.658 = h + \frac{0.1305}{h^2}$$

Rearranging for the supercritical (shallow) solution:

$$h = \sqrt{\frac{0.1305}{1.658 - h}}$$

Iteration (from, e.g., $h = 0$) gives

$$h_2 = 0.3113 \text{ m}$$

To show that this supercritical flow actually occurs we must show that it is less than the depth upstream of the hydraulic jump (since supercritical GVF on a mild slope increases in depth with distance). As the channel is "long", the hydraulic jump must cause a return to the normal depth and the depth on the upstream side of the jump is the sequent depth. With $\text{Fr}_n = 0.6305$ as calculated in part (a),

$$h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8\text{Fr}_n^2}\right) \quad = 0.4539 \text{ m}$$

Since $h_2 < h_J$ a depth-increasing length of supercritical flow exists.

Answer: 0.311 m.

(c) GVF from just downstream of the narrow section ($h_{\text{start}} = 0.3113$ m) to the upstream side of the hydraulic jump ($h_{\text{end}} = 0.4539$ m). The flow is supercritical so work in the direction of the flow. Using two steps the depth increment is

$$\Delta h = \frac{h_{\text{end}} - h_{\text{start}}}{N_{\text{steps}}} = \frac{0.4539 - 0.3113}{2} = 0.0713 \text{ m}$$

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right) \Delta h$$

For the working, write the derivative as a function of h.

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} = \frac{Q}{bh\sqrt{gh}} \qquad \Rightarrow \qquad \mathrm{Fr}^2 = \frac{Q^2}{b^2 g h^3} = \frac{0.2610}{h^3}$$

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 (1 + 2h/b)^{4/3} = 1.239 \times 10^{-3} \frac{(1 + 0.8h)^{4/3}}{h^{10/3}}$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\text{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.2610}{h^3}}{\left[4 - 1.239 \dfrac{(1 + 0.8h)^{4/3}}{h^{10/3}}\right] \times 10^{-3}}, \qquad \Delta h = 0.0713$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\text{mid}}$	Δx
0	0.3113	0			
			0.34695	96.27	6.864
1	0.3826	6.864			
			0.41825	87.70	6.253
2	0.4539	**13.12**			

Answer: 13.1 m.

Example. (Exam 2021)
A long rectangular channel of width 2.2 m, streamwise slope 1:100 and Chézy coefficient 80 $m^{1/2}$ s^{-1} carries a discharge of 4.5 m^3 s^{-1}.

(a) Find the normal depth and critical depth and show that the slope is steep at this discharge.

(b) An undershot sluice gate causes a hydraulic transition in this flow. The depth of parallel flow downstream of the gate is 0.35 m. Find the depth immediately upstream of the gate and sketch the flow.

(c) Using 2 steps in the gradually-varied-flow equation, find the distance between the gate and the hydraulic jump.

(a)
$b = 2.2$ m
$S_0 = 0.01$
$C = 80 \text{ m}^{1/2} \text{ s}^{-1}$
$Q = 4.5 \text{ m}^3 \text{ s}^{-1}$

For the normal depth,

$$Q = VA$$

where

$$V = CR_h^{1/2}S_0^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b+2h} \quad = \frac{h}{1+2h/b}$$

Hence,

$$Q = C\left(\frac{h}{1+2h/b}\right)^{1/2} S_0^{1/2}bh$$

$$\Rightarrow \qquad \frac{Q}{Cb\sqrt{S_0}} = \frac{h^{3/2}}{(1+2h/b)^{1/2}}$$

$$\Rightarrow \qquad h = \left(\frac{Q}{Cb\sqrt{S_0}}\right)^{2/3}(1+2h/b)^{1/3}$$

Here,

$$h = 0.4028(1 + 0.9091h)^{1/3}$$

Iterate (from, e.g., $h = 0.4028$) to get normal depth

$$h_n = 0.4518 \text{ m}$$

For the critical depth,

$$q = \frac{Q}{b} \quad = 2.045 \text{ m}^2 \text{ s}^{-1}$$

Then,

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad = 0.7526 \text{ m}$$

The normal depth is supercritical since $h_n < h_c$ (i.e. shallower and faster than the case Fr = 1, and hence its Froude number must be > 1). Hence, by definition, the slope is "steep" at this discharge.

Answer: normal depth = 0.452 m; critical depth = 0.753 m.

(b) First find the depth just upstream of the sluice gate by requiring the same head on both sides:

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$

$$h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$$

With $h_2 = 0.35$ m:

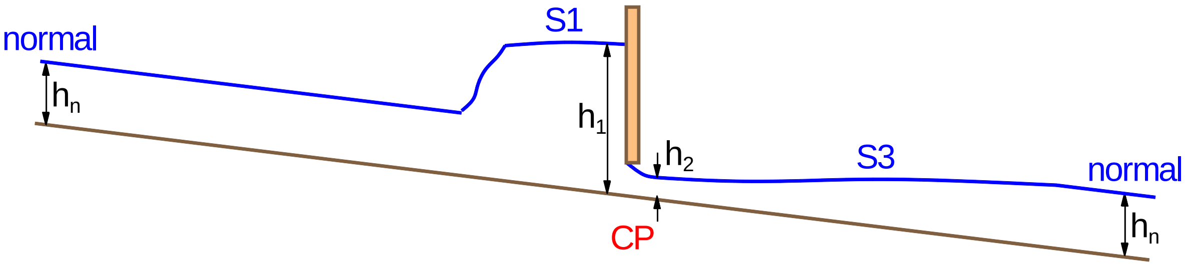
$$h_1 + \frac{0.2132}{h_1^2} = 2.090$$

Rearrange for iteration to the subcritical (deep) solution:

$$h_1 = 2.090 - \frac{0.2132}{h_1^2}$$

Iterating from, e.g., $h_1 = 2.090$ gives $h_1 = 2.039$ m.

The depth far upstream is supercritical, that upstream of the sluice is subcritical. Hence, there must be a hydraulic jump upstream, followed by GVF up to the gate.



Answer: 2.04 m.

(c) Upstream, since the channel is long, the jump goes from normal depth to the corresponding sequent depth in the hydraulic jump:

$$h_n = 0.4518 \text{ m}$$

$$V_n = \frac{Q}{bh_n} \quad = 4.527 \text{ m s}^{-1}$$

$$\text{Fr}_n = \frac{V_n}{\sqrt{gh_n}} \quad = 2.150$$

$$h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8\text{Fr}_n^2}\right) \quad = 1.166 \text{ m}$$

The GVF is subcritical, so work *upstream* from $h_0 = 2.039$ m just upstream of the sluice to $h_2 = 1.166$ m just downstream of the jump, using 2 steps in the GVF equation. (Note the renumbering of depths for this part.)

$$\Delta h = \frac{1.166 - 2.039}{2} = -0.4365 \text{ m}$$

With lengths in m throughout:

$$\text{Fr}^2 = \frac{V^2}{gh} \quad = \frac{Q^2}{gb^2h^3} \quad = \frac{0.4263}{h^3}$$

$$S_f = \left(\frac{Q}{Cb}\right)^2 \frac{(1 + 2h/b)}{h^3} \quad = 6.537 \times 10^{-4} \frac{(1 + 0.9091h)}{h^3}$$

Hence,

$$\frac{\text{d}x}{\text{d}h} = \frac{1 - \text{Fr}^2}{S_0 - S_f} \quad = \frac{1 - \frac{0.4263}{h^3}}{0.01 - 6.537 \times 10^{-4} \frac{1 + 0.9091h}{h^3}}$$

and

$$\Delta x = \left(\frac{\text{d}x}{\text{d}h}\right)_{\text{mid}} \Delta h$$

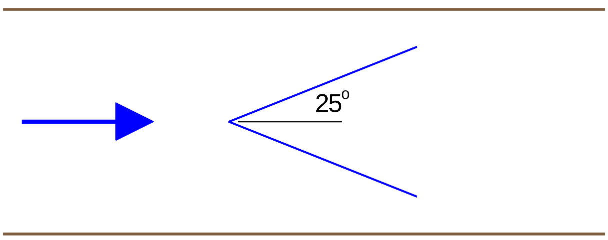
Working is set out in the following table.

i	h_i	x_i	h_{mid}	$(\text{d}x/\text{d}h)_{\text{mid}}$	Δx
0	2.039	0			
			1.821	95.69	–41.77
1	1.6025	–41.77			
			1.384	88.87	–38.79
2	1.166	**–80.56**			

Answer: distance to the (upstream) hydraulic jump = 80.6 m.

Section 4.2

Example. (From White, 2006)
A pencil point piercing the surface of a wide rectangular channel flow creates a wedgelike 25° half-angle wave, as in the figure right. If the channel has a Manning's *n* of 0.014 $m^{-1/3}$ s and the depth is 350 mm, determine: (a) the Froude number; (b) the critical depth; and (c) the critical slope.



$\alpha = 25°$
$n = 0.014 \text{ m}^{-1/3} \text{ s}$
$h = 0.35 \text{ m}$

(a)

$$\sin\alpha = \frac{1}{\text{Fr}}$$

$$\Rightarrow \quad \text{Fr} = \frac{1}{\sin\alpha} \quad = \frac{1}{\sin 25°} \quad = 2.366$$

Answer: 2.37.

(b)

$$\text{Fr} \equiv \frac{V}{\sqrt{gh}}$$

$$\Rightarrow \quad V = \text{Fr}\sqrt{gh} \quad = 2.366 \times \sqrt{9.81 \times 0.35} \quad = 4.384 \text{ m s}^{-1}$$

$$\Rightarrow \quad q = Vh \quad = 4.384 \times 0.35 \quad = 1.534 \text{ m}^2 \text{ s}^{-1}$$

$$\Rightarrow \quad h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = 0.6213 \text{ m}$$

Answer: 0.621 m.

(c) The critical slope is that at which the normal depth is equal to the critical depth.

Normal flow:

$$q = Vh \quad \text{where} \quad V = \frac{1}{n} R_h^{2/3} S^{1/2} \quad \text{(Manning)} \quad R_h = h \quad \text{(“wide” channel)}$$

$$\Rightarrow \qquad q = \frac{1}{n} h^{2/3} S^{1/2} h$$

Rearranging, and setting a depth *h* equal to the critical depth ($h_c = 0.6213$ m),

$$\Rightarrow \qquad S = \frac{(nq)^2}{h^{10/3}} = \frac{(0.014 \times 1.534)^2}{0.6213^{10/3}} = 2.254 \times 10^{-3}$$

Answer: 2.25×10^{-3}.

Sediment Transport; Section 2.1
Note: removed from the present curriculum because of the reduction in teaching hours.

Example.

An undershot sluice is placed in a channel with a horizontal bed covered by gravel with a median diameter of 5 cm and density 2650 kg m^{-3}. The flow rate is 4 m^3 s^{-1} per metre width and initially the depth below the sluice is 0.5 m. Assuming a critical Shields parameter τ^*_{crit} of 0.06 and friction coefficient c_f of 0.01:

(a) find the depth just upstream of the sluice and show that the bed there is stationary;

(b) show that the bed below the sluice will erode and determine the depth of scour.

$d = 0.05$ m
$\rho_s = 2650 \text{ kg m}^{-3} \quad (s = 2.65)$
$q = 4 \text{ m}^2 \text{ s}^{-1}$
$\tau^*_{\text{crit}} = 0.06$
$c_f = 0.01$

(a) The bed is mobile if and only if the bed shear stress exceeds the critical shear stress for incipient motion. (Alternatively, one could find the velocity at which incipent motion occurs.)

First find the critical shear stress for incipient motion. The local shear stress can be found by finding velocities using open-channel flow theory and using the friction coefficient.

The critical shear stress for incipient motion is

$$\tau_{\text{crit}} = \tau^*_{\text{crit}}(\rho_s - \rho)gd \quad = 0.06 \times (2650 - 1000) \times 9.81 \times 0.05 \quad = 48.56 \text{ Pa}$$

In the accelerated flow just below the sluice gate,

$$h = 0.5 \text{ m}$$

$$V = \frac{q}{h} \quad = \frac{4}{0.5} \quad = 8 \text{ m s}^{-1}$$

The total head relative to the undisturbed bed is thus (since $z_s = h$ initially):

$$H = z_s + \frac{V^2}{2g} \quad = 0.5 + \frac{8^2}{2 \times 9.81} \quad = 3.762 \text{ m}$$

Upstream of the sluice one must have the same total head:

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{q^2}{2gh^2}$$

but now we look for the subcritical (deep) solution:

$$h = H - \frac{q^2}{2gh^2}$$

Substituting numerical values, in metre-second units:

$$h = 3.762 - \frac{0.8155}{h^2}$$

Iterating (from, e.g., $h = 3.762$) gives

$$h = 3.703 \text{ m}$$

The velocity upstream of the sluice is

$$V = \frac{q}{h} \quad = \frac{4}{3.703} \quad = 1.080 \text{ m s}^{-1}$$

and the bed shear stress upstream of the sluice is

$$\tau_b = c_f \left(\frac{1}{2}\rho V^2\right) \quad = 0.01 \times \frac{1}{2} \times 1000 \times 1.080^2 \quad = 5.832 \text{ N m}^{-2}$$

Since $\tau_b < \tau_{\text{crit}}$ the bed here is stationary.

Answer: depth upstream of sluice = 3.70 m;   as demonstrated, the bed here is stationary.

(b) Initially, beneath the sluice,

$$V = 8 \text{ m s}^{-1}$$

$$\tau_b = c_f \left(\frac{1}{2}\rho V^2\right) \quad = 0.01 \times \frac{1}{2} \times 1000 \times 8^2 \quad = 320 \text{ N m}^{-2}$$

Since $\tau_b > \tau_{\text{crit}}$ the bed here is initially mobile.

The bed will continue to erode, the flow depth h increasing and velocity V decreasing until

$$\tau_b = \tau_{\text{crit}}$$

At this point,

$$c_f(\frac{1}{2}\rho V^2) = \tau_{\text{crit}}$$

whence

$$V = \sqrt{\frac{2}{c_f}\frac{\tau_{\text{crit}}}{\rho}} \quad = \sqrt{\frac{2}{0.01} \times \frac{48.56}{1000}} \quad = 3.116 \text{ m s}^{-1}$$

$$h = \frac{q}{V} \quad = \frac{4}{3.116} \quad = 1.284 \text{ m}$$

This is the depth of flow. Since the downstream water level is set (at 0.5 m above the undisturbed bed) by the gate the depth of the scour hole is

$$1.284 - 0.5 = 0.784 \text{ m}$$

Answer: 0.784 m.