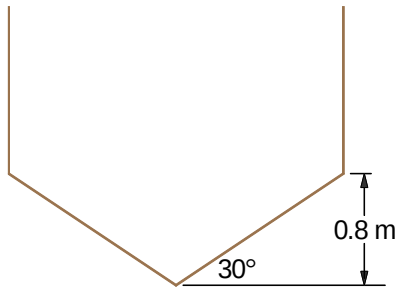
Normal and Critical Depths

Q1.
If the discharge in a channel of width 5 m is 20 $m^3\ s^{-1}$ and Manning's n is 0.02 $m^{-1/3}$ s, find:
(a) the normal depth and Froude number for a streamwise slope of 0.001;
(b) the normal depth and Froude number for a streamwise slope of 0.01;
(c) the critical depth;
(d) the critical slope.

Q2.
A prismatic channel of symmetric trapezoidal section, 1600 mm deep and with top and bottom widths 3 m and 0.6 m respectively carries water at a rate of 2.6 $m^3\ s^{-1}$. Manning's n may be taken as 0.012 $m^{-1/3}$ s. Find:
(a) the normal depth at a slope of 1 in 2500;
(b) the Froude number at the normal depth;
(c) the critical depth;
(d) the critical slope.

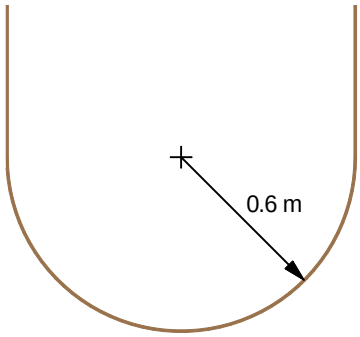
Q3.
A channel of semi-circular cross-section, radius 0.7 m, carries water at a rate of 0.8 $m^3\ s^{-1}$. Manning's n is 0.013 $m^{-1/3}$ s. Find:
(a) the normal depth (relative to the bottom of the channel) at a slope of 2%;
(b) the Froude number at the normal depth;
(c) the critical depth.

Q4.
A prismatic channel with the symmetric cross-section shown carries water at a rate of 1.5 $m^3\ s^{-1}$. Manning's n may be taken as 0.02 $m^{-1/3}$ s and the streamwise slope is 0.1%. Find:
(a) the normal depth (relative to the lowest point);
(b) the Froude number at the normal depth;
(c) the critical depth at this flow rate.



Q5.

A prismatic channel, with the cross-section shown, has a streamwise slope of 1 in 50.



(a) At a flow rate of 2 $m^3\ s^{-1}$ the flow depth (measured from the lowest point of the channel) is 0.6 m. Estimate the value of Manning's n.

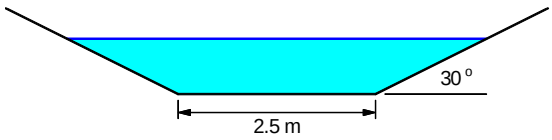
(b) Find the depth in the channel at a flow rate of 3 $m^3\ s^{-1}$.

(c) Find the Froude number at the flow rate in part (b).

(d) State whether the channel slope is steep or mild for the flow rate in part (b), justifying your answer.

Q6.

A trapezoidal channel with the geometry shown carries a flow of 25 $m^3\ s^{-1}$ down a slope of 0.003. Find the normal and critical depths (relative to the horizontal bed of the channel), assuming a *Chézy* coefficient $C = 45\ \mathrm{m}^{1/2}\ \mathrm{s}^{-1}$.



Rapidly-Varied Flow

Q7.

A broad-crested weir is placed in a wide channel with a slope of 2×10^{-4} with a discharge of 1.5 $m^3\ s^{-1}$ per metre width. Manning's n is 0.015 $m^{-1/3}$ s. Assuming that the flow far upstream is normal, hydraulic jumps do not occur and energy losses over the weir may be neglected:

(a) find depths upstream, over, and downstream of the weir if the weir height is 0.2 m;

(b) find depths upstream, over, and downstream of the weir if the weir height is 0.5 m;

(c) the weir height that will just make the flow critical.

Q8.

A discharge of 9 $m^3\ s^{-1}$ flows down a long channel with streamwise slope 1 in 1000 and Manning's roughness coefficient $n = 0.024\ \mathrm{m}^{-1/3}$ s. The channel cross-section is rectangular with bottom width 4 m and side height 2.5 m.

(a) Find the normal depth and critical depth in the channel.

A broad-crested weir is constructed at one point in the channel. Find the weir height that will:

(b) just make the flow go critical;

(c) cause the flow to overtop the banks.

Q9. (Exam 2021)

(a) Water flows down a long, wide channel of slope 0.1% at 3 $m^3 s^{-1}$ per metre width. Manning's n may be taken as 0.015 $m^{-1/3}$ s. The flow passes over a smooth depression of depth 0.3 m, as shown in figure (a). Find the water depth at stations A, C and E. Stations A and E are, respectively, just upstream and downstream of the depression.

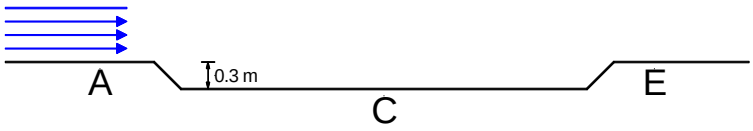


Figure (a)

(b) A broad-crested weir of height 0.7 m is now constructed at the centre of the depression (figure (b)). Find the water depths at stations A, B, C, D and E, assuming no hydraulic jump and the same discharge as part (a).

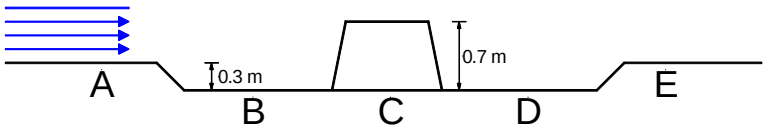


Figure (b)

Q10.

A long wide rectangular channel has a slope of 2×10^{-5}, a Manning's n of 0.01 $m^{-1/3}$ s and a flow rate of 0.5 $m^3 s^{-1}$ per metre width. A broad-crested weir with a height of 0.7 m is placed in the channel. Determine:

(a) the normal depth in the channel;
(b) the depth over the weir;
(c) the depth downstream of the weir assuming that the hydraulic jump occurs well downstream;
(d) the depth upstream of the hydraulic jump, and thus …
(e) the actual position of the hydraulic jump.

Q11.

A rectangular channel of width 5 m carries a discharge of 8 $m^3 s^{-1}$. The streamwise slope of the channel is 1.0×10^{-4} and Manning's roughness coefficient may be taken as 0.015 $m^{-1/3}$ s. At one point there is a localised narrowing to width 2 m.

(a) Assuming a long undisturbed fetch upstream, find the depth of flow far upstream of the narrow point.
(b) Find the critical depth and the critical specific energy at the narrow point.
(c) Determine the water depths at the narrow point and at stations just up and downstream of the contracted section if the channel bed in the contracted section is:
    (i) the same as the main channel;
    (ii) raised by 0.75 m;
    (iii) lowered by 0.75 m.

(You may assume no hydraulic jump occurs immediately downstream of the narrow section.)

Q12.
Water is conveyed at 12 $m^3 s^{-1}$ through a rectangular channel of width 5 m, streamwise slope 0.01% and Manning's $n = 0.016\ m^{-1/3}\ s$. At one point the channel narrows to a width of 2 m. Assuming that normal-flow conditions prevail upstream, calculate:

(a) the normal depth in the main channel;
(b) the critical depth at the narrow point; show that the flow does *not* become critical here;
(c) the actual water depth at the narrow point;
(d) the minimum height by which the bed of the constricted section must be raised locally in order to force critical conditions there;
(e) the depths at the narrow point and in the main channel just up- and downstream of the constricted section if the bed is raised as in part (d). (Assume that no hydraulic jumps occur here.)

Q13. (Exam 2019 – part)
A rectangular channel, whose width along most of its length is 4 m, undergoes a local narrowing at one point to a width of 2 m. The velocity at the narrow point is 2.7 m s^{-1}.

(a) If a smooth hydraulic transition takes place through the narrowed section, find the flow velocities just upstream and downstream in the full-width channel.
(b) If a hydraulic transition does *not* take place through the narrow section, and the depth there is 1.0 m, find the flow velocities just upstream and downstream in the full-width channel.

Assume no loss of energy and no hydraulic jump in the vicinity of the narrowed section.

Q14.
A river consists of a rectangular channel of width 5 m. At one point the piers of a simple beam bridge cause a local narrowing to width 3 m. The bottom of the bridge deck is 1.7 m above the bed of the river.

(a) When the river flow is 11 $m^3 s^{-1}$ the constriction of the channel by the bridge causes a subcritical to supercritical flow transition. Calculate the depths of water upstream, downstream and under the centre of the bridge, stating any assumptions made.
(b) At the flow rate above, a hydraulic jump occurs a short distance downstream of the bridge. Find the depth of flow immediately downstream of the hydraulic jump.
(c) Show that if the river flow is 22 $m^3 s^{-1}$ then the flow passage beneath the bridge will be completely choked.

Q15. (Exam 2022)
An undershot sluice gate controls the flow in a channel of width 0.8 m. The depth just downstream of the sluice gate is 0.25 m and the flow rate is 0.9 $m^3\ s^{-1}$. Neglecting frictional losses, find:

(a) the total head (relative to the bed of the channel);
(b) the depth and velocity of flow just upstream of the gate;
(c) the Froude number on either side of the gate;
(d) the force on the gate.

Q16.
Water passes under a sluice gate in a horizontal channel of width 2 m. The depths of flow on either side of the sluice gate are 1.8 m and 0.3 m. A hydraulic jump occurs a short distance downstream. Assuming no energy loss at the gate, calculate:

(a) the force on the gate;
(b) the depth of flow downstream of the hydraulic jump;
(c) the fraction of the fluid energy that is dissipated in the jump.

Q17.
A sluice controls the flow in a uniform rectangular channel of width 3 m, where the flow rate is 1.8 $m^3\ s^{-1}$.

(a) If the parallel-flow depth just downstream of the sluice is 0.22 m, calculate the depth just upstream of the sluice.
(b) The channel slope is 1 in 40 and the lining of the channel has Manning's roughness parameter $n = 0.03\ \mathrm{m}^{-1/3}\ \mathrm{s}$. Calculate the normal depth and the critical depth in the channel at the given flow rate.
(c) In the absence of any other local flow controls, will a hydraulic jump occur upstream or downstream of the sluice? Justify your answer and sketch the depth profile along the channel.
(d) In the situation discussed in part (c), what will be the depths on either side of the hydraulic jump?

Q18.
An undershot sluice controls the flow in a channel of width 1.5 m. If the flow rate is 3 $m^3\ s^{-1}$ and the upstream depth is 1.8 m calculate the minimum depth and Froude number just downstream of the sluice if:

(a) there is no energy loss;
(b) there is a 10% loss in specific energy through the sluice.

Forces on Obstacles

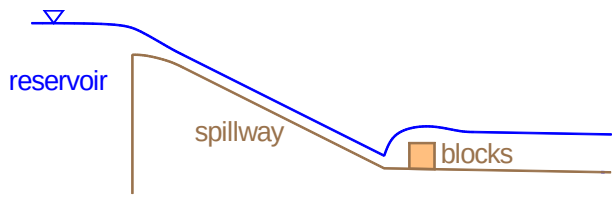
Q19.
Two rows of baffle blocks are installed in a stilling basin in order to force a hydraulic jump within the basin. <u>Each</u> row of blocks has drag coefficient $c_D = 0.3$, defined here by

$$c_D = \frac{\text{force}}{\frac{1}{2}\rho V^2 A}$$

where A is the frontal area (blocks + gaps) facing the flow and V is the upstream velocity. The effective height of the blocks is 0.3 m and the width of the basin is 6 m. If the discharge is 28 $m^3\ s^{-1}$ and the upstream depth is 0.6 m, determine the downstream depth if:

(a) a hydraulic jump *does not* occur;
(b) a hydraulic jump *does* occur.

Q20.



The spillway of a reservoir is a rectangular channel of width 4 m and slope 1 in 20. Manning's n may be taken as 0.012 $\mathrm{m}^{-1/3}$ s. Shortly after a storm the water depth in the reservoir is 0.5 m above the top of the spillway.

(a) Assuming critical flow at the entrance to the spillway calculate the discharge.

(b) Calculate the normal depth on the spillway and confirm that the slope is hydraulically "steep" at this discharge.

The spillway leads to a channel of the same cross-section and roughness but slope 1 in 1000. A set of blocks is placed at the start of this channel, causing a hydraulic jump to occur there.

(c) Calculate the normal depth in the downstream channel.

(d) Assuming that the blocks cause a hydraulic jump to the downstream normal depth, calculate the total force on the blocks.

Q21.

(a) A sluice gate controls the flow in a rectangular channel of width 3 m. If the depth of parallel flow just upstream of the sluice is 2 m and that just downstream of the sluice is 0.3 m calculate the discharge in the channel.

(b) If the channel has slope 1 in 1000 and a Manning's roughness coefficient $n = 0.014\ \mathrm{m}^{-1/3}$ s, calculate the normal depth at this discharge.

(c) A short distance downstream of the sluice a set of blocks provokes a hydraulic jump. If the flow depth immediately downstream of the blocks is the normal depth, calculate the total force on the blocks.

Hydraulic Jumps

Q22.

A rectangular channel carrying 10 $\mathrm{m}^3\ \mathrm{s}^{-1}$ undergoes an abrupt expansion from width 4 m to width 8 m, triggering a hydraulic jump. The upstream depth is 0.5 m. Stating assumptions clearly, find the downstream depth.

Q23. (Exam 2024)

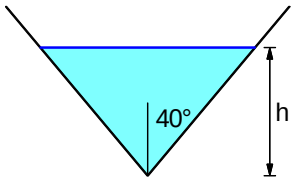
A long rectangular channel of width 3.5 m, streamwise slope 0.003 and Manning's roughness coefficient $n = 0.016\ \mathrm{m}^{-1/3}$ s carries water at a volume flow rate of 8 $\mathrm{m}^3\ \mathrm{s}^{-1}$.

(a) Determine the critical and normal depths in this channel and show that the normal depth is subcritical.

(b) A short section of the channel is narrowed to a width of 2.2 m. Show that this will cause a hydraulic transition and find the water depth just downstream of the contracted section.

(c) A short distance downstream of the contracted section the channel widens abruptly to a width of 5 m, provoking a hydraulic jump. Find the water depth downstream of the hydraulic jump.

Q24.



An artificial channel has a V-shaped cross-section with semi-angle 40°. Depths are measured from the bottom of the vee.

(a) If the flow rate is 16 $m^3\ s^{-1}$, find the critical depth.

(b) A hydraulic jump is observed to occur at this flow rate. If the depth on one side of the jump is 1.85 m, find the depth on the other side of the jump.

Note: you would be well-advised to work from first principles, not inapplicable formulae.

Q25.

Water flows in a V-shaped channel, whose cross-sectional shape is that of an equilateral triangle. If a hydraulic jump occurs and the water depths on either side of the jump are 1.2 m and 2.1 m (measured from the bottom of the vee), find the volume flow rate and the head lost in the jump.

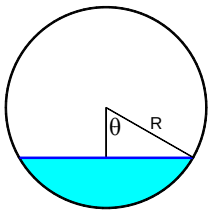
Q26.

A hydraulic jump occurs in a horizontal culvert of circular cross section and radius 2 m, carrying water at 1.5 $m^3\ s^{-1}$. The depth of water (relative to the invert) is 0.3 m. Find:

(a) the water depth downstream of the jump;

(b) the Froude numbers upstream and downstream of the jump.

Data. The distance $\bar{d}$ from the chord to the centroid of a circular segment is



$$\bar{d} = R\left[\frac{\frac{2}{3}\sin^3\theta}{\theta - \frac{1}{2}\sin(2\theta)} - \cos\theta\right]$$

where R is radius and θ is the semi-angle subtended at the centre (see figure).

Gradually-Varied Flow

Q27.
A wide channel has a slope of 2×10^{-5}, Manning's n of 0.01 $m^{-1/3}$ s and flow rate of 0.5 m^3 s^{-1} per metre width. At the end of the channel there is a free overfall. Using 2 steps in the gradually-varied-flow equation, determine the distance from the overfall to where the depth is 1 m. Then code your calculations in a spreadsheet and repeat with 2, 5, 10, 50 and 100 steps.

Q28.
A long river channel may be modelled as a rectangular section of width 5 m and streamwise slope 0.0006, carrying a flow of 7 m^3 s^{-1}. Manning's roughness parameter may be taken as $n = 0.035\ \mathrm{m}^{-1/3}$ s.

(a) Calculate the normal depth in the channel.

(b) A weir of height 1.75 m is constructed at one point in the channel. Demonstrate that the flow will undergo a subcritical to supercritical flow transition at the weir and calculate the water depth just upstream of the weir.

(c) Use the gradually-varied-flow equation with two steps to determine for how far upstream the water-level rise caused by the construction of the weir exceeds 0.25 m.

Q29.
A rectangular chute of width 1.5 m conveys water to a free overfall. The flow is drawn down at the overfall and the depth near the end is 0.6 m. The chute has streamwise slope 0.002 and Manning's $n = 0.014\ \mathrm{m}^{-1/3}\ \mathrm{s}$.

(a) Calculate the flow rate.

(b) Use two steps in the GVF equation to estimate the distance upstream to the point where the depth is 0.8 m.

Q30.
A wide river has streamwise slope 8×10^{-4}, a Manning's roughness coefficient $n = 0.03\ \mathrm{m}^{-1/3}\ \mathrm{s}$ and flows into the sea at $0.7\ \mathrm{m}^3\ \mathrm{s}^{-1}$ per metre width. At high tide the depth of water in the river just before outflow at the coast is 2.0 m.

(a) Calculate the normal depth and critical depth in the river and demonstrate that the river slope is hydrodynamically mild at this flow rate.

(b) Assuming the river depth to be uniform far upstream, sketch the depth profile in the river as it approaches outflow to the sea, indicating carefully depths with respect to normal and critical depths.

(c) Using two steps in the gradually-varied-flow equation, determine the distance from the coast at which the water depth in the river is 1.0 m.

(d) If the river slope had been hydrodynamically steep instead of mild, sketch the depth profile as it approaches outflow.

Q31. (Exam 2023)

(a) A river can be approximated as a long rectangular channel of width 6 m, streamwise slope 0.01 and Manning's $n = 0.035\ \mathrm{m}^{-1/3}\ \mathrm{s}$. Calculate the normal depth and critical depth at a flow rate of $9\ \mathrm{m}^3\mathrm{s}^{-1}$ and determine whether the normal flow is subcritical or supercritical.

(b) A bank collapse reduces the width of the river to 2.4 m. Show that, for the flow rate in part (a), this will cause a hydraulic transition, and compute depths just upstream and downstream of the damaged section.

(c) Calculate the distance from the end of the bank collapse to a downstream hydraulic jump, using two steps in the gradually-varied-flow equation.

Q32.

(a) A broad-crested weir with a crest height of 1.5 m controls the flow in a wide channel. If the discharge is 0.9 $m^3 s^{-1}$ per metre width find the depths of flow over, just upstream and just downstream of the weir.

(b) If the channel has slope 3×10^{-4} and Manning's roughness coefficient 0.012 $m^{-1/3}$ s find the normal depth at this discharge.

(c) Using *two* steps in the gradually-varied-flow equation, find the distance upstream of the weir at which the depth is 0.1 m greater than the normal depth.

(d) Downstream of the weir the flow undergoes a hydraulic jump back to normal flow. Use *one* step in the gradually-varied-flow equation to estimate the distance from weir to hydraulic jump.

(e) Sketch the surface profile, indicating the main flow features and, in particular, showing the 2-character designation for each of the GVF curves (S1, M2 etc.).

Q33.

A long rectangular channel of width 5 m, slope 5×10^{-4} and Chézy coefficient 100 $m^{1/2} s^{-1}$ carries a discharge of 15 $m^3 s^{-1}$. A broad-crested weir is installed with a height of 1.0 m, which causes a critical-flow transition at the weir, with a hydraulic jump further downstream. Calculate:

(a) the normal and critical depths;

(b) the depth of flow at the downstream end of the weir;

(c) the depth of flow just upstream of the hydraulic jump;

(d) the distance of the hydraulic jump from the downstream end of the weir.

You may assume that the flow downstream of the hydraulic jump is normal flow. Use two steps in the GVF equation to locate the hydraulic jump.

Q34. (Exam 2019)

A long river channel has a rectangular section of width 7 m and streamwise slope 0.005. Manning's roughness parameter may be taken as $n = 0.035\ \mathrm{m}^{-1/3}\ \mathrm{s}$.

(a) In undisturbed flow, the depth is found to be 1.6 m. Calculate the flow rate.

(b) Calculate the critical depth in the channel. State whether the channel slope is mild or steep, justifying your answer.

(c) A broad-crested weir of height 1.8 m is constructed at one point in the channel. Demonstrate that the flow will undergo a subcritical to supercritical flow transition here and, assuming no immediate hydraulic jump, calculate the water depths just upstream and downstream of the weir.

(d) Use the gradually-varied-flow equation with one step to estimate the position of the hydraulic jump, stating whether it is upstream or downstream of the weir.

Q35. (Exam 2018)
An undershot sluice controls the flow in a long rectangular channel of width 2.5 m, Manning's roughness coefficient $n = 0.012\ \mathrm{m}^{-1/3}\ \mathrm{s}$ and streamwise slope 0.002. The depths of parallel flow upstream and downstream of the gate are 1.8 m and 0.3 m, respectively.

(a) Assuming no losses at the sluice, find the volume flow rate Q.
(b) Find the normal and critical depths in the channel.
(c) Compute the distance from the sluice gate to the hydraulic jump, assuming normal depth downstream of the jump. Use two steps in the gradually-varied-flow equation.

Q36. (Exam 2017)
Water flows along an irrigation channel of rectangular cross section and width 4 m. The streamwise slope is 0.004 and Manning's roughness coefficient n is 0.022 $\mathrm{m}^{-1/3}$ s. The flow rate is determined by measuring the depth of flow in a local constricted section of width 2.0 m, which provokes a hydraulic transition and is long enough for parallel flow to be established.

(a) If the depth of parallel flow in the constricted section is 2.5 m, find the volumetric flow rate and the depths of flow immediately upstream and downstream of the constricted section.
(b) A hydraulic jump occurs some distance downstream of the constricted section. If there is a long undisturbed fetch further downstream, find the depth of flow on either side of the jump.
(c) Use one step in the gradually-varied flow equation to estimate the distance from the end of the channel constriction to the hydraulic jump.

Q37. (Exam 2024)
A broad-crested weir of height 1.8 m is constructed to maintain adequate water depth in the upper reaches of a wide river. The river has slope 0.001 and Manning's roughness coefficient $n = 0.025\ \mathrm{m}^{-1/3}\ \mathrm{s}$.

(a) If the volume flow rate is $0.8\ \mathrm{m}^3\ \mathrm{s}^{-1}$ per metre width and a hydraulic transition occurs over the weir, find:
    (i) the depth of flow over the top of the weir;
    (ii) the depth of flow just upstream of the weir.
(b) Use two steps in the gradually-varied-flow equation to find at what distance upstream the water depth is equal to the weir height.
(c) If the weir were to fail catastrophically, what would be the depth of flow in the river?

Wave Speed and Supercritical Flow

Q38. (Exam 2017)

(a) Define the Froude number, Fr, in open-channel flow, and explain how it can be interpreted in terms of current speed and wave speed.

(b) By means of a suitably-annotated diagram, or otherwise, derive the result

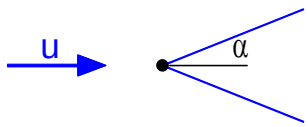
$$\sin\alpha = \frac{1}{\mathrm{Fr}}$$

for the semi-angle α of the bounding wedge made by surface waves downstream of a small disturbance in supercritical flow (A).

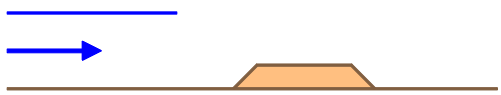
(c) A long channel carries a discharge of 2.5 $m^3\ s^{-1}$ per metre width. A pole in midstream causes a wedge-shaped disturbance on the surface downstream, with semi-angle 20°. Find the depth and speed of flow.

(d) The bed of the channel is raised for a short distance (B). Find the minimum height by which it must be raised in order to provoke a hydraulic jump upstream of the bed rise.

(e) If the bed rise is half the value in (d), find the depth of parallel flow over the raised section.



(A)



(B)