Open-channel flow is an important area of fluid mechanics for civil engineers. It describes the flow in rivers, man-made channels and partially-full pipes (sewers, drains), as well as the behaviour of hydraulic structures such as weirs, spillways and sluices.

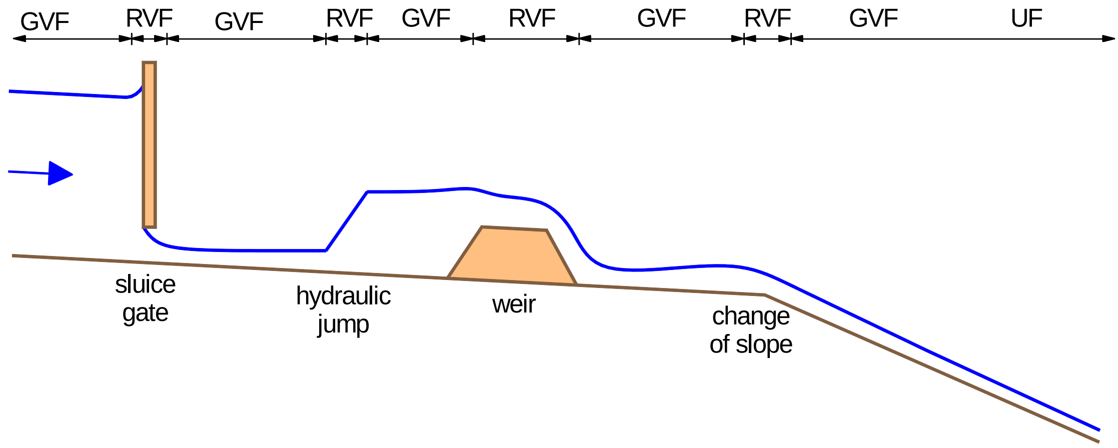
The common feature of all open-channel flows is the free surface, where the gauge pressure $p = 0$. All such flows are gravity-driven, with the discharge Q and flow depth h dependent on the balance between the downslope component of gravity and bed friction.

1.1 Classification

Steady or Unsteady

Open-channel flow is *steady* if all flow properties are independent of time. The most important examples of *unsteady* flow are waves, surges and tidal flows. Waves are covered in the second half of Hydraulics 3.

In this part of the course we consider only steady flow. For a given channel this consists of various fetches considered as *uniform*, *rapidly-varied* or *gradually-varied* flow.



Uniform Flow

In uniform flow the depth and velocity do not vary in the direction of flow. This can only occur in long straight channels of uniform cross-section, constant slope and no side streams. (These are called *prismatic channels*; they are always an approximation for natural water courses like rivers). Here, the downslope component of weight exactly balances bed friction. Steady uniform flow is called *normal flow*. Steady downslope flows in uniform channels tend to normal flow if there is sufficient undisturbed length.

Rapidly-Varied Flow (RVF)

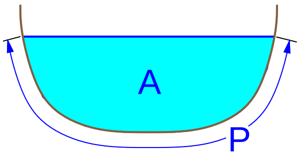
Rapidly-varied flow occurs when the flow adjusts over relatively short distances (a few times the flow depth). Examples are hydraulic jumps and flow past hydraulic structures such as weirs (local bed rise), venturis (local narrowing) and sluices (variable-opening gates). As the streamwise distance is short, changes to the flow are obtained by neglecting bed friction.

Gradually-Varied Flow (GVF)

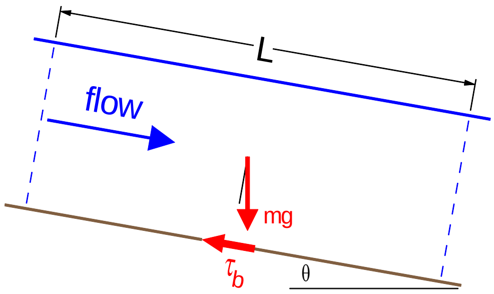
In gradually-varied flow the water depth changes slowly with streamwise distance (typically over distances of hundreds or thousands of times the flow depth) because of an imbalance between gravitational and friction forces. This may occur as the result of a change in channel properties (slope, cross-section or roughness) or an adjustment brought about by upstream or downstream disturbances such as weirs and sluices. Because the variation is gradual the flow can still be treated as *one-dimensional* (varying only with *x*) and the pressure as *hydrostatic*.

1.2 Normal Flow

General Friction Law



Let the cross-sectional area of flow be A and the wetted perimeter be P.



The bed shear stress is τ_b. Since bed friction (stress × wetted area) balances the downslope component of weight ($mg\sin\theta$), then, for a streamwise length L,

$$\tau_b \times (PL) = (\rho AL) \times g\sin\theta$$

Hence,

$$\tau_b = \rho g R_h S \qquad (1)$$

where the *hydraulic radius* R_h is defined as[1]

$$R_h = \frac{A}{P} \qquad (2)$$

and S is the streamwise slope. (Strictly, $S = \tan\theta$, but $\tan\theta \approx \theta \approx \sin\theta$ for small slopes).

The skin-friction coefficient c_f is *defined* as the ratio of bed shear stress τ_b to dynamic pressure $\frac{1}{2}\rho V^2$, where V is the average velocity over the cross-section. i.e.

$$\tau_b = c_f(\tfrac{1}{2}\rho V^2) \qquad (3)$$

Equating the two expressions (1) and (3) for τ_b, and rearranging for V:

$$V = \sqrt{\frac{2g}{c_f} R_h S} \qquad (4)$$

[1] Just be warned: some textbooks call this the *hydraulic mean depth* and give it the symbol m. I don't like either.

There are various possible friction laws, including:

- Darcy friction factor ($\lambda = 4c_f$, $D_h = 4R_h$) $\Rightarrow$ $V = \sqrt{2g\dfrac{D_h}{\lambda}S}$ (pipe flow)
- Chézy ($C = \sqrt{2g/c_f}$) $\Rightarrow$ $V = C\sqrt{R_h S}$
- Manning ($\sqrt{2g/c_f} = \frac{1}{n}R_h^{1/6}$) $\Rightarrow$ $V = \dfrac{1}{n}R_h^{2/3}S^{1/2}$

We will usually use Manning's equation in this course, but see the Examples for alternatives.

Main Calculation Formulae

Discharge: $Q = VA$

Manning's equation: $V = \dfrac{1}{n}R_h^{2/3}S^{1/2}$

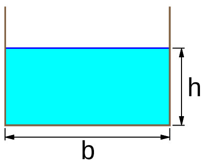
Hydraulic radius: $R_h = \dfrac{A}{P}$

Hydraulic Radius For Particular Channel Shapes

In each case h is the depth of flow (measured from the lowest point, or *invert*).

- Rectangular (width b)

$$R_h = \frac{bh}{b+2h} \quad = \frac{h}{1+\frac{2h}{b}}$$

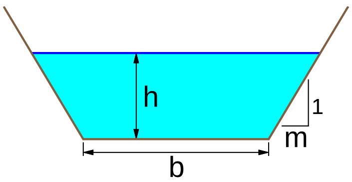


- Wide (obtained from the above in the limit $h/b \ll 1$):

$$R_h = h$$

- Trapezoidal (bottom width b; side slope expressed here as $horizontal{:}\,vertical = m{:}\,1$, but could be stated many ways.)

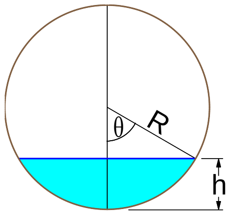
$$R_h = \frac{h(b+mh)}{b+2h\sqrt{1+m^2}}$$



- Circular (radius R)

$$R_h = \frac{2(\frac{1}{2}R^2\theta - \frac{1}{2}R\sin\theta\,.\,R\cos\theta)}{2R\theta} \quad = \frac{R}{2}\left(1 - \frac{\sin 2\theta}{2\theta}\right)$$

where $h = R - R\cos\theta$



Normal Depth

For any given discharge Q there will be a particular *normal depth*, h_n. (We will drop the subscript n when the context is clear.) The relationship between them arises from:

$$Q = VA$$

where

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

and A and R_h are functions of h.

Thus,

$$Q = f(h) \tag{5}$$

where function $f(h)$ depends on the shape of the channel, roughness n and slope S.

In most cases, however, we need to know depth h for a particular discharge Q, not vice versa. Only for a limited number of channel shapes (e.g. wide or V-shaped) can (5) be rearranged explicitly for h. More generally, for a given discharge Q it may be solved numerically by

- repeated trial for values of h, or, after suitable rearrangement,
- iteration.

For *wide* channels it is usual to work in terms of the *flow per unit width*,

$$q = \frac{Q}{b} \tag{6}$$

The corresponding area per unit width is the flow depth h. Then, by Manning, per unit width:

$$q = Vh \quad = \frac{1}{n} R_h^{2/3} S^{1/2} \times h \qquad \text{where} \qquad R_h = h$$

Hence,

$$q = \frac{h^{5/3} S^{1/2}}{n}$$

or, by inversion,

$$h = \left(\frac{nq}{\sqrt{S}} \right)^{3/5}$$

For *rectangular* channels, $A = bh$ and $R_h = h/(1 + 2h/b)$. Then,

$$Q = VA \quad = \frac{1}{n} \left(\frac{h}{1 + 2h/b} \right)^{2/3} S^{1/2} \times bh \quad = \frac{b\sqrt{S}}{n} \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

This can be solved by either repeated trial or rearranged to give an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}} \right)^{3/5} (1 + 2h/b)^{2/5}$$

Similar iteration formulae may be derived for trapezoidal channels and many other shapes.

Example.
The discharge in a channel with bottom width 3 m is 12 $m^3 s^{-1}$. If Manning's n is 0.013 $m^{-1/3}$ s and the streamwise slope is 1 in 200, find the normal depth if:
(a) the channel has vertical sides (i.e. rectangular channel);
(b) the channel is trapezoidal with side slopes 2H:1V.

1.3 Flow Energy: Fluid Head

In hydraulics, because many flows are gravity-driven, it is common to express energy or pressure in height units:

total pressure: $$p + \rho g z + \frac{1}{2}\rho V^2$$ (à la Bernoulli's equation) (7)

Divide by ρg:

total head (H): $$\frac{p}{\rho g} + z + \frac{V^2}{2g}$$ (8)

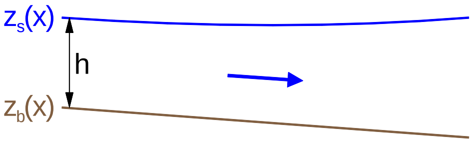
The total head is the energy per unit weight (mgH divided by mg) and is convenient because it is easily determined from still-water levels ($H = z$ when $p = V = 0$).

If there is no vertical acceleration then the pressure at any streamwise location is *hydrostatic*:

$p + \rho g z$ is constant along a vertical line (9)

or, dividing by ρg,

$$\frac{p}{\rho g} + z$$ is constant along a vertical line (10)



But $p = 0$ at the free surface. Hence,

$$\frac{p}{\rho g} + z = \left(\frac{p}{\rho g} + z\right)_{\text{surface}} = z_s$$

So, if the pressure is hydrostatic, the sum of pressure and elevation heads is just the level of the free surface.

Hence, in regions of *uniform* or *gradually-varied* flow the total head is given by[2]

$$H = z_s + \frac{V^2}{2g} \qquad (11)$$

The first term is a measure of potential energy; the second term (the *dynamic head*) is a measure of kinetic energy.

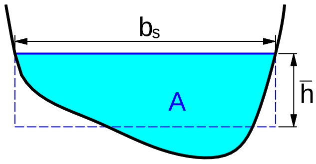
[2] In advanced analysis it is necessary to precede the kinetic energy term $V^2/2g$ by a corrective multiplicative factor α (the *kinetic energy correction coefficient*) to account for the fact that the velocity profile is not uniform, and hence the mean squared velocity $< U^2 >$ is not equal to the square of the mean velocity $< U >^2$. For fully turbulent flow, α is typically about 1.02; i.e. very close to 1. Hence, this factor will be ignored here.

1.4 Froude Number

In Hydraulics 2 we defined a Froude number in general as $\mathrm{Fr} = V/\sqrt{gL}$, where V and L are "representative" velocity and length scales. In open-channel flow we invariably take V to be the average velocity over a cross section and L to be the average depth; i.e.

$$\mathrm{Fr} \equiv \frac{V}{\sqrt{g\bar{h}}} \qquad (12)$$

For a wide or rectangular channel $\bar{h}$ is simply equal to h. For a non-rectangular channel $\bar{h}$ is the *mean* depth: the depth of a rectangle with the same cross-sectional area and surface width:



$$\bar{h} = \frac{A}{b_s} \qquad (13)$$

where A is the water cross-section and b_s is the *surface* width.

Where $\mathrm{Fr} < 1$ the flow is said to be *subcritical* or *tranquil*.
Where $\mathrm{Fr} > 1$ the flow is said to be *supercritical* or *rapid*.
Reasons for the term "*critical*" and this definition of $\bar{h}$ will be explained in Section 2. Note that Fr is not constant, but changes along the channel as the depth changes.

Interpretations of the Froude Number

(1) (Square root of) the ratio of inertial forces (i.e. mass × acceleration) to gravitational forces.

For mass m, velocity V, lengthscale h (and hence timescale h/V):

$$\text{mass} \times \text{acceleration} \sim m \times \frac{V}{h/V} = m\frac{V^2}{h}$$

$$\text{gravitational force} \sim mg$$

Hence,

$$\frac{\text{inertial force}}{\text{gravitational force}} \sim \frac{V^2}{gh} = \mathrm{Fr}^2$$

(2) Ratio of water velocity (V) to long-wave speed ($\sqrt{gh}$) – see Section 4.
This is important because information can only propagate *upstream* if the water velocity is less than the wave speed; i.e. if $\mathrm{Fr} < 1$.

Subcritical flow	⇔	$\mathrm{Fr} < 1$	⇔	downstream control
Supercritical flow	⇔	$\mathrm{Fr} > 1$	⇔	upstream control

Where $\mathrm{Fr} = 1$ the flow is said to be *critical*.

(3) The minimum *specific energy* (see Section 2) for a given discharge occurs at the *critical depth* where $\text{Fr} = 1$, and separates regions of deep, slow, subcritical flow ($\text{Fr} < 1$) and shallow, fast, supercritical flow ($\text{Fr} > 1$).

The flow may pass through such a region at a broad-crested weir, venturi flume or free overfall, providing a *control point* where the relationship between fluid depth and discharge is known.

Example.
The discharge in a rectangular channel of width 6 m with Manning's $n = 0.012\ \text{m}^{-1/3}\ \text{s}$ is 24 $\text{m}^3\ \text{s}^{-1}$. If the streamwise slope is 1 in 200 find:
(a) the normal depth;
(b) the Froude number at the normal depth;
(c) the critical depth.

State whether the normal flow is subcritical or supercritical.

Appendix: Typical values of Manning's n (from White, 2011)

		n ($m^{-1/3}$ s)
Artificial lined channels	Glass	0.01
	Brass	0.011
	Steel, smooth	0.012
	painted	0.014
	riveted	0.015
	Cast iron	0.013
	Concrete, finished	0.012
	unfinished	0.014
	Planed wood	0.012
	Clay tile	0.014
	Brickwork	0.015
	Asphalt	0.016
	Corrugated metal	0.022
	Rubble masonry	0.025
Excavated earth channels	Clean	0.022
	Gravelly	0.025
	Weedy	0.03
	Stony, cobbles	0.035
Natural channels	Clean and straight	0.03
	Sluggish, deep pools	0.04
	Major rivers	0.035
Floodplains	Pasture, farmland	0.035
	Light brush	0.05
	Heavy brush	0.075
	Trees	0.15