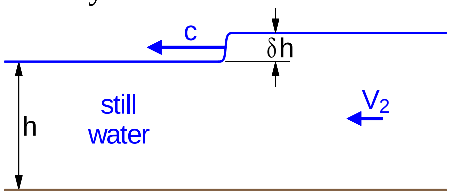
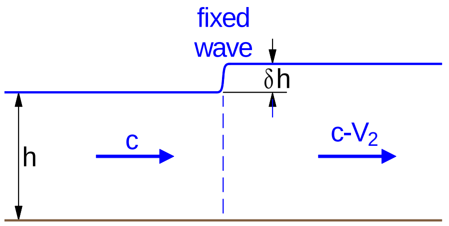
4.1 Long-Wave Speed on Shallow Water

Surge waves can be generated in channels by the raising or lowering of gates. (Compare water-hammer waves due to rapid opening or closing of valves in pipelines.) They can also appear in rivers as *tidal bores* when the incoming tide is funnelled into an estuary.



Consider a surge wave of additional height δh propagating from right to left into still water at speed c.



To analyse this as a steady case consider the continuity and momentum balance in the inertial frame of the moving wave (i.e. add a right-directed velocity c to both sides). This gives the following (*per unit width*).

Continuity (velocity × area = constant):

$$ch = (c - V_2)(h + \delta h)$$

$$\Rightarrow \qquad V_2 = \frac{\delta h}{h + \delta h} c \qquad (1)$$

Momentum (net hydrostatic pressure force = mass flux × change in velocity):

$$\frac{1}{2}\rho g[h^2 - (h + \delta h)^2] = \rho ch\,[(c - V_2) - c]$$

$$\Rightarrow \qquad V_2 = \frac{g\delta h}{c}(1 + \frac{1}{2}\frac{\delta h}{h}) \qquad (2)$$

Comparing the two expressions for V_2:

$$c^2 = gh(1 + \frac{1}{2}\frac{\delta h}{h})(1 + \frac{\delta h}{h}) \qquad (3)$$

This shows that:

(i) the bigger the surface rise δh then the faster the wave (which is why, in practice, such waves tend to steepen);

(ii) in the small-amplitude limit ($\delta h/h \to 0$):

$$c = \sqrt{gh} \qquad (4)$$

The Froude number is thus the ratio

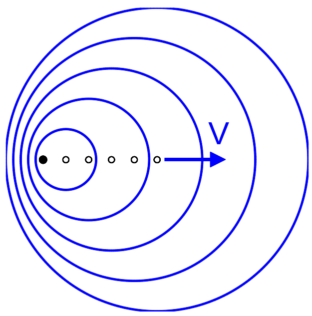
$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} = \frac{\text{water velocity}}{\text{wave speed}} \qquad (5)$$

Note.
$\sqrt{gh}$ is the speed of *small-amplitude*, *long waves* on *shallow* water. It is derived by assuming *hydrostatic pressure* in the momentum equation. In the latter half of Hydraulics 3 you will learn about waves of arbitrary wavelength in arbitrary depths, where the hydrostatic assumption is no longer valid and the wave speed is dependent on wavelength.

4.2 Zone of Influence

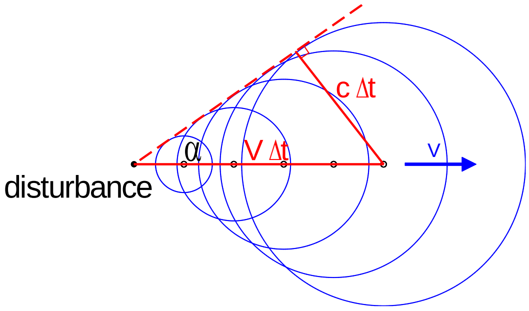
Individual waves emitted from a point spread out as circles, swept along at the flow speed V.

If $\mathrm{Fr} < 1$ (i.e. $V < c$) then waves can travel upstream. Hence, downstream conditions can influence the flow upstream.



If $\mathrm{Fr} > 1$ (i.e. $V > c$) then waves are swept downstream and downstream disturbances cannot influence behaviour upstream.

The zone of influence is a wedge-shaped region with semi-vertex angle α (see the diagram for waves emitted a time Δt apart) by

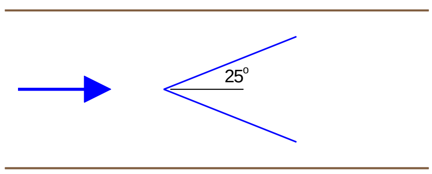


$$\sin\alpha = \frac{c\Delta t}{V\Delta t} \quad = \frac{c}{V} \quad = \frac{\sqrt{gh}}{V}$$

i.e.

$$\sin\alpha = \frac{1}{\mathrm{Fr}}$$

Example. (From White, 2006)
A pencil point piercing the surface of a wide rectangular channel flow creates a wedgelike 25° half-angle wave, as in the figure right. If the channel has a Manning's n of 0.014 m$^{-1/3}$ s and the depth is 350 mm, determine:
(a) the Froude number;
(b) the critical depth;
(c) the critical slope.



4.3 Analogy With Compressible Flow

Ratio of Flow Velocity to Wave Speed

Froude number $\mathrm{Fr} = \dfrac{V}{c}$ wave speed $c = \sqrt{gh}$

Mach number $\mathrm{Ma} = \dfrac{V}{c}$ wave speed $c = \sqrt{\gamma p/\rho} \quad = \sqrt{\gamma RT}$

Zone of Influence

Wedge-shaped region of surface waves.
Mach cone.

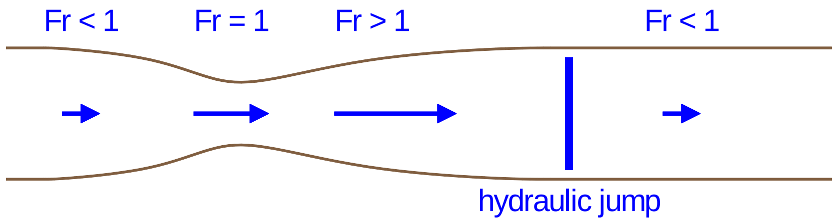
Discontinuity (rapid transition; loss of energy)

Hydraulic jump (super- to subcritical flow).
Normal shock (super- to subsonic flow).

Choked Flow (smooth transition)

Venturi flume (sub- to supercritical flow; Fr = 1 at the throat).
Transonic nozzle (sub- to supersonic flow; Ma = 1 at the throat).

Venturi flume



Transonic nozzle