Q1.
$b = 5$ m
$Q = 20$ m^3 s^{-1}
$n = 0.02$ m$^{-1/3}$ s

Discharge:

$$Q = VA$$

where, in normal flow:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1 + 2h/b)^{2/3}} S^{1/2}$$

or, rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5} \qquad (*)$$

(a) When $S = 0.001$, substitution of numerical values in (*) yields an iterative formula

$$h = 1.745(1 + 0.4h)^{2/5}$$

Iteration (from, e.g., $h = 1.745$) gives

$$h = 2.257 \text{ m}$$

Then,

$$V = \frac{Q}{A} = \frac{Q}{bh} = \frac{20}{5 \times 2.257} = 1.772 \text{ m s}^{-1}$$

$$\text{Fr} = \frac{V}{\sqrt{gh}} = \frac{1.772}{\sqrt{9.81 \times 2.257}} = 0.3766$$

Answer: normal depth = 2.26 m; Froude number = 0.377

(b) When $S = 0.01$, substitution of numerical values in (*) yields an iterative formula

$$h = 0.8747(1 + 0.4h)^{2/5}$$

Iteration (from, e.g., $h = 0.8747$) gives

$$h = 1.001 \text{ m}$$

Then,

$$V = \frac{Q}{A} \quad = \frac{Q}{bh} \quad = \frac{20}{5 \times 1.001} \quad = 3.996 \text{ m s}^{-1}$$

$$\text{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{3.996}{\sqrt{9.81 \times 1.001}} \quad = 1.275$$

Answer: normal depth = 1.00 m;   Froude number = 1.28

(c) For a rectangular channel the critical depth is

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

where the discharge per unit width is

$$q = \frac{Q}{b} \quad = \frac{20}{5} \quad = 4 \text{ m}^2 \text{ s}^{-1}$$

Hence,

$$h_c = \left(\frac{4^2}{9.81}\right)^{1/3} \quad = 1.177 \text{ m}$$

Answer: 1.18 m

(d) The critical slope is that slope S at which the normal depth equals the critical depth h_c. Here,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1 + 2h/b)^{2/3}} S^{1/2}$$

with $h = h_c = 1.177$ m.

Rearranging for S:

$$S = \left(\frac{nQ}{b}\right)^2 \times \frac{(1 + 2h/b)^{4/3}}{h^{10/3}}$$

$$= \left(\frac{0.02 \times 20}{5}\right)^2 \times \frac{(1 + 2 \times 1.177/5)^{4/3}}{1.177^{10/3}} \quad = 6.218 \times 10^{-3}$$

Answer: 0.00622

Q2.
(a)
$n = 0.012 \text{ m}^{-1/3} \text{ s}$
$Q = 2.6 \text{ m}^3 \text{ s}^{-1}$
$S = 0.0004$

Let the *bottom* width be b (= 0.6 m) and the reciprocal of the side slope be m. The half-width changes from 0.3 m to 1.5 m over a depth of 1.6 m. Hence,

$$m = \frac{1.2}{1.6} \quad = 0.75$$

Area and wetted perimeter:

$$A = \frac{1}{2}(b \quad + b + 2 \times mh) \times h \quad = hb(1 + 0.75h/b)$$

$$P = b + 2\sqrt{1 + m^2}h \quad = b(1 + 2.5h/b\,)$$

Hydraulic radius:

$$R_h \equiv \frac{A}{P} = h\left(\frac{1 + 0.75h/b}{1 + 2.5h/b}\right)$$

In normal flow,

$$Q = VA \quad = \frac{1}{n}R_h^{2/3}S^{1/2}A$$

Hence,

$$Q = \frac{1}{n}h^{2/3}\left(\frac{1 + 0.75h/b}{1 + 2.5h/b}\right)^{2/3} S^{1/2}hb(1 + 0.75h/b)$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = h^{5/3}\frac{(1 + 0.75h/b)^{5/3}}{(1 + 2.5h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5}\frac{(1 + 2.5h/b)^{2/5}}{1 + 0.75h/b}$$

Here, with lengths in metres,

$$h = 1.774\frac{(1 + 4.167h)^{2/5}}{1 + 1.25h}$$

Iteration (from, e.g., $h = 1.774$) gives

$$h_n = 1.393 \text{ m}$$

Answer: 1.39 m

(b) Froude number $\text{Fr} = V/\sqrt{g\bar{h}}$. Need velocity V (= Q/A) and mean depth $\bar{h}$ (= A/b_s).

$$A = hb(1 + 0.75h/b)$$
$$b_s = b + 2mh \quad = b + 1.5h$$

At depth $h = 1.393$ m,

$$A = hb(1 + 0.75h/b) \quad = 2.291 \text{ m}^2$$
$$b_s = b + 1.5h \quad = 2.690 \text{ m}$$

Then,

$$V = \frac{Q}{A} \quad = \frac{2.6}{2.291} \quad = 1.135 \text{ m s}^{-1}$$
$$\bar{h} = \frac{A}{b_s} \quad = \frac{2.291}{2.690} \quad = 0.8517 \text{ m}$$

Froude number:

$$\text{Fr} = \frac{V}{\sqrt{g\bar{h}}} \quad = 0.3927$$

Answer: 0.393

(c) At the critical depth, $\text{Fr} = 1$. But

$$\text{Fr}^2 = \frac{V^2}{g\bar{h}} = \frac{Q^2/A^2}{gA/b_s} \quad = \frac{Q^2 b_s}{gA^3}$$

$$\Rightarrow \qquad \frac{Q^2(b + 1.5h)}{gh^3b^3(1 + 0.75h/b)^3} = 1$$

$$\Rightarrow \qquad \frac{Q^2(1 + 1.5h/b)}{gb^2(1 + 0.75h/b)^3} = h^3$$

$$\Rightarrow \qquad h = \left(\frac{Q^2}{gb^2}\right)^{1/3} \frac{(1 + 1.5h/b)^{1/3}}{1 + 0.75h/b}$$

Here, with lengths in metres,

$$h = 1.242\,\frac{(1 + 2.5h)^{1/3}}{1 + 1.25h}$$

Iteration (from, e.g., $h = 1.242$) gives

$$h_c = 0.8735 \text{ m}$$

Answer: 0.874 m

(d) The critical slope is that slope S at which the normal depth equals the critical depth h_c.

From the earlier working for the normal depth:

$$\frac{nQ}{b\sqrt{S}} = h^{5/3}\frac{(1+0.75h/b)^{5/3}}{(1+2.5h/b)^{2/3}}$$

Making S the subject:

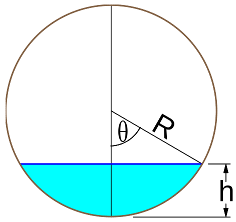
$$S = \left(\frac{nQ}{b}\right)^2 \frac{1}{h^{10/3}}\frac{(1+2.5h/b)^{4/3}}{(1+0.75h/b)^{10/3}}$$

Putting $h = h_c = 0.8735$ m gives

$$S = 2.805 \times 10^{-3}$$

Answer: 0.00281

Q3.
(a)
$R = 0.7$ m
$S = 0.02$
$Q = 0.8$ m^3 s^{-1}
$n = 0.013$ m$^{-1/3}$ s



The simplest indicator of fill is the semi-angle θ, which is related to the depth by

$$h = R - R\cos\theta$$

Area (found by subtracting a triangle from a sector) and wetted perimeter are given by

$$A = 2 \times [\frac{1}{2}R^2\theta - \frac{1}{2}(R\sin\theta)(R\cos\theta)] \quad = R^2(\theta - \sin\theta\cos\theta)$$

$$P = 2R\theta$$

where θ is in radians, $0 \le \theta \le \pi$.

A series of trial solutions of θ is used to target a flow rate of $Q = 0.8$ m^3 s^{-1}. The sequence of calculations is:

$$R_h \equiv \frac{A}{P}$$

$$V = \frac{1}{n}R_h^{2/3}S^{1/2}$$

$$Q = VA$$

The working is set out in the table below. All values are in metre-second units. Trial values of θ after the first two are by interpolation/extrapolation. Obviously, the Solver tool in Excel, or similar, could be used to expedite this.

θ	$A = 0.49(\theta - \sin\theta\cos\theta)$	$P = 1.4\theta$	$R_h = A/P$	$V = 10.879R_h^{2/3}$	$Q = VA$
1	0.2672	1.400	0.1909	3.607	0.9638
0.5	0.0388	0.700	0.0554	1.554	0.0585
0.9095	0.2082	1.273	0.1636	3.254	0.6775
0.9482	0.2325	1.327	0.1752	3.406	0.7919
0.9509	0.2342	1.331	0.1760	3.417	**0.8003**

$\theta = 0.9509$ radians gives

$$h = 0.7 - 0.7\cos 0.9509 \quad = 0.2933 \text{ m}$$

Answer: 0.293 m

(b)
For the Froude number we need the mean depth, $\bar{h}$, and for that we require first a formula for the water-surface width, b_s . Here,

$$b_s = 2R\sin\theta \quad = 2\times 0.7\times\sin 0.9509 \quad = 1.140 \text{ m}$$

The mean depth is then

$$\bar{h} = \frac{A}{b_s} \quad = \frac{0.2342}{1.140} \quad = 0.2054 \text{ m}$$

and the Froude number is

$$\text{Fr} = \frac{V}{\sqrt{g\bar{h}}} \quad = \frac{3.417}{\sqrt{9.81\times 0.2054}} \quad = 2.407$$

Answer: 2.41

(c) For the critical depth we use a set of trial values of θ to get $\text{Fr} = 1$.

$$\frac{V}{\sqrt{g\bar{h}}} = 1$$

$$\Rightarrow \qquad \frac{V^2}{g\bar{h}} = 1$$

$$\Rightarrow \qquad \frac{(Q/A)^2}{gA/b_s} = 1$$

$$\Rightarrow \qquad \frac{Q^2}{g}\frac{b_s}{A^3} = 1$$

Substituting expressions for area A and width of water surface b_s:

$$\frac{Q^2}{g}\frac{2R\sin\theta}{R^6(\theta-\sin\theta\cos\theta)^3} = 1$$

Substituting numerical values ($Q = 0.8 \text{ m}^3 \text{ s}^{-1}$, $R = 0.7$ m):

$$0.7763\times\frac{\sin\theta}{(\theta-\sin\theta\cos\theta)^3} = 1$$

Try a sequence of values of θ and work out the LHS:

θ	$f(\theta)$
1	4.028
1.5	0.2651
1.2	1.129
1.22	1.009
1.222	**0.998**

$\theta = 1.222$ radians gives

$$h = 0.7 - 0.7\cos(1.222\text{ rad}) \quad = 0.4608\text{ m}$$

Answer: 0.461 m

Q4.
$Q = 1.5 \text{ m}^3 \text{ s}^{-1}$
$n = 0.02 \text{ m}^{-1/3} \text{ s}$
$S = 0.001$

Preliminary

To avoid having to consider alternative formulae for depths above and below the vee a quick calculation of flow rate and Froude number *with water depth precisely equal to the top of the vee* (0.8 m) will tell you whether the normal depth (part (a)) and critical depth (part (c)) lie above or below this level.

For part (a) calculate the flow rate in normal flow at depth h = 0.8 m (sloping side 1.6 m and surface width $2 \times 0.8\sqrt{3}$):

$$A = \frac{1}{2} \times 0.8 \times 1.6\sqrt{3} \quad = 1.109 \text{ m}^2$$

$$P = 2 \times 1.6 \quad = 3.2 \text{ m}$$

$$R_h = \frac{A}{P} \quad = 0.3466 \text{ m}$$

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \quad = \frac{1}{0.02} \times 0.3466^{2/3} \times \sqrt{0.001} \quad = 0.7802 \text{ m s}^{-1}$$

$$Q = VA \quad = 0.7802 \times 1.109 \quad = 0.8652 \text{ m}^3 \text{ s}^{-1}$$

This is less than the required 1.5 $\text{m}^3 \text{ s}^{-1}$. Hence, *the normal depth is above the vee.*

For part (c) calculate the Froude number for the given flow rate and depth 0.8 m:

$$V = \frac{Q}{A} \quad = \frac{1.5}{1.109} \quad = 1.353 \text{ m s}^{-1} \qquad \text{(note: \textit{not} normal flow)}$$

$$\bar{h} = \frac{A}{b_s} \quad = \frac{0.64\sqrt{3}}{1.6\sqrt{3}} \quad = 0.4 \text{ m}$$

$$\text{Fr} = \frac{V}{\sqrt{g\bar{h}}} \quad = \frac{1.353}{\sqrt{9.81 \times 0.4}} \quad = 0.6830$$

This is subcritical ("deep and slow"). Hence, *the critical depth is below the top of the vee.*

(a) From the preliminary calculation above, the normal depth h is greater than 0.8 m:

$$A = 1.109 + (h - 0.8) \times 1.6\sqrt{3} \quad = 2.771h - 1.108$$

$$P = 3.2 + 2(h - 0.8) \quad = 1.6 + 2h$$

$$Q = VA \qquad \text{where} \qquad V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad R_h = \frac{A}{P}$$

$$\Rightarrow \qquad Q = \frac{S^{1/2}}{n} \frac{A^{5/3}}{P^{2/3}}$$

$$\Rightarrow \qquad Q = 1.581\frac{(2.771h - 1.108)^{5/3}}{(1.6 + 2h)^{2/3}} \qquad\qquad (*)$$

There are many ways of rearranging this for iteration; however, in this case it is straightforward to solve by repeated trial of h, aiming for $Q = 1.5$ m^3 s^{-1}. Trial values of h after the first two are guided by interpolation.

h (m)	Q (m^3 s^{-1})
0.800	0.865
1.000	1.571
0.980	1.496
0.981	**1.500**

Answer: 0.981 m

(b) When $h = 0.981$ m,

$$A = 2.771h - 1.108 \quad = 1.610 \text{ m}^2$$

$$V = \frac{Q}{A} \quad = \frac{1.5}{1.610} \quad = 0.9317 \text{ m s}^{-1}$$

$$b_s = 1.6 \times \sqrt{3} \text{ m}$$

$$\bar{h} = \frac{A}{b_s} \quad = \frac{1.610}{1.6\sqrt{3}} \quad = 0.5810 \text{ m}$$

$$\text{Fr} = \frac{V}{\sqrt{g\bar{h}}} \quad = \frac{0.9317}{\sqrt{9.81 \times 0.5810}} \quad = 0.3903$$

Answer: 0.390

(c) The critical depth is defined as that for which $\text{Fr} = 1$ at the given flow rate.

$$\text{Fr} \equiv \frac{V}{\sqrt{g\bar{h}}} \quad = \frac{Q/A}{\sqrt{gA/b_s}}$$

Because of the square root it is easier to work with Fr^2:

$$\text{Fr}^2 = \frac{Q^2}{g}\frac{b_s}{A^3}$$

From the preliminary calculations at the top, we know that the critical depth lies inside the vee. By geometry:

$$b_s = 2 \times h\sqrt{3}$$

$$A = \frac{1}{2} \times h \times 2h\sqrt{3} \quad = \sqrt{3}h^2$$

$$\Rightarrow \qquad \mathrm{Fr}^2 = \frac{1.5^2 \times 2\sqrt{3}h}{9.81 \times 3\sqrt{3}h^6} \quad = \frac{0.1529}{h^5}$$

Solving for h when $\mathrm{Fr}^2 = 1$,

$$h = 0.1529^{1/5} \quad = 0.6869 \text{ m}$$

Answer: 0.687 m

Q5.
(a) From the channel geometry:

$$A = \frac{1}{2}\pi R^2 \quad = \frac{1}{2}\pi \times 0.6^2 \quad = 0.5655 \text{ m}^2$$

$$P = \pi R \quad = 1.885 \text{ m}$$

$$R_h \equiv \frac{A}{P} \quad = \frac{1}{2}R \quad = 0.3 \text{ m}$$

Using Manning's equation:

$$V = \frac{1}{n}R_h^{2/3}S^{1/2} \quad = \frac{1}{n} \times 0.3^{2/3} \times \sqrt{0.02} \quad = \frac{0.06338}{n}$$

$$\Rightarrow \qquad Q = VA \quad = \frac{0.06338}{n} \times 0.5655 \quad = \frac{0.03584}{n}$$

But $Q = 2 \text{ m}^3 \text{ s}^{-1}$; hence,

$$\frac{0.03584}{n} = 2$$

$$\Rightarrow \qquad n = 0.01792 \text{ m}^{-1/3} \text{ s}$$

Alternative method:
From the flow rate:

$$V = \frac{Q}{A} \quad = \frac{2}{0.5655} \quad = 3.537 \text{ m s}^{-1}$$

Equating the two values of V:

$$\frac{0.06338}{n} = 3.537 \text{ m s}^{-1}$$

$$\Rightarrow \qquad n = 0.01792 \text{ m}^{-1/3} \text{ s}$$

Answer: $0.0179 \text{ m}^{-1/3} \text{ s}$

(b) The cross-sectional area and wetted perimeter are augmented by the section with straight sides. Write expressions for A and P in terms of *additional* depth x (so that $h = 0.6 + x$) and solve for x to get a flow rate of $Q = 3 \text{ m}^3 \text{ s}^{-1}$.

$$A = 0.5655 + 1.2x$$

$$P = 1.885 + 2x$$

The remaining formulae, in sequence, are

$$R_h \equiv \frac{A}{P}$$

$$V = \frac{1}{n}R_h^{2/3}S^{1/2} \quad = 7.892R_h^{2/3}$$

$$Q = VA$$

Vary x by repeated trial to find the value at which $Q = 3 \text{ m}^3 \text{ s}^{-1}$. (Successive trials after the first two are guided by linear interpolation.)

x (m)	A (m^2)	P (m)	R_h (m)	V (m s^{-1})	Q (m^3 s^{-1})
0.0	0.5655	1.885	0.3000	3.537	2.000
1.0	1.766	3.885	0.4546	4.666	8.240
0.1603	0.7579	2.206	0.3436	3.872	2.935
0.1714	**0.7712**	**2.228**	**0.3461**	**3.890**	**3.000**

The total depth is

$$0.6 + 0.1714 = 0.7714 \text{ m}$$

Answer: 0.771 m

(c) Using area and velocity from the solution line of the table above, the mean depth is

$$\bar{h} = \frac{A}{b_s} = \frac{0.7712}{1.2} = 0.6427 \text{ m}$$

and the Froude number is

$$\text{Fr} \equiv \frac{V}{\sqrt{g\bar{h}}} = \frac{3.890}{\sqrt{9.81 \times 0.6427}} = 1.549$$

Answer: 1.55

(d) The normal flow at this discharge is supercritical ($\text{Fr} > 1$); hence, the channel is hydraulically **steep**.

Q6.
$b = 2.5$ m
$Q = 25$ m^3 s^{-1}
$S = 0.003$
$C = 45$ m$^{1/2}$ s^{-1}

Normal Depth
From the geometry, the area and wetted perimeter are given by:

$$A = \frac{1}{2}(b \quad + b + 2 \times h\sqrt{3}) \times h \quad = hb(1 + \sqrt{3}\, h/b)$$

$$P = b + 2 \times 2h \quad = b(1 + 4h/b)$$

Hydraulic radius:

$$R_h \equiv \frac{A}{P} = h\left(\frac{1 + \sqrt{3}\, h/b}{1 + 4h/b}\right)$$

In normal flow,

$$Q = VA \quad = C\sqrt{R_h S}\, A$$

Hence,

$$Q = Ch^{1/2}\left(\frac{1 + \sqrt{3}\, h/b}{1 + 4h/b}\right)^{1/2} S^{1/2} hb(1 + \sqrt{3}\, h/b)$$

$$\Rightarrow \qquad \frac{Q}{bC\sqrt{S}} = h^{3/2}\frac{(1 + \sqrt{3}\, h/b)^{3/2}}{(1 + 4h/b)^{1/2}}$$

$$\Rightarrow \qquad h = \left(\frac{Q}{bC\sqrt{S}}\right)^{2/3}\frac{(1 + 4h/b)^{1/3}}{1 + \sqrt{3}\, h/b}$$

Here, with lengths in metres,

$$h = 2.544\frac{(1 + 1.6h)^{1/3}}{1 + 0.6928h}$$

Iteration (from, e.g., $h = 2.544$) gives

$$h_n = 1.784 \text{ m}$$

Critical Depth
Froude number $\text{Fr} = V/\sqrt{g\bar{h}}$. Need velocity $V\ (= Q/A)$ and mean depth $\bar{h}\ (= A/b_s)$:

$$A = hb(1 + \sqrt{3}\, h/b) \qquad \text{(as above)}$$

$$b_s = b + 2\sqrt{3}\, h$$

At the critical depth, $\text{Fr} = 1$. But

$$\mathrm{Fr}^2 = \frac{V^2}{g\bar{h}} = \frac{Q^2/A^2}{gA/b_s} \quad = \frac{Q^2 b_s}{gA^3}$$

$$\Rightarrow \qquad \frac{Q^2(b + 2\sqrt{3}\,h)}{gh^3 b^3 (1 + \sqrt{3}\,h/b)^3} = 1$$

$$\Rightarrow \qquad \frac{Q^2(1 + 2\sqrt{3}\,h/b)}{gb^2(1 + \sqrt{3}\,h/b)^3} = h^3$$

$$\Rightarrow \qquad h = \left(\frac{Q^2}{gb^2}\right)^{1/3} \frac{(1 + 2\sqrt{3}\,h/b)^{1/3}}{1 + \sqrt{3}\,h/b}$$

Here, with lengths in metres,

$$h = 2.168\,\frac{(1 + 1.386h)^{1/3}}{1 + 0.6928h}$$

Iteration (from, e.g., $h = 2.168$) gives

$$h_c = 1.536 \text{ m}$$

Answer: normal depth = 1.78 m;   critical depth = 1.54 m

Q7.
$S = 2 \times 10^{-4}$
$q = 1.5 \text{ m}^2 \text{ s}^{-1}$
$n = 0.015 \text{ m}^{-1/3} \text{ s}$

<u>Approach Flow</u> (normal)

$$Q = VA \quad = \frac{1}{n} R_h^{2/3} S^{1/2} A, \qquad \text{where} \qquad R_h = h \quad \text{(wide channel)}$$

Per unit width ($A = h$):

$$q = \frac{1}{n} h^{5/3} S^{1/2}$$

Inverting for h:

$$h = \left(\frac{nq}{\sqrt{S}}\right)^{3/5}$$

Substituting numerical values:

$$h_n = \left(\frac{0.015 \times 1.5}{\sqrt{2 \times 10^{-4}}}\right)^{3/5} \quad = 1.321 \text{ m}$$

$$V_n = \frac{q}{h_n} \quad = \frac{1.5}{1.321} \quad = 1.136 \text{ m s}^{-1}$$

The approach-flow specific energy is

$$E_a = h_n + \frac{V_n^2}{2g} \quad = 1.321 + \frac{1.136^2}{2 \times 9.81} \quad = 1.387 \text{ m}$$

<u>Critical Conditions</u>
Critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{1.5^2}{9.81}\right)^{1/3} \quad = 0.6121 \text{ m}$$

Critical specific energy:

$$E_c = \frac{3}{2} h_c \quad = \frac{3}{2} \times 0.6121 \quad = 0.9182 \text{ m}$$

(a) The total head required for critical conditions at the weir is

$$H_c = z_{\text{weir}} + E_c \quad = 0.2 + 0.9182 \quad = 1.1182 \text{ m}$$

But this is less than the available head from the normal flow ($H_a = E_a = 1.387$ m). Hence, the flow does *not* go critical over the weir and the total head in the vicinity of the weir is that from the approach flow; i.e. $H = 1.387$ m.

Upstream and downstream of the weir we have normal depth; i.e.

$$h = 1.321 \text{ m}$$

Over the weir itself the specific energy is

$$E = H - z_{\text{weir}} \quad = 1.387 - 0.2 \quad = 1.187 \text{ m}$$

and the flow is subcritical.

Now,

$$E = h + \frac{V^2}{2g} \quad = h + \frac{q^2}{2gh^2}$$

This can be arranged (for a subcritical solution) as the iterative formula

$$h = E - \frac{q^2}{2gh^2}$$

Substituting numerical values,

$$h = 1.187 - \frac{0.1147}{h^2}$$

Iteration (from, e.g., $h = 1.187$) gives

$$h = 1.091 \text{ m}$$

Answer: depths upstream, over, downstream of the weir: 1.32 m, 1.09 m, 1.32 m

(b) The total head required for critical conditions at the weir is

$$H_c = z_{\text{weir}} + E_c \quad = 0.5 + 0.9182 \quad = 1.418 \text{ m}$$

This is greater than the available head from the approach flow ($H_a = E_a = 1.387$ m). Hence, the flow *does* go critical over the weir, the flow backs up and the total head in the vicinity of the weir is the critical head; i.e. $H = 1.418$ m.

<u>Over the weir</u> the flow is critical:

$$h = h_c = 0.6121 \text{ m}$$

Just upstream or downstream of the weir (where $z_b = 0$):

$$H = E \quad = h + \frac{q^2}{2gh^2}$$

<u>On the upstream side</u> we will have subcritical flow. The first term on the RHS dominates so rearrange as

$$h = H - \frac{q^2}{2gh^2}$$

Substituting numerical values,

$$h = 1.418 - \frac{0.1147}{h^2}$$

Iteration (from, e.g., $h = 1.418$) gives

$$h = 1.356 \text{ m}$$

<u>On the downstream side</u> we will have supercritical flow. The second term on the RHS dominates so rearrange as

$$h = \frac{q}{\sqrt{2g(H-h)}}$$

Substituting numerical values,

$$h = \frac{1.5}{\sqrt{19.62(1.418-h)}}$$

Iteration (from, e.g., $h = 0$) gives

$$h = 0.3237 \text{ m}$$

Answer: depths upstream, over and downstream of the weir: 1.36 m, 0.612 m, 0.324 m

(c) For the flow *just* to go critical, the total head over the weir must be precisely equal to the head in the approach flow. Hence, measuring bed level relative to the undisturbed channel at the position of the weir,

$$H_c = H_a$$

or

$$z_{\text{weir}} + E_c = H_a$$

Hence,

$$z_{\text{weir}} = H_a - E_c \quad = 1.387 - 0.9182 \quad = 0.4688 \text{ m}$$

Answer: 0.469 m

Q8.
$Q = 9 \text{ m}^3 \text{ s}^{-1}$
$S = 0.001$
$n = 0.024 \text{ m}^{-1/3} \text{ s}$
$b = 4 \text{ m}$

(a)
<u>Normal Depth</u>

$$Q = VA$$

where

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} \quad = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n} \left(\frac{h}{1 + 2h/b} \right)^{2/3} S^{1/2} bh$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}} \right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 1.379(1 + 0.5h)^{2/5}$$

Iteration (from, e.g., $h = 1.379$) gives

$$h_n = 1.779 \text{ m}$$

<u>Critical Depth</u>

Flow rate per unit width:

$$q = \frac{Q}{b} \quad = \frac{9}{4} \quad = 2.25 \text{ m}^2 \text{ s}^{-1}$$

Critical depth:

$$h_c = \left(\frac{q^2}{g} \right)^{1/3} \quad = \left(\frac{2.25^2}{9.81} \right)^{1/3} \quad = 0.8021 \text{ m}$$

Answer: normal depth = 1.78 m; critical depth = 0.802 m

(b) The flow will just go critical if the total head over the weir assuming critical conditions is exactly equal to the available head in the approach flow (here, normal flow, since the channel is "long").

The specific energy in the approach flow is

$$E_a = h_n + \frac{V_n^2}{2g} \quad = h_n + \frac{Q^2}{2gb^2h_n^2} \quad = 1.779 + \frac{9^2}{2 \times 9.81 \times 4^2 \times 1.779^2} \quad = 1.861 \text{ m}$$

Hence, relative to the bed in the vicinity of the weir:

$$H_a = E_a \quad = 1.861 \text{ m}$$

If the flow just goes critical:

$$H_c = H_a$$

$$\Rightarrow \qquad z_{\text{weir}} + \frac{3}{2}h_c = H_a$$

$$\Rightarrow \qquad z_{\text{weir}} = H_a - \frac{3}{2}h_c \quad = 1.861 - \frac{3}{2} \times 0.8021 \quad = 0.6579 \text{ m}$$

Answer: 0.658 m

(c) The flow will be on the verge of overtopping if the highest water surface (which occurs just upstream of the weir) equals that of the sides of the channel. This depth will set the total head, which will be the same as the critical head over the weir at that particular weir height.

Total head when $h = 2.5$ m is:

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2} \quad = 2.5 + \frac{9^2}{2 \times 9.81 \times 4^2 \times 2.5^2} \quad = 2.541 \text{ m}$$

Since this is the same as the critical head over the weir:

$$z_{\text{weir}} + \frac{3}{2}h_c = 2.541$$

$$\Rightarrow \qquad z_{\text{weir}} = 2.541 - \frac{3}{2}h_c \quad = 2.541 - 1.5 \times 0.8021 \quad = 1.338 \text{ m}$$

Answer: 1.34 m

Q9.
$S = 0.001$
$q = 3 \text{ m}^2 \text{ s}^{-1}$
$n = 0.015 \text{ m}^{-1/3} \text{ s}$

<u>Normal Depth</u>

$$Q = VA \quad = \frac{1}{n} R_h^{2/3} S^{1/2} A, \qquad \text{where} \qquad R_h = h \quad \text{(wide channel)}$$

Per unit width ($A = h$):

$$q = \frac{1}{n} h^{5/3} S^{1/2}$$

Inverting for h:

$$h = \left(\frac{nq}{\sqrt{S}}\right)^{3/5}$$

Substituting numerical values:

$$h_n = \left(\frac{0.015 \times 3}{\sqrt{0.001}}\right)^{3/5} \quad = 1.236 \text{ m}$$

In the depression specific energy must increase and there would be no hydraulic transition in either subcritical or supercritical flow. The channel is "long", so the approach flow is normal. Hence, the depths at stations A and E are normal.

The total head can be determined from the conditions in the approach flow:

$$H = H_a \quad = z_{sa} + \frac{V_n^2}{2g} \quad = h_n + \frac{q^2}{2gh_n^2} \quad = 1.236 + \frac{3^2}{2 \times 9.81 \times 1.236^2} \quad = 1.536 \text{ m}$$

The flow at C has the same total head, but we need to know whether it is subcritical or supercritical. Since there is no hydraulic transition this depends on whether the normal flow is subcritical or supercritical. We could determine this by either finding the Froude number or comparing with the critical depth. Since we need the latter in part (b) we'll find and use it now:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{3^2}{9.81}\right)^{1/3} \quad = 0.9717 \text{ m}$$

Since $h_n > h_c$, the normal depth, and hence the flow throughout, is subcritical. The depth at C is thus the subcritical depth in the depression with the same total head. At C the water surface height is $z_s = -0.3 + h$, so that

$$H = -0.3 + h + \frac{q^2}{2gh^2}$$

For the deep, subcritical solution, rearrange for iteration as

$$h = 1.836 - \frac{0.4587}{h^2}$$

Iteration from, e.g., $h = 1.836$ gives

$$h = 1.672 \text{ m}$$

Answer: depths at A, C, E are, respectively, 1.24 m, 1.67 m, 1.24 m

(b) If a weir is installed then the minimum head required to surmount it is the critical head

$$H_c = -0.3 + 0.7 + \frac{3}{2}h_c \quad = 1.858 \text{ m}$$

Note that the height datum must be the same as that of H_a. The critical head H_c exceeds the available head ($H_a = 1.536$ m), so the flow must back up, a hydraulic transition takes place at the weir and the head throughout is $H = H_c = 1.858$ m.

At all stations,

$$H = z_s + \frac{V^2}{2g}$$

$$\Rightarrow \qquad H = z_b + h + \frac{q^2}{2gh^2}$$

$$\Rightarrow \qquad 1.858 - z_b = h + \frac{0.4587}{h^2}$$

Station A: subcritical, $z_b = 0$:

$$h = 1.858 - \frac{0.4587}{h^2} \qquad \rightarrow \qquad h = 1.699 \text{ m}$$

Station B: subcritical, $z_b = -0.3$:

$$h = 2.158 - \frac{0.4587}{h^2} \qquad \rightarrow \qquad h = 2.049 \text{ m}$$

Station C: critical

$$h = h_c \quad = 0.9717 \text{ m}$$

Station D: supercritical, $z_b = -0.3$:

$$h = \sqrt{\frac{0.4587}{2.158 - h}} \qquad \rightarrow \qquad h = 0.5310 \text{ m}$$

Station E: supercritical, $z_b = 0$:

$$h = \sqrt{\frac{0.4587}{1.858 - h}} \qquad \rightarrow \qquad h = 0.6051 \text{ m}$$

Answer: depths at stations A, B, C, D, E are 1.70 m, 2.05 m, 0.972 m, 0.531 m, 0.605 m

Q10.
$S_0 = 2 \times 10^{-5}$
$n = 0.01 \text{ m}^{-1/3} \text{ s}$
$q = 0.5 \text{ m}^2 \text{ s}^{-1}$
$z_{\text{weir}} = 0.7 \text{ m}$

(a) Discharge (per unit width) in normal flow:

$$q = Vh \quad = \frac{1}{n} R_h^{2/3} S^{1/2} h, \qquad \text{where} \qquad R_h = h \qquad \text{(wide channel)}$$

$$\Rightarrow \qquad \frac{nq}{\sqrt{S}} = h^{5/3} \qquad (*)$$

For the given slope this gives a normal depth

$$h_n = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} \quad = \left(\frac{0.01 \times 0.5}{\sqrt{2 \times 10^{-5}}}\right)^{3/5} \quad = 1.069 \text{ m}$$

Answer: 1.07 m

(b) At the normal depth the velocity is

$$V_n = \frac{q}{h_n} \quad = \frac{0.5}{1.069} \quad = 0.4677 \text{ m s}^{-1}$$

and the approach-flow specific energy is

$$E_a = h_n + \frac{V_n^2}{2g} \quad = 1.069 + \frac{0.4677^2}{2 \times 9.81} \quad = 1.080 \text{ m}$$

The critical depth and critical specific energy are:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{0.5^2}{9.81}\right)^{1/3} \quad = 0.2943 \text{ m}$$

$$E_c = \frac{3}{2} h_c \quad = \frac{3}{2} \times 0.2943 \quad = 0.4415 \text{ m}$$

Under critical conditions the total head over the weir is then

$$H_c = z_{\text{weir}} + E_c \quad = 0.7 + 0.4415 \quad = 1.141 \text{ m}$$

This is the minimum possible head for this flow rate over the weir and is greater than the head available in the approach flow ($H_a = E_a = 1.080$ m). Hence the flow backs up, the water level rises just upstream of the weir and the flow over the top is critical. The total head in the vicinity of the weir is determined by critical conditions: $H = 1.141$ m.

Answer: 0.294 m

(c) Assuming that the flow in the vicinity of the weir is unaffected by the hydraulic jump the flow goes smoothly supercritical on the downstream side, with total head $H = 1.141$ m and discharge per metre width $q = 0.5 \text{ m}^2 \text{ s}^{-1}$. Downstream of the weir (where $z_b = 0$):

$$H = h + \frac{q^2}{2gh^2}$$

Rearrange for an iterative formula for the supercritical solution:

$$h = \frac{q}{\sqrt{2g(H-h)}}$$

Substituting numerical values:

$$h = \frac{0.5}{\sqrt{19.62(1.141-h)}}$$

Iteration (from, e.g., $h = 0$) gives

$$h = 0.1112 \text{ m}$$

Answer: 0.111 m

(d) Denote conditions upstream and downstream of the hydraulic jump by subscripts A and B respectively. From the downstream conditions (assuming normal flow since the channel is described as "long"):

$$h_B = 1.069 \text{ m}$$

$$\text{Fr}_B = \frac{V_B}{\sqrt{gh_B}} = \frac{q}{\sqrt{gh_B^3}} = \frac{0.5}{\sqrt{9.81 \times 1.069^3}} = 0.1444$$

Hence, from the hydraulic-jump relation for the sequent depths:

$$h_A = \frac{h_B}{2}\left(-1 + \sqrt{1 + 8Fr_B^2}\right) = \frac{1.069}{2}\left(-1 + \sqrt{1 + 8 \times 0.1444^2}\right) = 0.04287 \text{ m}$$

Answer: 0.043 m

(e) Unless subject to control, the supercritical flow at the downstream end of the weir would gradually increase in depth until a hydraulic jump occurred (see the lectures on GVF). Since the sequent depth upstream of the hydraulic jump is less than the supercritical depth downstream of the weir, no such increasing-depth GVF is possible and the hydraulic jump must actually occur at (or just before) the downstream end of the weir.

Q11.
(a)
$Q = 8\ \mathrm{m^3\ s^{-1}}$
$b = 5$ m (main channel); $b_{\min} = 2$ m
$S = 1 \times 10^{-4}$
$n = 0.015\ \mathrm{m^{-1/3}\ s}$

(a) Far upstream of any disturbance the depth will be normal:

$$Q = VA, \quad \text{where} \quad V = \frac{1}{n} R_h^{2/3} S^{1/2}, \quad A = bh, \quad R_h = \frac{bh}{b+2h} \quad = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n}\left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} bh$$

$$\Rightarrow \quad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\Rightarrow \quad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 1.691(1 + 0.4h)^{2/5}$$

Iterate (from, e.g., $h = 1.691$) to get

$$h_n = 2.171 \text{ m}$$

Answer: 2.17 m

(b) At the narrow point:

$$q_m = \frac{Q}{b_{\min}} \quad = \frac{8}{2} \quad = 4\ \mathrm{m^2\ s^{-1}}$$

Critical depth:

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \quad = \left(\frac{4^2}{9.81}\right)^{1/3} \quad = 1.177 \text{ m}$$

Critical specific energy:

$$E_c = \frac{3}{2} h_c \quad = 1.766 \text{ m}$$

Answer: $h_c = 1.18$ m; $E_c = 1.77$ m

(c) To determine the behaviour at the narrow point compare the head available in the approach flow with critical conditions at the narrow point.

Approach flow:

$$h_a = h_n \quad = 2.171 \text{ m}$$

$$V_a = \frac{Q}{bh_a} \quad = \frac{8}{5 \times 2.171} \quad = 0.7370 \text{ m s}^{-1}$$

$$H_a = E_a \quad = h_a + \frac{V_a^2}{2g} \quad = 2.171 + \frac{0.7370^2}{2 \times 9.81} \quad = 2.199 \text{ m}$$

(The Froude number $Fr_a \equiv V_a/\sqrt{gh_a} = 0.1597$, so that the approach flow is subcritical.)

The critical head, the minimum head necessary to pass this flow through the throat, is:

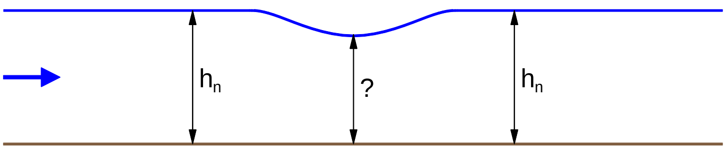
$$H_c = E_c + \Delta z_b \quad = 1.766 + \Delta z_b$$

First establish the flow behaviour and total head through the venturi in the three cases.

Case (i): $\Delta z_b = 0$

$H_c = 1.766$ m, so that $H_a > H_c$

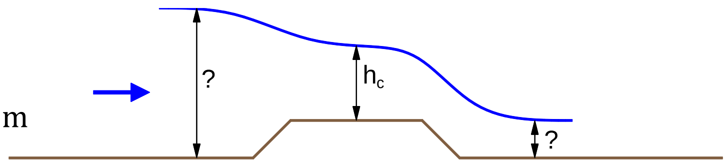
$\Rightarrow$ Doesn't go critical; $H = H_a = 2.199$ m



Case (ii): $\Delta z_b = +0.75$ m

$H_c = 2.516$ m, so that $H_a < H_c$

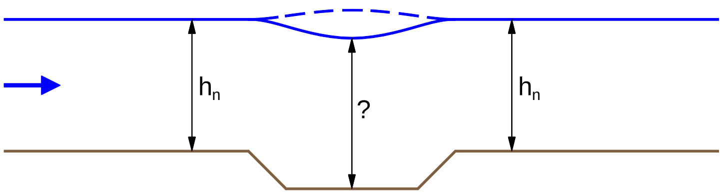
$\Rightarrow$ Critical flow transition; $H = H_c = 2.516$ m



Case (iii): $\Delta z_b = -0.75$ m

$H_c = 1.016$ m, so that $H_a > H_c$

$\Rightarrow$ Doesn't go critical; $H = H_c = 2.199$ m



Only in Case (ii) is there a sub- to supercritical flow transition. Only the depths marked '?' need be found, as the remaining ones are either h_n or h_c (found earlier). Because of the combination of bed-level change <u>and</u> narrowing in Case (iii) we cannot determine in advance whether the surface level rises or falls at the narrow point.

<u>Case (i):</u> $\Delta z_b = 0$

Since $H = H_a$ throughout, the depths <u>just upstream and downstream</u> are the same as in the approach flow; i.e. 2.171 m.

At the <u>narrow point</u> ($\Delta z_b = 0$; $\quad b_m = 2$ m, $\quad q_m = 4 \text{ m}^2 \text{ s}^{-1}$):

$$H = z_s + \frac{V^2}{2g} \quad = 0 + h + \frac{Q^2}{2gb_m^2h^2}$$

$$\Rightarrow \qquad 2.199 = h + \frac{0.8155}{h^2}$$

As no transition occurs we require the subcritical solution. Rearrange for iteration as

$$h = 2.199 - \frac{0.8155}{h^2}$$

Iteration (from, e.g., $h = 2.199$) gives $h = 1.994$ m at the throat.

Answer: depths (upstream, throat, downstream) are (2.17, 1.99, 2.17) m

Case (ii): $\Delta z_b = +0.75$ m

At the narrow point the flow is critical, so the depth here is $h = h_c = 1.177$ m.

Just upstream and downstream (but at full channel width, $b = 5$ m) the total head is the same as the critical head at the throat; i.e. 2.516 m.

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2}$$

Hence, we need the sub- and supercritical solutions of

$$2.516 = h + \frac{8^2}{2 \times 9.81 \times 5^2 \times h^2} \quad = h + \frac{0.1305}{h^2}$$

Upstream (subcritical):

$$h = 2.516 - \frac{0.1305}{h^2}$$

Iteration (from, e.g., $h = 2.516$) gives $h = 2.495$ m.

Downstream (supercritical):

$$h = \sqrt{\frac{0.1305}{2.516 - h}}$$

Iteration (from, e.g., $h = 0$) gives $h = 0.2394$ m.

Answer: depths (upstream, throat, downstream) are (2.50, 1.18, 0.239) m

Case (iii): $\Delta z_b = -0.75$ m

Since $H = H_a$ throughout, the depths just upstream and downstream are the same as in the approach flow; i.e. 2.171 m.

At the narrow point ($\Delta z_b = -0.75$ m; $b_m = 2$ m, $q_m = 4$ m^2 s^{-1}):

$$H = z_s + \frac{V^2}{2g} \quad = -0.75 + h + \frac{Q^2}{2gb_m^2h^2}$$

$$\Rightarrow \qquad 2.949 = h + \frac{0.8155}{h^2}$$

As no transition occurs we require the subcritical solution. Rearrange for iteration as

$$h = 2.949 - \frac{0.8155}{h^2}$$

Iteration (from, e.g., $h = 2.921$) gives $h = 2.848$ m at the throat.

Answer: depths (upstream, throat, downstream) are (2.17, 2.85, 2.17) m

Q12.
$Q = 12 \text{ m}^3 \text{ s}^{-1}$
$b = 5$ m (main channel); $b_{\text{min}} = 2$ m
$S = 10^{-4}$
$n = 0.016 \text{ m}^{-1/3} \text{ s}$

(a) Discharge:

$$Q = VA$$

where, in normal flow,

$$V = \tfrac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b+2h} \quad = \frac{h}{1+2h/b}$$

Hence,

$$Q = \frac{1}{n}\left(\frac{h}{1+2h/b}\right)^{2/3} S^{1/2} bh$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1+2h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1+2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 2.242(1+0.4h)^{2/5}$$

Iteration (from, e.g., $h = 2.242$) gives

$$h_n = 3.094 \text{ m}$$

Answer: 3.09 m

(b) The total head in the main channel at the position of the venturi is

$$H_a \quad = z_{sn} + \frac{V_n^2}{2g} \quad = h_n + \frac{Q^2}{2gb^2h_n^2} \quad = 3.125 \text{ m}$$

At the narrow point,

$$q_m = \frac{Q}{b_{min}} \quad = 6 \text{ m}^2 \text{ s}^{-1}$$

Hence the critical depth is

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \quad = 1.542 \text{ m}$$

The corresponding critical specific energy is

$$E_c = \frac{3}{2}h_c \quad = 2.313 \text{ m}$$

This is the same as the total head H_c (since the bed is flat) and is less than the approach-flow head H_a. Hence, the flow does not go critical.

Answer: 1.54 m

(c) The head in the narrow section is the same as that in the main channel; i.e. $H = 3.125$ m. At the throat, $q = 6 \text{ m}^2 \text{ s}^{-1}$.

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{q^2}{2gh^2}$$

Rearrange as an iterative formula for the subcritical (i.e. deep, slow) solution,

$$h = H - \frac{q^2}{2gh^2}$$

Here, with lengths in metres,

$$h = 3.125 - \frac{1.835}{h^2}$$

Iterate (from, e.g., $h = 3.125$) to get

$$h = 2.908 \text{ m}$$

Answer: 2.91 m

(d) The bed must be raised so that the total head under critical conditions equals that in normal flow; i.e.

$$\Delta z_b + E_c = H_a$$

Hence,

$$\Delta z_b = H_a - E_c \quad = 3.125 - 2.313 \quad = 0.812 \text{ m}$$

Answer: 0.812 m

(e) The total head is $H = 3.125$ m. The depth at the narrow point is $h_c = 1.542$ m. The depth just upstream is the normal depth, $h_n = 3.094$ m. Up and downstream of the constricted section:

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2}$$

Here,

$$3.125 = h + \frac{0.2936}{h^2}$$

The downstream (supercritical) solution can be obtained from

$$h = \sqrt{\frac{0.2936}{3.125 - h}}$$

Iterate (e.g. from 0) to get

$$h = 0.3237 \text{ m}$$

Answer: depths upstream: 3.09 m; in narrow section: 1.54 m; downstream: 0.324 m

Q13.
(a) If a hydraulic transition occurs at the narrow point then critical conditions occur here:

$$\text{Fr} = \frac{V_c}{\sqrt{gh_c}} = 1$$

$$\Rightarrow \qquad h_c = \frac{V_c^2}{g} \quad = \frac{2.7^2}{9.81} \quad = 0.7431 \text{ m}$$

The critical head is

$$H_c = z_b + E_c \quad = 0 + \frac{3}{2}h_c \quad = 1.115 \text{ m}$$

and the flow rate is

$$Q = V_c(b_m h_c) \quad = 2.7 \times 2.0 \times 0.7431 \quad = 4.013 \text{ m}^3 \text{ s}^{-1}$$

In the upstream and downstream full channel width $b = 4$ m:

$$H = z_s + \frac{V^2}{2g} \qquad\qquad \text{with } z_s = h$$

As there is no energy loss, the total head equals the critical head ($H = H_c = 1.115$ m).

We now have two choices: (i) write h in terms of V using $Q = V(bh)$ and solve for the two V solutions directly; or (ii) solve for h and deduce V from continuity at the end.

<u>Method (i): Solve for V directly</u>

Write h in terms of velocity V via $Q = V \times bh$:

$$H_c = \frac{Q}{bV} + \frac{V^2}{2g}$$

Here, with $b = 4$ m in the main channel:

$$1.115 = \frac{1.003}{V} + \frac{V^2}{19.62}$$

Upstream, we require the deep, slow solution, so rearrange for iteration with the first term on the RHS dominating:

$$V = \frac{1.003}{1.115 - V^2/19.62}$$

Iteration (from a "slow" value, e.g., $V = 0$) gives

$$V = 0.9372 \text{ m s}^{-1}$$

Downstream, we require the shallow, fast solution, so rearrange for iteration with the second term on the RHS dominating:

$$V = \sqrt{19.62\left(1.115 - \frac{1.003}{V}\right)}$$

Iteration, starting with the value obtained by neglecting the last term gives

$$V = 4.138 \text{ m s}^{-1}$$

<u>Method (ii): Solve for h first</u>

Write V in terms of depth h via $Q = V \times bh$:

$$H_c = h + \frac{Q^2}{2gb^2h^2}$$

Here, with $b = 4$ m in the main channel:

$$1.115 = h + \frac{0.05130}{h^2}$$

Upstream, we require the deep, slow solution, so rearrange for iteration with the first term on the RHS dominating:

$$h = 1.115 - \frac{0.05130}{h^2}$$

Iteration (from, e.g., $h = 1.115$) gives $h = 1.070$, and hence

$$V = \frac{Q}{bh} \quad = \frac{4.013}{4 \times 1.070} \quad = 0.9376 \text{ m s}^{-1}$$

Downstream, we require the shallow, fast solution, so rearrange for iteration with the second term on the RHS dominating:

$$h = \sqrt{\frac{0.05130}{1.115 - h}}$$

Iteration (from, e.g., $h = 0$) gives $h = 0.2425$ m, and hence

$$V = \frac{Q}{bh} \quad = \frac{4.013}{4 \times 0.2425} \quad = 4.137 \text{ m s}^{-1}$$

Answer: upstream: 0.937 m s^{-1};   downstream: 4.14 m s^{-1}

(b) The total head and the flow rate throughout the device can be found from the given depth and velocity at the narrow point:

$$H = z_s + \frac{V^2}{2g} \quad = 1.0 + \frac{2.7^2}{2 \times 9.81} \quad = 1.372 \text{ m}$$

$$Q = VA \quad = Vb_m h \quad = 2.7 \times 2.0 \times 1.0 \quad = 5.4 \text{ m}^3 \text{ s}^{-1}$$

In the absence of a flow transition, the flow must stay either subcritical or supercritical throughout. Depths and velocities will be the same at upstream and downstream locations. The Froude number at the narrow point is

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{2.7}{\sqrt{9.81 \times 1}} \quad = 0.8620$$

Hence, we shall require only the subcritical solution.

Method (i): Solve for V directly

At arbitrary width b as above:

$$H = \frac{Q}{bV} + \frac{V^2}{2g}$$

or, with $b = 4$ m in the main channel:

$$1.372 = \frac{1.35}{V} + \frac{V^2}{19.62}$$

Rearranging for iteration for the slower, subcritical, solution:

$$V = \frac{1.35}{1.372 - V^2/19.62}$$

Iteration (from, e.g., $V = 0$) gives

$$V = 1.024 \text{ m s}^{-1}$$

Method (ii): Solve for h first

Write V in terms of depth h via $Q = V(bh)$:

$$H = h + \frac{Q^2}{2gb^2h^2}$$

Here, with $b = 4$ m in the main channel:

$$1.372 = h + \frac{0.09289}{h^2}$$

We require only the deep, slow solution, so rearrange for iteration with the first term on the RHS dominating:

$$h = 1.372 - \frac{0.09289}{h^2}$$

Iteration (from, e.g., $h = 1.372$) gives $h = 1.319$, and hence

$$V = \frac{Q}{bh} \quad = \frac{5.4}{4 \times 1.319} \quad = 1.024 \text{ m s}^{-1}$$

Answer: 1.02 m s^{-1} (both locations)

Q14.

Assumptions:

- constant total head (main assumption);
- constricted section long enough to establish parallel flow with critical depth;
- downstream controls do not prevent supercritical flow being established.

Find the total head from the critical conditions. At the narrow point the critical depth is

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \quad = \left(\frac{(11/3)^2}{9.81}\right)^{1/3} \quad = 1.111 \text{ m}$$

For a flat bed, the total head relative to the bed is the corresponding critical specific energy:

$$E_c \quad = \frac{3}{2}h_c \quad = \frac{3}{2} \times 1.111 \quad = 1.667 \text{ m}$$

As a flow transition is stated to occur this must be the total head (H) throughout the device.

In any parallel-flow region:

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2}$$

Outside the contracted section, $b = 5$ m. Here, with lengths in m:

$$1.667 = h + \frac{0.2467}{h^2}$$

Just <u>upstream</u>, rearrange as an iterative formula for the subcritical solution:

$$h = 1.667 - \frac{0.2467}{h^2}$$

Iterate (from, e.g., $h = 1.667$) to get

$$h = 1.566 \text{ m}$$

Just <u>downstream</u>, rearrange as an iterative formula for the supercritical solution:

$$h = \sqrt{\frac{0.2467}{1.667 - h}}$$

Iterate (from, e.g., $h = 0$) to get

$$h = 0.4503 \text{ m}$$

Answer: depth upstream = 1.57 m; under the bridge = 1.11 m; downstream = 0.450 m

(b) Conditions upstream of the hydraulic jump:

$$h = 0.4503 \text{ m}$$

$$V = \frac{Q}{bh} \quad = \frac{11}{5 \times 0.4503} \quad = 4.886 \text{ m s}^{-1}$$

$$\text{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{4.886}{\sqrt{9.81 \times 0.4503}} \quad = 2.325$$

Hence, from the hydraulic-jump formula, the downstream depth is:

$$h_{\text{downstream}} = \frac{0.4503}{2}(-1 + \sqrt{1 + 8 \times 2.325^2}) \quad = 1.272 \text{ m}$$

Answer: 1.27 m

(c) If the discharge increases to 22 m^3 s^{-1} then the critical depth at the narrow point is

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \quad = \left(\frac{(22/3)^2}{9.81}\right)^{1/3} = 1.763 \text{ m}$$

This exceeds the clearance of the bridge deck (1.70 m). Hence, critical conditions cannot be attained and the flow must be choked.

(Alternatively, you could rearrange to find the flow rate that corresponds to a critical depth of exactly 1.7 m and show that it is less than 22 m^3 s^{-1}.)

Q15.
$b = 0.8$ m
$h_2 = 0.25$ m
$Q = 0.9\ \text{m}^3\text{s}^{-1}$

(a) Total head from given downstream depth:

$$H = z_{s2} + \frac{V_2^2}{2g} \quad = h_2 + \frac{Q^2}{2gb^2h_2^2} \quad = 0.25 + \frac{0.9^2}{2 \times 9.81 \times 0.8^2 \times 0.25^2} \quad = 1.282\text{ m}$$

Answer: 1.28 m

(b) In the absence of losses the head is the same on the upstream side:

$$H = h_1 + \frac{Q^2}{2gb^2h_1^2}$$

i.e.

$$1.282 = h_1 + \frac{0.06451}{h_1^2}$$

Rearranging for the subcritical (deep) solution gives, in m:

$$h_1 = 1.282 - \frac{0.06451}{h_1^2}$$

Iteration (from, e.g., $h_1 = 1.282$) gives

$$h_1 = 1.240\text{ m}$$

Then

$$V_1 = \frac{Q}{bh_1} \quad = \frac{0.9}{0.8 \times 1.240} \quad = 0.9073$$

Answer: $h_1 = 1.24$ m, $V_1 = 0.907\text{ m s}^{-1}$

(c) The Froude number on each side is given by

$$\text{Fr} \equiv \frac{V}{\sqrt{gh}} \quad = \frac{Q}{bh\sqrt{gh}} \quad = \frac{0.9}{0.8h\sqrt{9.81h}} \quad = \frac{0.3592}{h^{3/2}} \qquad (h\text{ in metres})$$

Upstream:

$$h = 1.240 \quad \Rightarrow \quad \text{Fr}_1 = 0.2601$$

Downstream:

$$h = 0.25 \quad \Rightarrow \quad \text{Fr}_2 = 2.874$$

Answer: upstream $\text{Fr} = 0.260$; downstream $\text{Fr} = 2.87$

(d) Momentum principle:

net (hydrostatic) pressure force + force from gate on fluid = change in momentum flux

$$\rho g\left(\frac{1}{2}h_1\right)h_1 b - \rho g\left(\frac{1}{2}h_2\right)h_2 b - F = \rho Q(V_2 - V_1)$$

Here, $V = Q/bh$ on each side. Hence,

$$F = \frac{1}{2}\rho g b(h_1^2 - h_2^2) - \rho\frac{Q^2}{b}\left(\frac{1}{h_2} - \frac{1}{h_1}\right)$$

$$= \frac{1}{2}\times 1000 \times 9.81 \times 0.8 \times (1.240^2 - 0.25^2) - 1000 \times \frac{0.9^2}{0.8}\left(\frac{1}{0.25} - \frac{1}{1.240}\right)$$

$$= 5788 - 3233 \quad = 2555 \text{ N}$$

Answer: 2.56 kN

Q16.

(a) Find the force on the gate by using the momentum principle. Depths are given; the velocities are found by first finding the flow rate.

Assuming the same total head on either side of the gate:

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$

$$\Rightarrow \qquad h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$$

$$\Rightarrow \qquad h_1 - h_2 = \frac{q^2}{2g}\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)$$

Substituting values $h_1 = 1.8$ m, $h_2 = 0.3$ m:

$$1.5 = 0.5506q^2$$

Hence, the flow per unit width is

$$q = \sqrt{\frac{1.5}{0.5506}} \quad = 1.651 \text{ m}^2 \text{ s}^{-1}$$

The corresponding velocities are:

$$V_1 = \frac{q}{h_1} \quad = \frac{1.651}{1.8} = 0.9172 \text{ m s}^{-1}, \qquad V_2 = \frac{q}{h_2} \quad = \frac{1.651}{0.3} \quad = 5.503 \text{ m s}^{-1}$$

If F is the force on the gate then, by the momentum principle:

$$\frac{1}{2}\rho g(h_1^2 - h_2^2)b - F = \rho q b(V_2 - V_1)$$

Rearranging for F:

$$F = \frac{1}{2}\rho g(h_1^2 - h_2^2)b - \rho q b(V_2 - V_1)$$

$$= \frac{1}{2} \times 1000 \times 9.81 \times (1.8^2 - 0.3^2) \times 2 - 1000 \times 1.651 \times 2 \times (5.503 - 0.9172)$$

$$= 15760 \text{ N}$$

Answer: 15.7 kN

(b) Denote conditions upstream and downstream of the hydraulic jump by subscripts A and B respectively. Since the hydraulic jump is only a short distance downstream, the depth and velocity upstream of the jump are the same as those downstream of the gate; i.e. $h_A = 0.3$ m, $V_A = 5.503 \text{ m s}^{-1}$. The corresponding Froude number is

$$\text{Fr}_A = \frac{V_A}{\sqrt{gh_A}} \quad = \frac{5.503}{\sqrt{9.81 \times 0.3}} \quad = 3.208$$

From the hydraulic jump relation:

$$h_B = \frac{h_A}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_A^2}\right) \quad = \frac{0.3}{2}\left(-1 + \sqrt{1 + 8 \times 3.208^2}\right) \quad = 1.219 \text{ m}$$

Answer: 1.22 m

(c) Velocity downstream of the jump:

$$V_B = \frac{q}{h_B} \quad = \frac{1.651}{1.219} \quad = 1.354 \text{ m s}^{-1}$$

The total heads on either side of the jump are, relative to the bed:

$$H_A = z_{sA} + \frac{V_A^2}{2g} \quad = 0.3 + \frac{5.503^2}{2 \times 9.81} \quad = 1.843 \text{ m}$$

$$H_B = z_{sB} + \frac{V_B^2}{2g} \quad = 1.219 + \frac{1.354^2}{2 \times 9.81} \quad = 1.312 \text{ m}$$

The fraction of the total head that is lost is

$$\frac{\Delta H}{H_A} = \frac{1.843 - 1.312}{1.843} \quad = 0.2881$$

Answer: 28.8%

Q17.
(a) Assuming the same total head on either side of the sluice:

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$

$$\Rightarrow \qquad h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$$

Substituting values $Q = 1.8 \text{ m}^3 \text{ s}^{-1}$, $b = 3$ m, $h_2 = 0.22$ m:

$$h_1 + \frac{0.01835}{h_1^2} = 0.5991$$

Rearranging for the deep solution upstream:

$$h_1 = 0.5991 - \frac{0.01835}{h_1^2}$$

Iteration (from, e.g., $h_1 = 0.5991$) gives

$$h_1 = 0.5350 \text{ m}$$

Answer: 0.535 m

(b)
$b = 3$ m
$Q = 1.8 \text{ m}^3 \text{ s}^{-1}$
$S = 0.025$
$n = 0.03 \text{ m}^{-1/3} \text{ s}$

<u>Normal Depth</u>

$$Q = VA, \qquad \text{where} \qquad V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad R_h = \frac{h}{1 + 2h/b}, \qquad A = bh$$

$$\Rightarrow \qquad Q = \frac{1}{n}\left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} bh$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 0.2715(1 + 2h/3)^{2/5}$$

Iteration (from, e.g., $h = 0.2715$) gives

$$h_n = 0.2915 \text{ m}$$

Critical Depth

Flow rate per unit width:

$$q = \frac{Q}{b} \quad = \frac{1.8}{3} \quad = 0.6 \text{ m}^2 \text{ s}^{-1}$$

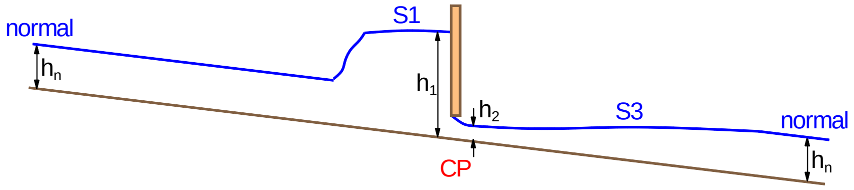
Critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{0.6^2}{9.81}\right)^{1/3} \quad = 0.3323 \text{ m}$$

Answer: normal depth = 0.291 m; critical depth = 0.332 m

(c) The depth far upstream is normal and supercritical ($h_n < h_c$). The depth just upstream of the sluice is subcritical (0.535 m $> h_c$). Hence there must be an *upstream* hydraulic jump.

Downstream, the flow exiting the sluice is shallower than both normal and critical depths and will deepen asymptotically toward the normal depth. Since both the depth downstream of the gate and the normal depth are supercritical, there is no downstream hydraulic jump.



Answer: there is an upstream hydraulic jump.

(d) Because of the absence of any other flow control, the depth on the upstream side of the jump is normal:

$$h_n = 0.2915 \text{ m}$$

The corresponding velocity and Froude number are

$$V_n = \frac{Q}{bh_n} \quad = \frac{1.8}{3 \times 0.2915} \quad = 2.058 \text{ m s}^{-1}$$

$$\text{Fr}_n = \frac{V_n}{\sqrt{gh_n}} \quad = \frac{2.058}{\sqrt{9.81 \times 0.2915}} \quad = 1.217$$

and, from the hydraulic-jump formula, the depth on the downstream side of the jump is

$$h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8Fr_n^2}\right) \quad = \frac{0.2915}{2}\left(-1 + \sqrt{1 + 8 \times 1.217^2}\right) \quad = 0.3767 \text{ m}$$

Answer: depths on upstream and downstream sides of the hydraulic jump are 0.291 m and 0.377 m respectively

Q18.

The specific energy (= head relative to the bed of the channel) is

$$E = h + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2}$$

With $Q = 3\ \mathrm{m^3\ s^{-1}}$ and $b = 1.5$ m, this gives, in metre-second units:

$$E = h + \frac{0.2039}{h^2}$$

Upstream,

$$E_1 = 1.8 + \frac{0.2039}{1.8^2} \quad = 1.863\ \mathrm{m}$$

Downstream, since the sluice is "controlling the flow" we require the supercritical solution of

$$E_2 = h + \frac{0.2039}{h^2}$$

i.e. an appropriate iterative solution of

$$h = \sqrt{\frac{0.2039}{E_2 - h}}$$

(a) If there is no energy loss, $E_2 = E_1 = 1.863$ m. Hence,

$$h = \sqrt{\frac{0.2039}{1.863 - h}}$$

Iterating from $h = 0$ gives

$$h = 0.3695\ \mathrm{m}$$

Then,

$$V = \frac{Q}{bh} \quad = \frac{3}{1.5 \times 0.3695} \quad = 5.413\ \mathrm{m\ s^{-1}}$$

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{5.413}{\sqrt{9.81 \times 0.3695}} \quad = 2.843$$

Answer: $h_2 = 0.369$ m; $\mathrm{Fr} = 2.84$

(b) If there is 10% specific energy loss, $E_2 = 0.9E_1 = 1.677$ m. Hence,

$$h = \sqrt{\frac{0.2039}{1.677 - h}}$$

Iterating from $h = 0$ gives

$$h = 0.3995\ \mathrm{m}$$

Then,

$$V = \frac{Q}{bh} \quad = \frac{3}{1.5 \times 0.3995} \quad = 5.006 \text{ m s}^{-1}$$

$$\text{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{5.006}{\sqrt{9.81 \times 0.3995}} \quad = 2.529$$

Answer: $h_2 = 0.400$ m; $\text{Fr} = 2.53$

Q19.
$\rho = 1000 \text{ kg m}^{-3}$
$Q = 28 \text{ m}^3 \text{ s}^{-1}$
$b = 6 \text{ m}$
$h_1 = 0.6 \text{ m}$
$h_{\text{block}} = 0.3 \text{ m}$
$c_D = 0.3$

Approach-flow velocity:

$$V_1 = \frac{Q}{bh_1} \quad = \frac{28}{6 \times 0.6} \quad = 7.778 \text{ m s}^{-1}$$

Force on 2 rows of blocks (height 0.3 m and total width 6 m):

$$F = 2 \times c_D \left(\frac{1}{2}\rho V_1^2\right)(h_{\text{block}}b) = 2 \times 0.3 \times \left(\frac{1}{2} \times 1000 \times 7.778^2\right) \times (0.3 \times 6)$$
$$= 32670 \text{ N}$$

Steady-state momentum principle:

$$\rho Q(V_2 - V_1) = -F + \frac{1}{2}\rho g(h_1^2 - h_2^2)b$$

Noting that, by continuity, $V_2(bh_2) = V_1(bh_1)$, so that $V_2 = V_1(h_1/h_2)$ this can be rearranged to keep all the terms in the unknown h_2 on the LHS:

$$\rho Q V_1(h_1/h_2) + \frac{1}{2}\rho g h_2^2 b = -F + \rho Q V_1 + \frac{1}{2}\rho g h_1^2 b$$

Substituting numerical values:

$$\frac{130700}{h_2} + 29430h_2^2 = 195700 \qquad (*)$$

(a) If a hydraulic jump *does not* occur, look for the smaller-h_2 (supercritical) solution of (*) by making the h_2 from the first term on the LHS the subject of an iterative formula:

$$h_2 = \frac{130700}{195700 - 29430h_2^2}$$

Iteration (from, e.g., $h_2 = 0$) gives:

$$h_2 = 0.7252 \text{ m}$$

Answer: 0.725 m

(b) If a hydraulic jump *does* occur, look for the larger-h_2 (subcritical) solution of (*) by making the h_2 from the second term on the LHS the subject of an iterative formula:

$$h_2 = \sqrt{\frac{195700 - \dfrac{130700}{h_2}}{29430}}$$

i.e.

$$h_2 = \sqrt{6.650 - \frac{4.441}{h_2}}$$

Iteration (from, e.g., $h_2 = \sqrt{6.650} = 2.579$) gives

$$h_2 = 2.139 \text{ m}$$

Answer: 2.14 m

Q20.

(a) Assume rapidly-varied flow with negligible upstream dynamic head in the reservoir. If the flow is supercritical on the spillway (check in part(b)) then the flow goes through a critical point at the top of the spillway.

Measure head relative to the top of the weir; the head H is then the freeboard in the reservoir and this is 3/2 times critical depth:

$$H = \frac{3}{2} h_c$$

$$\Rightarrow \qquad h_c = \frac{2}{3} H \quad = \frac{2}{3} \times 0.5 \quad = 0.3333 \text{ m}$$

But

$$h_c = (\frac{q^2}{g})^{1/3}$$

$$\Rightarrow \qquad q = g^{1/2} h_c^{3/2} \quad = 9.81^{1/2} \times 0.3333^{3/2} \quad = 0.6027 \text{ m}^2 \text{ s}^{-1}$$

$$\Rightarrow \qquad Q = qb \quad = 0.6027 \times 4 \quad = 2.411 \text{ m}^3 \text{ s}^{-1}$$

Answer: 2.41 $\text{m}^3 \text{ s}^{-1}$

(b)

$b = 4$ m

$S = 0.05$

$n = 0.012 \text{ m}^{-1/3}$ s

$Q = 2.411 \text{ m}^3 \text{ s}^{-1}$

<u>Normal depth</u>

Discharge:

$$Q = VA$$

where:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} \quad = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n} \frac{bh^{5/3}}{(1 + 2h/b)^{2/3}} S^{1/2}$$

Rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 0.1276(1 + 0.5h)^{2/5}$$

Iteration (from, e.g., $h = 0.1276$) gives

$$h_n = 0.1309 \text{ m}$$

Criticality

Since $h_n < h_c$ the normal depth is supercritical; i.e. the slope is steep at this discharge. (Alternatively, you could demonstrate this by finding the Froude number.)

Answer: 0.131 m

(c) As before:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

With $S = 0.001$ this gives:

$$h = 0.4127(1 + 0.5h)^{2/5}$$

Iteration (from, e.g., $h = 0.4127$) gives

$$h_n = 0.4474 \text{ m}$$

(This is less than h_c, hence subcritical.)

Answer: 0.447 m

(d) Consider a control volume enclosing the blocks. The flow has depth $h_1 = 0.1309$ m upstream and $h_2 = 0.4474$ m downstream. By the steady-state momentum principle:

$$\frac{1}{2}\rho g h_1^2 b - \frac{1}{2}\rho g h_2^2 b - F = \rho Q(V_2 - V_1)$$

$$\Rightarrow \qquad F = \frac{1}{2}\rho g b(h_1^2 - h_2^2) + \rho Q(V_1 - V_2)$$

Here,

$$V_1 = \frac{q}{h_1} \quad = \frac{0.6027}{0.1309} \quad = 4.604 \text{ m s}^{-1}$$

$$V_2 = \frac{q}{h_2} \quad = \frac{0.6027}{0.4474} \quad = 1.347 \text{ m s}^{-1}$$

Hence,

$$F = \frac{1}{2} \times 1000 \times 9.81 \times 4 \times (0.1309^2 - 0.4474^2)$$
$$+1000 \times 2.411 \times (4.604 - 1.347)$$
$$= 4262 \text{ N}$$

Answer: 4.26 kN

Q21.

(a) Total head is constant, so

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$

or, in terms of the discharge per unit width, q:

$$h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$$

Rearranging to collect terms in q:

$$2g(h_1 - h_2) = q^2\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)$$

$$\Rightarrow \qquad 2 \times 9.81 \times (2 - 0.3) = q^2\left(\frac{1}{0.3^2} - \frac{1}{2^2}\right)$$

$$\Rightarrow \qquad 33.35 = 10.86q^2$$

Solving gives

$$q = 1.752 \text{ m}^2 \text{ s}^{-1}$$

The total discharge (for width $b = 3$ m) is then

$$Q = qb \quad = 1.752 \times 3 \quad = 5.256 \text{ m}^3 \text{ s}^{-1}$$

Answer: 5.26 m^3 s^{-1}

(b)

Discharge:

$$Q = VA$$

where, in normal flow:

$$V = \frac{1}{n}R_h^{2/3}S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} \quad = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n}\frac{bh^{5/3}}{(1 + 2h/b)^{2/3}}S^{1/2}$$

or, rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5}(1 + 2h/b)^{2/5}$$

Substituting values:

$$h = 0.8586(1 + 2h/3)^{2/5}$$

Iterating (from, e.g., $h = 0$) gives

$$h = 1.064 \text{ m}$$

Answer: 1.06 m

(c) Consider a control volume encompassing the blocks. Let subscripts A and B respectively denote conditions upstream and downstream of the blocks. Then:

$$h_A = 0.3 \text{ m}, \quad h_B = h_n = 1.064 \text{ m}$$

The corresponding velocities are:

$$V_A = \frac{q}{h_A} = \frac{1.752}{0.3} = 5.84 \text{ m s}^{-1}, \qquad V_B = \frac{q}{h_B} = \frac{1.752}{1.064} = 1.647 \text{ m s}^{-1}$$

If F is the force on the blocks then, by the momentum principle:

$$\frac{1}{2}\rho g(h_A^2 - h_B^2)b - F = \rho q b(V_B - V_A)$$

Rearranging for F:

$$F = \frac{1}{2}\rho g(h_A^2 - h_B^2)b + \rho Q(V_A - V_B)$$

$$= \frac{1}{2} \times 1000 \times 9.81 \times (0.3^2 - 1.064^2) \times 3 + 1000 \times 5.256 \times (5.84 - 1.647) = 6704 \text{ N}$$

Answer: 6.70 kN

Q22.
$Q = 10 \text{ m}^3 \text{ s}^{-1}$
$h_1 = 0.5$ m
$b = 4$ m (width prior to expansion)
$B = 8$ m (width after expansion)

Velocity upstream of the expansion:

$$V_1 = \frac{Q}{bh_1} \quad = \frac{10}{4 \times 0.5} \quad = 5 \text{ m s}^{-1}$$

Velocity downstream of the expansion:

$$V_2 = \frac{Q}{Bh_2} \quad = \frac{10}{8 \times h_2} \quad = \frac{1.25}{h_2}$$

Assume that a hydraulic jump is triggered immediately at the expansion and that reactions from the expansion end walls are in equilibrium with a hydrostatic pressure distribution. Apply the steady-state momentum principle from the point of expansion to downstream of the hydraulic jump:

$$\frac{1}{2}\rho g h_1^2 B - \frac{1}{2}\rho g h_2^2 B = \rho Q(V_2 - V_1)$$

(Effectively, it is assumed that a hydrostatic pressure force acts across the expanded width B at both sections.) Dividing by ρ:

$$\frac{1}{2} gB(h_1^2 - h_2^2) = Q(V_2 - V_1)$$

Substituting numerical values:

$$39.24(0.25 - h_2^2) = 10\left(\frac{1.25}{h_2} - 5\right)$$

$$\Rightarrow \qquad 59.81 - 39.24h_2^2 = \frac{12.5}{h_2}$$

Look for the larger-h_2 (subcritical) solution by making the h_2 on the LHS (which comes from the hydrostatic pressure terms) the subject of an iterative formula:

$$h_2 = \sqrt{\frac{59.81 - \frac{12.5}{h_2}}{39.24}}$$

or

$$h_2 = \sqrt{1.524 - \frac{0.3186}{h_2}}$$

Iteration (from, e.g., $h_2 = \sqrt{1.524}$) gives

$$h_2 = 1.112 \text{ m}$$

Answer: 1.11 m

Q23.
(a)
$n = 0.016 \text{ m}^{-1/3} \text{ s}$
$b = 3.5 \text{ m}$
$S = 0.003$
$Q = 8 \text{ m}^3 \text{ s}^{-1}$

Critical Depth
Flow rate per unit width:

$$q = \frac{Q}{b} \quad = \frac{8}{3.5} \quad = 2.286 \text{ m}^2 \text{ s}^{-1}$$

Critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{2.286^2}{9.81}\right)^{1/3} \quad = 0.8106 \text{ m}$$

Normal Depth

$$Q = VA$$

where

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} \quad = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n}\left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} bh$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 0.7848(1 + 2h/3.5)^{2/5}$$

Iteration (from, e.g., $h = 0.7848$) gives

$$h_n = 0.9308 \text{ m}$$

The normal depth is greater than the critical depth, so the Froude number is less than 1; i.e. subcritical.

Answer: critical depth = 0.811 m;   normal depth = 0.931 m

(b) The flow will go critical in the restricted section if the critical head required to pass the given flow rate exceeds that available in the approach flow.

The specific energy in the approach flow (normal, since the channel is “long”) is

$$E_a = h_n + \frac{V_n^2}{2g} \qquad = h_n + \frac{Q^2}{2gb^2h_n^2}$$

$$= 0.9308 + \frac{8^2}{2 \times 9.81 \times 3.5^2 \times 0.9308^2} \qquad = 1.238 \text{ m}$$

Hence, in the vicinity of the restricted section:

$$H_a = E_a \qquad = 1.238 \text{ m}$$

Flow rate per unit width in the restricted section:

$$q_m = \frac{Q}{b_{\text{min}}} \qquad = \frac{8}{2.2} \qquad = 3.636 \text{ m}^2 \text{ s}^{-1}$$

Critical depth in the restricted section:

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \qquad = \left(\frac{3.636^2}{9.81}\right)^{1/3} \qquad = 1.105 \text{ m}$$

Critical head:

$$H_c = \frac{3}{2} h_c \qquad = 1.658 \text{ m}$$

The critical head, H_c, exceeds the head available in the approach flow, H_a. Hence the flow *does* go critical and the total head in the vicinity of this section is the critical head:

$$H = H_c \qquad = 1.658 \text{ m}$$

The depths just up- and downstream of the restricted section are the sub- and supercritical depths with this total head and the main channel width ($b = 3.5$ m):

$$H = h + \frac{V^2}{2g} \qquad = h + \frac{Q^2}{2gb^2h^2}$$

For h in metres:

$$1.658 = h + \frac{0.2663}{h^2}$$

Rearranging for the shallow (supercritical) solution:

$$h = \sqrt{\frac{0.2663}{1.658 - h}}$$

Iteration (from, e.g., $h = 0$) gives

$$h_{\text{downstream}} = 0.4743 \text{ m}$$

Answer: 0.474 m

(c)

Velocity upstream of the expansion:

$$V_1 = \frac{Q}{bh_1} \qquad = \frac{8}{3.5 \times 0.4743} \qquad = 4.819 \text{ m s}^{-1}$$

Velocity downstream of the expansion:

$$V_2 = \frac{Q}{Bh_2} \quad = \frac{8}{5 \times h_2} \quad = \frac{1.6}{h_2}$$

Assume a hydraulic jump is triggered at the expansion and that reactions from the expansion end walls are in equilibrium with a hydrostatic pressure distribution. Apply the steady-state momentum principle from the point of expansion to downstream of the hydraulic jump:

$$\frac{1}{2}\rho g h_1^2 B - \frac{1}{2}\rho g h_2^2 B = \rho Q(V_2 - V_1)$$

(Effectively, it is assumed that a hydrostatic pressure force acts across the expanded width B at both sections.) Multiplying by $2/(\rho g B)$:

$$h_1^2 - h_2^2 = \frac{2Q}{gB}(V_2 - V_1)$$

Substituting numerical values:

$$0.2250 - h_2^2 = 0.3262\left(\frac{1.6}{h_2} - 4.819\right)$$

Look for the larger-h_2 (subcritical) solution by making the h_2 on the LHS the subject of an iterative formula. After multiplying out and rearranging:

$$h_2 = \sqrt{1.797 - \frac{0.5219}{h_2}}$$

Iteration (from, e.g., $h_2 = \sqrt{1.797}$) gives

$$h_2 = 1.161 \text{ m}$$

Answer: 1.16 m

Q24.
(a) With depth h measured from the bottom of the vee, the surface width b_s and cross-sectional area A are given by

$$b_s = 2h \tan\alpha$$

$$A = \frac{1}{2} b_s h \quad = h^2 \tan\alpha$$

where, in this instance, $\alpha = 40°$. Hence, the *average* depth (across the width of the channel) is

$$\bar{h} \equiv \frac{A}{b_s} \quad = \frac{1}{2} h$$

and the Froude number (squared) is given by

$$\mathrm{Fr}^2 = \frac{V^2}{g\bar{h}} \quad = \frac{(Q/A)^2}{g\bar{h}} \quad = \frac{Q^2}{(h^4 \tan^2\alpha) g(\frac{1}{2}h)} \quad = \frac{2Q^2}{g(\tan^2\alpha) h^5}$$

The critical depth corresponds to $\mathrm{Fr} = 1$, whence:

$$h_c = \left(\frac{2Q^2}{g\tan^2\alpha}\right)^{1/5} \quad = \left(\frac{2 \times 16^2}{9.81 \times \tan^2 40°}\right)^{1/5} \quad = 2.366 \text{ m}$$

Answer: 2.37 m

(b) From the momentum principle:

(pressure force)$_{in}$ – (pressure force)$_{out}$ = (momentum flux)$_{out}$ – (momentum flux)$_{in}$

⇒ pressure force + momentum flux = constant

From hydrostatics, with centroidal depth for a triangle, $\bar{d} = \frac{1}{3}h$:

$$\text{pressure force} = \rho g \bar{d} \times A \quad = (\rho g \frac{1}{3} h) \times (h^2 \tan\alpha) \quad = \frac{1}{3} \rho g h^3 \tan\alpha$$

The momentum flux, with uniform cross-sectional velocity, is

$$\text{momentum flux} = \rho Q V \quad = \rho Q \times \frac{Q}{A} \quad = \frac{\rho Q^2}{h^2 \tan\alpha}$$

Hence,

$$\frac{1}{3}\rho g h^3 \tan\alpha + \frac{\rho Q^2}{h^2 \tan\alpha} = \text{constant}$$

or, dividing by ρ,

$$\frac{1}{3} g h^3 \tan\alpha + \frac{Q^2}{h^2 \tan\alpha} = \text{constant}$$

Substituting values, fixing the constant from the depth $h = 1.85$ m on one side, gives:

$$2.744 h^3 + \frac{305.1}{h^2} = 106.5$$

Since the given depth ($h = 1.85$ m) is the supercritical upstream solution (since it is smaller than the critical depth found in part (a)), we now seek the subcritical ("large h") solution, for which we expect the first term in the equation to dominate. Rearranging for iteration:

$$h = \left[\frac{1}{2.744}\left(106.5 - \frac{305.1}{h^2}\right)\right]^{1/3}$$

Iteration (from any subcritical value, $h > 2.37$ m), gives $h = 2.970$ m.

Answer: 2.97 m

Q25.
From the momentum principle:

(pressure force)$_{in}$ – (pressure force)$_{out}$ = (momentum flux)$_{out}$ – (momentum flux)$_{in}$

$$\Rightarrow \quad \bar{p}_1 A_1 - \bar{p}_2 A_2 = \rho Q (V_2 - V_1)$$

$$\Rightarrow \quad \rho g \bar{d}_1 A_1 - \rho g \bar{d}_2 A_2 = \rho Q \left(\frac{Q}{A_2} - \frac{Q}{A_1}\right)$$

$$\Rightarrow \quad g(\bar{d}_1 A_1 - \bar{d}_2 A_2) = Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1}\right) \qquad (*)$$

where $\bar{d}$ is the depth of the centroid. Q is the only unknown and we simply need to find the centroidal depth d and cross-sectional area A for each given depth.

For a triangle,

$$\bar{d} = \frac{1}{3} h$$

Hence,

$$\bar{d}_1 = \frac{1}{3} h_1 \quad = \frac{1}{3} \times 1.2 \quad = 0.4 \text{ m}$$

$$\bar{d}_2 = \frac{1}{3} h_2 \quad = \frac{1}{3} \times 2.1 \quad = 0.7 \text{ m}$$

The surface width is $2 \times h \tan 30° = 2h/\sqrt{3}$ and hence the cross-sectional area A is

$$A = \frac{1}{2} \times h \times \frac{2h}{\sqrt{3}} \quad = \frac{h^2}{\sqrt{3}}$$

Hence,

$$A_1 = \frac{h_1^2}{\sqrt{3}} \quad = \frac{1.2^2}{\sqrt{3}} \quad = 0.8314 \text{ m}^2$$

$$A_2 = \frac{h_2^2}{\sqrt{3}} \quad = \frac{2.1^2}{\sqrt{3}} \quad = 2.546 \text{ m}^2$$

Substituting in (*):

$$9.81(0.4 \times 0.8314 - 0.7 \times 2.546) = Q^2 \left(\frac{1}{2.546} - \frac{1}{0.8314}\right)$$

$$\Rightarrow \quad -14.22 = Q^2 \times (-0.8100)$$

$$\Rightarrow \quad Q = 4.190 \text{ m}^3 \text{ s}^{-1}$$

Answer: 4.19 m^3 s^{-1}

The head *loss* is

$$H_1 - H_2 = z_{s1} - z_{s2} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

or, measuring *z* from the bottom of the vee:

$$H_1 - H_2 = h_1 - h_2 + \frac{1}{2g}(V_1^2 - V_2^2) \qquad (**)$$

The velocities either side of the jump are

$$V_1 = \frac{Q}{A_1} \quad = \frac{4.190}{0.8314} \quad = 5.040 \text{ m s}^{-1}$$

$$V_2 = \frac{Q}{A_2} \quad = \frac{4.190}{2.546} \quad = 1.646 \text{ m s}^{-1}$$

Hence, the head loss (**) is

$$H_1 - H_2 = 1.2 - 2.1 + \frac{1}{2 \times 9.81}(5.040^2 - 1.646^2) \quad = 0.2566 \text{ m}$$

Answer: 0.257 m

Q26.
Momentum principle:

$$\bar{p}_1 A_1 - \bar{p}_2 A_2 = \rho Q(V_2 - V_1)$$

With $\bar{p} = \rho g \bar{d}$ and $V = Q/A$,

$$\rho g(\bar{d}_1 A_1 - \bar{d}_2 A_2) = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

Rearranging, and dividing by ρ:

$$\left(g\bar{d}A + \frac{Q^2}{A}\right)_1 = \left(g\bar{d}A + \frac{Q^2}{A}\right)_2 \quad = C, \quad \text{say}$$

Here, for a circular cross-section:

$$A = 2 \times \left(\frac{1}{2}R^2\theta - \frac{1}{2}R\sin\theta\, R\cos\theta\right) \quad = R^2\left(\theta - \frac{1}{2}\sin 2\theta\right)$$

$$\bar{d} = R\left[\frac{\frac{2}{3}\sin^3\theta}{\theta - \frac{1}{2}\sin(2\theta)} - \cos\theta\right]$$

Hence, the following quantity has to be the same on both sides of the jump:

$$C = gR^3\left[\frac{2}{3}\sin^3\theta - \cos\theta\left(\theta - \frac{1}{2}\sin 2\theta\right)\right] + \frac{Q^2}{R^2\left(\theta - \frac{1}{2}\sin 2\theta\right)} \qquad (*)$$

On the upstream side:

$$h = R - R\cos\theta$$

$$\theta = \cos^{-1}\left(1 - \frac{h}{R}\right) \quad = 31.79° \quad (0.5548 \text{ rad})$$

With $R = 2$ m and $Q = 1.5$ m^3 s^{-1} (*) gives (in m-s units):

$$C = 5.762$$

Search (e.g. by repeated trial, which would benefit from a short computer program or spreadsheet – *an example in Python is given at the end of the question*) for a second value of θ that gives the same value of C. This gives

$$\theta = 0.8697 \text{ rad} \quad (49.83°)$$

$$h = R - R\cos\theta \quad = 0.7099 \text{ m}$$

Answer: 0.710 m

(b) The Froude number is given by

$$\text{Fr}^2 = \frac{V^2}{g\bar{h}} \quad = \frac{(Q/A)^2}{g(A/b_s)} \quad = \frac{Q^2 b_s}{gA^3}$$

whence

$$\text{Fr} = Q\sqrt{\frac{b_s}{gA^3}}$$

Here, the surface width is

$$b_s = 2R\sin\theta$$

For the upstream side:

$$\theta = 0.5548 \text{ rad } (= 31.79°), \quad b_s = 2.107 \text{ m}, \quad A = 0.4281 \text{ m}^2$$

For the downstream side:

$$\theta = 0.8697 \text{ rad } (= 49.83°), \quad b_s = 3.057 \text{ m}, \quad A = 1.507 \text{ m}^2$$

These give

$$\text{Fr}_1 = 2.482, \quad \text{Fr}_2 = 0.4526$$

Answer: upstream and downstream Froude numbers are 2.48 and 0.45, respectively

A simple Python search script is given overleaf for interest. Modify the search interval to home in on the required values.

There are plenty of other numerical or graphical ways of finding the answer.

```
from math import sin, cos, acos, sqrt, pi
import numpy as np

g = 9.81                               # gravity, m s^-2
R = 2.0                                # radius, m
Q = 1.5                                # volume flow rate (m^3 s^-1)

def angle( h ):                        # angle for given h
    return acos( 1 - h / R )

def depth( theta ):                    # depth for given angle
    return R * ( 1 - cos( theta ) )

def dbar( theta ):                     # centroidal depth
    return R * ( (2.0/3.0) * sin(theta)**3 / ( theta - 0.5*sin(2.0*theta) ) - cos(theta) )

def area( theta ):                     # area of segment
    return R * R * ( theta - 0.5 * sin( 2.0 * theta ) )

def surfaceWidth( theta ):             # surface width
    return 2 * R * sin( theta )

def averageDepth( theta ):             # average depth
    return area( theta ) / surfaceWidth( theta )

def Froude( theta ):                   # Froude number
    return Q / area( theta ) / sqrt( g * averageDepth( theta ) )

def Cfunc( theta ):
    A = area( theta )
    return g * dbar( theta ) * A + Q ** 2 / A

#----------------------

# Manual search via depth
h = float( input( "Input depth in m: " ) )
thetaRad = angle( h )
print( "theta (rad) = ", thetaRad )
print( "C = ", Cfunc( thetaRad ) )
print( "Check data ..." )
print( "theta (deg) = ", thetaRad * 180 / pi )
print( "dbar = ", dbar( thetaRad ) )
print( "A = ", area( thetaRad ) )
print( "bs = ", surfaceWidth( thetaRad ) )
print( "h = ", averageDepth( thetaRad ) )
print( "Fr = ", Froude( thetaRad ) )
print()

#----------------------

# Tabular search for theta in interval [a,b]
N, a, b = 51, thetaRad, 0.5 * pi             # search range - adjust as required
rad = np.linspace( a, b, N )
fmth, fmt = "{:>12}  ", "{:12.5f}  "
print( ( 4 * fmth ).format( "angle(rad)", "depth", "C", "Fr" ) )
for i in range( 0, N ):
    print( ( 4 * fmt ).format( rad[i], depth(rad[i]), Cfunc(rad[i]), Froude(rad[i]) ) )
```

Q27.
$S_0 = 2 \times 10^{-5}$
$n = 0.01 \text{ m}^{-1/3} \text{ s}$
$q = 0.5 \text{ m}^2 \text{ s}^{-1}$

Normal Depth

$$q = Vh \quad = \frac{1}{n} R_h^{2/3} S^{1/2} h, \qquad \text{where} \qquad R_h = h \qquad \text{(wide channel)}$$

$$\Rightarrow \qquad \frac{nq}{\sqrt{S}} = h^{5/3} \qquad (*)$$

For the given slope this gives a normal depth

$$h_n = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} \quad = \left(\frac{0.01 \times 0.5}{\sqrt{2 \times 10^{-5}}}\right)^{3/5} \quad = 1.069 \text{ m}$$

Critical Depth

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{0.5^2}{9.81}\right)^{1/3} \quad = 0.2943 \text{ m}$$

The normal depth is subcritical, so the slope is mild. The flow is subcritical, but goes critical at the free overfall.

Hence, we start a GVF calculation at the free overfall (where $h = h_c = 0.2943$ m) and work *upstream*.

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2} \qquad \text{or} \qquad \frac{\mathrm{d}E}{\mathrm{d}x} = S_0 - S_f$$

(*Continued overleaf*)

Method 1

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h}=\frac{1-\mathrm{Fr}^2}{S_0-S_f} \qquad \text{and} \qquad \Delta x\approx\left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)\Delta h$$

For the working, write the derivative as a function of h; (all lengths in metres).

$$\mathrm{Fr}=\frac{V}{\sqrt{gh}}=\frac{q}{\sqrt{gh^3}} \qquad \Rightarrow \qquad \mathrm{Fr}^2=\frac{q^2}{gh^3}=\frac{0.02548}{h^3}$$

$$S_f=\left(\frac{nq}{h^{5/3}}\right)^2=\frac{2.5\times10^{-5}}{h^{10/3}}$$

$$\Delta h=\frac{1.0-0.2943}{2}=0.35285$$

Working formulae:

$$\Delta x=\left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}}\Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h}=\frac{1-\dfrac{0.02548}{h^3}}{\left(2-\dfrac{2.5}{h^{10/3}}\right)\times10^{-5}} \qquad \Delta h=0.35285$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.2943	0			
			0.4707	–2622	–925
1	0.64715	–925.2			
			0.8236	–34400	–12140
2	1.0000	**–13065**			

Answer: 13.1 km

(*Alternative method overleaf*)

Method 2

Rewrite the GVF equation as:

$$\frac{dx}{dE} = \frac{1}{S_0 - S_f}, \qquad \text{whence} \qquad \Delta x \approx \frac{\Delta E}{(S_0 - S_f)_{\mathrm{av}}}$$

For the working, write the friction slope and specific energy as a function of h; (all lengths in metres).

$$S_f = \left(\frac{nq}{h^{5/3}}\right)^2 \quad = \frac{2.5 \times 10^{-5}}{h^{10/3}}$$

$$E = h + \frac{V^2}{2g} \quad = h + \frac{q^2}{2gh^2} \quad = h + \frac{0.01274}{h^2}$$

$$\Delta h = \frac{1.0 - 0.2943}{2} \quad = 0.35285$$

Working formulae:

$$\Delta x \approx \frac{\Delta E}{(S_0 - S_f)_{\mathrm{av}}}$$

where

$$S_0 - S_{av} = \left(2 - \frac{2.5}{h^{\frac{10}{3}}}\right) \times 10^{-5}, \qquad E = h + \frac{0.01274}{h^2}, \qquad \Delta h = 0.35285$$

i	h_i	x_i	E_i	$(S_0 - S_f)_i$	ΔE	$(S_0 - S_f)_{av}$	Δx
0	0.2943	0	0.4414	-145.4×10^{-5}			
					0.2362	-77.032×10^{-5}	–307
1	0.64715	–307	0.6776	-8.664×10^{-5}			
					0.3351	-4.582×10^{-5}	–7313
2	1.0000	**–7620**	1.0127	-0.500×10^{-5}			

Answer: 7.6 km.

Spreadsheet calculations (*exercise*) yield the following.

N_{steps}	x_{final} using $\Delta x = \left(\frac{1-\mathrm{Fr}^2}{S_0 - S_f}\right)_{\mathrm{mid}} \Delta h$	x_{final} using $\Delta x = \frac{\Delta E}{(S_0 - S_f)_{\mathrm{av}}}$
2	–13060	–7620
5	–17360	–15600
10	–18630	–18090
50	–19190	–19163
100	–19210	–19200

Q28.
$b = 5$ m
$S = 0.0006$
$Q = 7$ m^3 s^{-1}
$n = 0.035$ m$^{-1/3}$ s

(a)

$$Q = VA, \quad \text{where} \quad V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad R_h = \frac{h}{1 + 2h/b}, \qquad A = bh$$

$$\Rightarrow \qquad Q = \frac{1}{n}\left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} bh$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 1.516(1 + 0.4h)^{2/5}$$

Iteration (from, e.g., $h = 1.516$) gives

$$h_n = 1.901 \text{ m}$$

Answer: 1.90 m

(b) The specific energy in the approach flow (normal flow, since the channel is "long") is

$$E_a = h_n + \frac{V_n^2}{2g} \quad = h_n + \frac{Q^2}{2gb^2h_n^2} \quad = 1.901 + \frac{7^2}{2 \times 9.81 \times 5^2 \times 1.901^2} \quad = 1.929 \text{ m}$$

Hence, relative to the bed in the vicinity of the weir:

$$H_a = E_a \quad = 1.929 \text{ m}$$

The critical depth and critical specific energy are (with $q = Q/b = 1.4$ m^2 s^{-1}):

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{1.4^2}{9.81}\right)^{1/3} \quad = 0.5846 \text{ m}$$

$$E_c = \frac{3}{2} h_c \quad = \frac{3}{2} \times 0.5846 \quad = 0.8769 \text{ m}$$

Under critical conditions the total head over the weir would be

$$H_c = z_{\text{weir}} + E_c \quad = 1.75 + 0.8769 \quad = 2.627 \text{ m}$$

This is the minimum head for this flow rate over the weir and exceeds the head available in the approach flow. Hence, the flow backs up, the water level rises upstream of the weir and the flow over the top is critical. The total head in the vicinity of the weir is determined by critical conditions: $H = H_c = 2.627$ m.

Outside of the weir,

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2}$$

$$\Rightarrow \qquad 2.627 = h + \frac{0.09990}{h^2}$$

Upstream of the weir, rearrange for the deep (subcritical) solution:

$$h = 2.627 - \frac{0.09990}{h^2}$$

Iteration (from, e.g., $h = 2.627$) gives

$$h = 2.612 \text{ m}$$

Answer: 2.61 m

(c) GVF from just upstream of the weir ($h_{start} = 2.612$ m) to where the depth is 0.25 m greater than the normal depth ($h_{\text{end}} = 1.901 + 0.25 = 2.151$ m). The flow is subcritical here, so we will be working in the opposite direction to the flow. Using two steps the depth increment is

$$\Delta h = \frac{h_{\text{end}} - h_{\text{start}}}{N_{\text{steps}}} \quad = \frac{2.151 - 2.612}{2} \quad = -0.2305 \text{ m}$$

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)\Delta h$$

For the working, write the derivative as a function of h.

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{q}{\sqrt{gh^3}} \qquad \Rightarrow \qquad \mathrm{Fr}^2 = \frac{q^2}{gh^3} \quad = \frac{0.1998}{h^3}$$

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 (1 + 2h/b)^{4/3} \quad = 2.401 \times 10^{-3} \frac{(1 + 0.4h)^{4/3}}{h^{10/3}}$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.1998}{h^3}}{\left[6 - 24.01\,\dfrac{(1+0.4h)^{4/3}}{h^{10/3}}\right] \times 10^{-4}}, \qquad \Delta h = -0.2305$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	2.612	0			
			2.497	3146	–725.2
1	2.382	–725.2			
			2.267	4291	–989.1
2	2.151	**–1714**			

Answer: 1.71 km

Q29.
$b = 1.5$ m
$n = 0.014$ m$^{-1/3}$ s
$S = 0.002$
$h_{\text{overfall}} = 0.6$ m

(a) The flow is drawn down at the overfall, so must be subcritical leading to critical at the overfall. The critical depth is therefore 0.6 m.

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Rearranging, the flow rate per metre width is

$$q = \sqrt{gh_c^3} \quad = \sqrt{9.81 \times 0.6^3} \quad = 1.456 \text{ m}^2 \text{ s}^{-1}$$

and the total flow rate is

$$Q = qb \quad = 1.456 \times 1.5 \quad = 2.184 \text{ m}^3 \text{ s}^{-1}$$

Answer: 2.18 m^3 s^{-1}

(b) The flow is subcritical, so do a GVF calculation working upstream from the free overfall (where $h = 0.6$ m) to where the depth is $h = 0.8$ m. Using two steps,

$$\Delta h = \frac{0.8 - 0.6}{2} \quad = 0.1 \text{ m}$$

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)\Delta h$$

For the working, write the derivative as a function of h; (all lengths in metres).

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{Q/b}{\sqrt{gh^3}} \qquad \Rightarrow \qquad \mathrm{Fr}^2 = \frac{q^2}{gh^3} \quad = \frac{0.2161}{h^3}$$

For quasi-normal flow (used to find the friction slope):

$$Q = VA \qquad \text{where} \quad V = \frac{1}{n}R_h^{2/3}S_f^{1/2}, \quad A = bh, \quad R_h = \frac{bh}{b + 2h} \quad = \frac{h}{1 + 2h/b}$$

$$\Rightarrow \qquad Q = \frac{1}{n}\frac{bh^{5/3}}{(1 + 2h/b)^{2/3}}S_f^{1/2}$$

and, rearranging,

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 (1+2h/b)^{4/3} \quad = 4.155\times 10^{-4}\,\frac{(1+h/0.75)^{4/3}}{h^{10/3}}$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1-\dfrac{0.2161}{h^3}}{\left[20 - 4.155\times\dfrac{(1+h/0.75)^{4/3}}{h^{10/3}}\right]\times 10^{-4}}, \qquad\qquad \Delta h = 0.1$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.6	0			
			0.65	–105.8	–10.58
1	0.7	–10.58			
			0.75	–666.8	–66.68
2	0.8	**–77.26**			

Answer: 77 m

Q30.
(a)
Normal

$$q = Vh \quad = \frac{1}{n} R_h^{2/3} S^{1/2} h, \qquad \text{where} \qquad R_h = h \qquad \text{(wide channel)}$$

$$\Rightarrow \qquad \frac{nq}{\sqrt{S}} = h^{5/3} \qquad (*)$$

$$\Rightarrow \qquad h_n = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} \quad = \left(\frac{0.03 \times 0.7}{\sqrt{8 \times 10^{-4}}}\right)^{3/5} \quad = 0.8364 \text{ m}$$

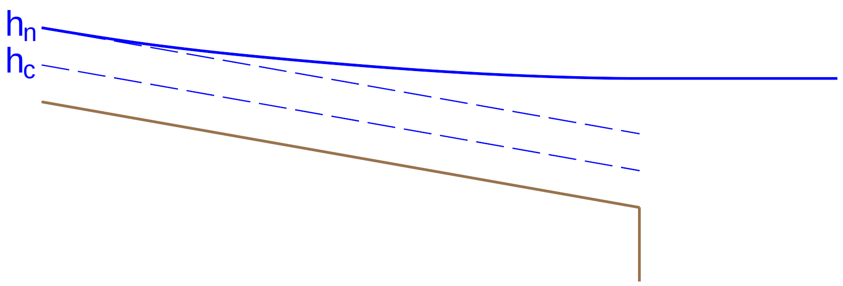
Critical

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{0.7^2}{9.81}\right)^{1/3} \quad = 0.3683 \text{ m}$$

The normal depth is greater than the critical depth. Hence, the normal flow is subcritical; i.e. the slope is mild.

Answer: normal depth = 0.836 m; critical depth = 0.368 m

(b) Note that the depth is everywhere above normal depth and depth is increasing. The depth asymptotes to normal depth far upstream and the water surface tends to horizontal approaching the sea. (The sea really spreads sidewards rather than straight down!)



(c) The flow is subcritical (part (a)). Hence, start a GVF calculation from the downstream end (where $h = 2.0$ m) and work *upstream* (to where $h = 1.0$ m).

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right) \Delta h$$

For the working, write the derivative as a function of h; (all lengths in metres).

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{q}{\sqrt{gh^3}} \qquad \Rightarrow \qquad \mathrm{Fr}^2 = \frac{q^2}{gh^3} \quad = \frac{0.04995}{h^3}$$

$$S_f = \left(\frac{nq}{h^{5/3}}\right)^2 \quad = \frac{4.41 \times 10^{-4}}{h^{10/3}}$$ (rearranging (*) in part (a) above)

$$\Delta h = \frac{1.0 - 2.0}{2} \quad = -0.5$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

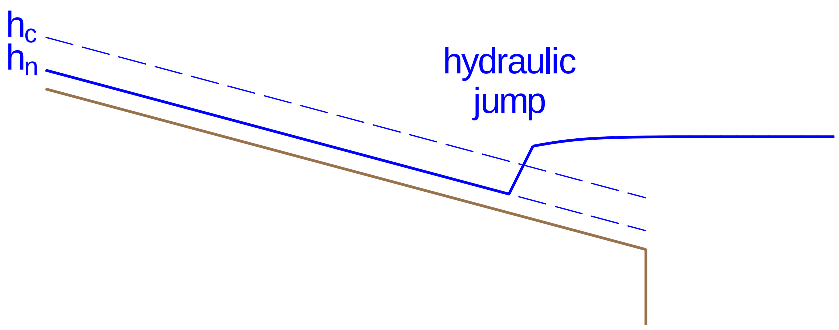
where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.04995}{h^3}}{\left(8 - \dfrac{4.41}{h^{10/3}}\right) \times 10^{-4}}, \qquad \Delta h = -0.5$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	2.0	0			
			1.75	1354	–677.0
1	1.5	–677.0			
			1.25	1650	–825.0
2	1.0	**–1502**			

Answer: 1.50 km

(d) If the slope changes then the only relevant depth that changes is the normal depth, which must now be less than the critical depth if the slope is to be "steep". As the normal flow is supercritical it must undergo a hydraulic jump before the river outfall to the sea. Given a long-enough upstream fetch the depth of flow upstream of the jump is normal flow. There is gradually-varied flow (S1) from the fixed depth at the coast upstream to where this matches the sequent depth of the normal flow. This fixes the position of the hydraulic jump.



Q31.
$b = 6$ m;   ($b_{\min} = 2.4$ m in part (b))
$S = 0.01$
$n = 0.035$ m$^{-1/3}$ s
$Q = 9$ m^3 s^{-1}

(a) For normal depth:

$$Q = VA$$

where

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b + 2h} \quad = \frac{h}{1 + 2h/b}$$

$$\Rightarrow \qquad Q = \frac{1}{n}\left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} bh$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1 + 2h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1 + 2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 0.6794(1 + 0.3333h)^{2/5}$$

Iteration (from, e.g., $h = 0.6794$) gives

$$h_n = 0.7422 \text{ m}$$

For critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{Q^2}{b^2 g}\right)^{1/3} \quad = 0.6121 \text{ m}$$

Normal flow is deeper, and hence slower, and hence has smaller Froude number than in critical flow (where $\mathrm{Fr} = 1$). Hence the Froude number is less than 1; i.e. subcritical.

Answer: normal depth = 0.742 m;   critical depth = 0.612 m

(b) The flow will go critical in the restricted section if the critical head required to pass the given flow rate exceeds that available in the approach flow.

The specific energy in the approach flow (normal, since the channel is "long") is

$$E_a = h_n + \frac{V_n^2}{2g} \quad = h_n + \frac{Q^2}{2gb^2h_n^2} \quad = 0.7422 + \frac{9^2}{2 \times 9.81 \times 6^2 \times 0.7422^2}$$
$$= 0.9504 \text{ m}$$

Hence, in the vicinity of the restricted section:

$$H_a = E_a \quad = 0.9504 \text{ m}$$

Flow rate per unit width in the restricted section:

$$q_m = \frac{Q}{b_{\text{min}}} \quad = \frac{9}{2.4} \quad = 3.75 \text{ m}^2 \text{ s}^{-1}$$

Critical depth:

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3} \quad = \left(\frac{3.75^2}{9.81}\right)^{1/3} \quad = 1.128 \text{ m}$$

Critical head:

$$H_c = \frac{3}{2} h_c \quad = 1.692 \text{ m}$$

The critical head, H_c, exceeds the head available in the approach flow, H_a. Hence the flow *does* go critical and the total head in the vicinity of this section is the critical head:

$$H = H_c \quad = 1.692 \text{ m}$$

The depths just up- and downstream of the restricted section are the sub- and supercritical depths with this total head and the main channel width ($b = 6$ m):

$$H = h + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2}$$

For h in metres:

$$1.692 = h + \frac{0.1147}{h^2}$$

Rearranging for the deep (subcritical) solution:

$$h = 1.692 - \frac{0.1147}{h^2}$$

Iteration (from, e.g., h =1.692) gives

$$h_{\text{upstream}} = 1.650 \text{ m}$$

Rearranging for the shallow (supercritical) solution:

$$h = \sqrt{\frac{0.1147}{1.692 - h}}$$

Iteration (from, e.g., h =0) gives

$$h_{\text{downstream}} = 0.2856 \text{ m}$$

Answer: upstream depth = 1.65 m; downstream depth = 0.286 m

(c) Unless the jump occurs immediately, there is a region of supercritical GVF until a hydraulic jump back to the preferred regime: subcritical flow.

Downstream of the hydraulic jump is normal flow (since the channel is "long"):

$$h_n = 0.7422 \text{ m}$$

$$\text{Fr}_n^2 = \frac{V_n^2}{gh_n} = \frac{Q^2}{b^2 g h_n^3} = \frac{9^2}{6^2 \times 9.81 \times 0.7422^3} = 0.5610$$

The depth just upstream of the jump is

$$h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8\text{Fr}_n^2}\right) = \frac{0.7422}{2}\left(-1 + \sqrt{1 + 8 \times 0.5610}\right) = 0.4983 \text{ m}$$

(This is larger than the depth just downstream of the contracted section, confirming that a length of GVF *does* occur.)

We must therefore do a supercritical GVF calculation from just downstream of the restricted section (where $h = 0.2856$ m) to just upstream of the hydraulic jump (where $h = 0.4983$ m).

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \text{Fr}^2}$$

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \text{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)\Delta h$$

For the working, write the derivative as a function of *h*; (all lengths in metres).

$$\text{Fr} = \frac{V}{\sqrt{gh}} = \frac{Q/b}{\sqrt{gh^3}} \qquad \Rightarrow \qquad \text{Fr}^2 = \frac{(Q/b)^2}{gh^3} = \frac{0.2294}{h^3}$$

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 \left(1 + \frac{2h}{b}\right)^{4/3} = 2.756 \times 10^{-3} \frac{(1 + 0.3333h)^{4/3}}{h^{10/3}}$$

$$\Delta h = \frac{0.4983 - 0.2856}{2} = 0.1064$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\text{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.2294}{h^3}}{0.01 - 2.756 \times 10^{-3} \times \dfrac{(1 + 0.3333h)^{4/3}}{h^{10/3}}} \qquad \Delta h = 0.1064$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.2856	0			
			0.3388	45.68	4.860
1	0.3920	4.860			
			0.4452	40.82	4.343
2	0.4983	**9.203**			

Answer: 9.20 m

Q32.

(a) $q = 0.9\ \mathrm{m}^2\ \mathrm{s}^{-1}$, so the critical depth (which is the depth over the weir because it is known to be "controlling the flow") is

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{0.9^2}{9.81}\right)^{1/3} \quad = 0.4355\ \mathrm{m}$$

The critical specific energy is

$$E_c = \frac{3}{2}h_c \quad = \frac{3}{2} \times 0.4355 \quad = 0.6533\ \mathrm{m}$$

The total head is

$$H = z_{\mathrm{weir}} + E_c \quad = 1.5 + 0.6533 \quad = 2.153\ \mathrm{m}$$

The upstream and downstream depths are solutions of

$$h + \frac{q^2}{2gh^2} = H$$

For the upstream (subcritical) depth, rearrange as

$$h = H - \frac{q^2}{2gh^2}$$

$$\Rightarrow \qquad h = 2.153 - \frac{0.04128}{h^2}$$

Iterating from $h = 2.153$ m gives

$$h = 2.144\ \mathrm{m}$$

For the downstream (supercritical) depth, rearrange as

$$h = \frac{q}{\sqrt{2g(H-h)}}$$

$$\Rightarrow \qquad h = \frac{0.9}{\sqrt{19.62(2.153-h)}}$$

Iterating from $h = 0$ gives

$$h = 0.1433\ \mathrm{m}$$

Answer: depths upstream, over weir, downstream = 2.14 m, 0.435 m, 0.143 m

(b)

$$q = Vh$$

where

$$V = \frac{1}{n}R_h^{2/3}S^{1/2} \qquad\qquad R_h = h$$

Hence,

$$q = \frac{1}{n} h^{5/3} S^{1/2}$$

$$\Rightarrow \qquad h = \left(\frac{nq}{\sqrt{S}}\right)^{3/5} \quad = \left(\frac{0.012 \times 0.9}{\sqrt{3 \times 10^{-4}}}\right)^{3/5} \quad = 0.7532 \text{ m}$$

Answer: 0.753 m

(c) Inverting the gradually-varied-flow equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f}$$

or, in discrete form:

$$\Delta x = \left(\frac{1 - \mathrm{Fr}^2}{S_0 - S_f}\right)_{\mathrm{mid}} \Delta h$$

To do the numerical integration it is convenient to have expressions for Fr and S_f as functions of h (in m):

$$\mathrm{Fr}^2 = \frac{V^2}{gh} \quad = \frac{q^2}{gh^3} \quad = \frac{0.08257}{h^3}$$

$$S_f = \left(\frac{nq}{h^{5/3}}\right)^2 \quad = \frac{1.166 \times 10^{-4}}{h^{10/3}}$$

Integration is carried out (in the *upstream* direction, because the flow here is subcritical) from $h = 2.144$ m to $h = 0.853$ m with 2 steps; i.e. $\Delta h = -0.6455$ m.

Working formulae (all lengths in m):

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.08257}{h^3}}{10^{-4}\left(3 - \dfrac{1.166}{h^{10/3}}\right)} \qquad\qquad \Delta h = -0.6455$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	2.144	0			
			1.821	3471	–2241
1	1.499	–2241			
			1.176	4090	–2640
2	0.853	**–4881**			

Answer: 4.9 km

(d) Downstream of the hydraulic jump (subscript 2) the depth is the normal depth ($h_2 = 0.7532$ m); the corresponding velocity and Froude number are

$$V_2 = \frac{q}{h_2} = \frac{0.9}{0.7532} = 1.195 \text{ m s}^{-1}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gh_2}} = \frac{1.195}{\sqrt{9.81 \times 0.7532}} = 0.4396$$

From the hydraulic-jump relations the sequent depth upstream is

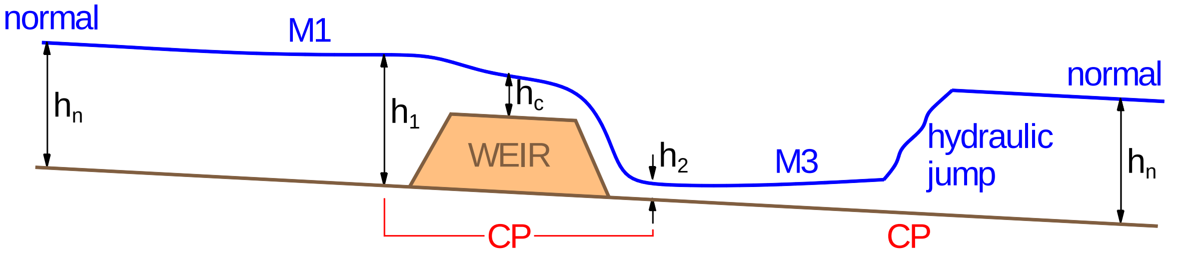
$$h_1 = \frac{h_2}{2}(-1 + \sqrt{1 + 8Fr_2^2}) = \frac{0.7532}{2}(-1 + \sqrt{1 + 8 \times 0.4396^2}) = 0.2243 \text{ m}$$

Integration is carried out (in the *downstream* direction, because the flow here is supercritical) from $h = 0.1433$ to $h = 0.2243$ with 1 step; i.e. $\Delta h = 0.0810$, all lengths being in m. Working is shown in the table below.

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.1433	0			
			0.1838	375.8	30.44
1	0.2243	**30.44**			

Answer: 30 m

(e)



Q33.
$b = 5$ m
$S = 5 \times 10^{-4}$
$C = 100\ \mathrm{m}^{1/2}\ \mathrm{s}^{-1}$
$Q = 15\ \mathrm{m}^3\ \mathrm{s}^{-1}$
$z_{\mathrm{weir}} = 1$ m

(a) For the normal depth,

$$Q = VA$$

where

$$V = CR_h^{1/2}S^{1/2}, \qquad A = bh, \qquad R_h = \frac{bh}{b+2h} \quad = \frac{h}{1+2h/b}$$

Hence,

$$Q = C\ \left(\frac{h}{1+2h/b}\right)^{1/2} S^{1/2}bh$$

$$\Rightarrow \qquad \frac{Q}{Cb\sqrt{S}} = \frac{h^{3/2}}{(1+2h/b)^{1/2}}$$

$$\Rightarrow \qquad h = \left(\frac{Q}{Cb\sqrt{S}}\right)^{2/3} (1+2h/b)^{1/3}$$

Here,

$$h = 1.216(1+0.4h)^{1/3}$$

Iterate (from, e.g., $h = 1.216$) to get normal depth

$$h_n = 1.412\ \mathrm{m}$$

For the critical depth,

$$q = \frac{Q}{b} \quad = 3\ \mathrm{m}^2\ \mathrm{s}^{-1}$$

Then,

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = 0.9717\ \mathrm{m}$$

Answer: normal depth = 1.41 m;   critical depth = 0.972 m

(b) The approach-flow head (relative to the bed of the channel) is

$$H_a = z_{sn} + \frac{V_n^2}{2g} \quad = h_n + \frac{q^2}{2gh_n^2} \quad = 1.642\ \mathrm{m}$$

The head assuming critical conditions at the weir is

$$H_c = z_{\text{weir}} + \frac{3}{2}h_c \quad = 2.458 \text{ m}$$

The latter is larger, so the flow does go critical and the total head is 2.458 m in the vicinity of the weir. At the downstream end:

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{q^2}{2gh^2}$$

Rearranging for the supercritical solution:

$$h = \frac{q}{\sqrt{2g(H-h)}}$$

Here (with lengths in m):

$$h = \frac{3}{\sqrt{19.62(2.458-h)}}$$

Iterate (from 0) to get

$$h = 0.4818 \text{ m}$$

Answer: 0.482 m

(c) The hydraulic jump occurs to normal depth downstream (subscript 2); i.e $h_2 = h_n = 1.412$ m. Then,

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gh_2}} \quad = \frac{q}{\sqrt{gh_2^3}} \quad = 0.5709$$

Hence, on the upstream side (subscript 1):

$$h_1 = \frac{h_2}{2}\left(-1 + \sqrt{1 + 8Fr_2^2}\right) \quad = 0.6349 \text{ m}$$

Answer: 0.635 m

(d) There is gradually-varied flow from $h = 0.4818$ m at the downstream end of the weir to $h = 0.6349$ m at the hydraulic jump. With 2 steps this gives

$$\Delta h = \frac{0.6349 - 0.4818}{2} = 0.07655 \text{ m}$$

With lengths in m throughout:

$$\text{Fr}^2 = \frac{V^2}{gh} \quad = \frac{q^2}{gh^3} \quad = \frac{0.9174}{h^3}$$

$$S_f = \left(\frac{Q}{Cb}\right)^2 \frac{(1+2h/b)}{h^3} \quad = 9 \times 10^{-4} \frac{(1+0.4h)}{h^3}$$

Hence,

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1-\mathrm{Fr}^2}{S_0 - S_f} \quad = \frac{1 - \frac{0.9174}{h^3}}{5 \times 10^{-4} - 9 \times 10^{-4} \frac{1+0.4h}{h^3}}$$

and

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

Working is set out in the following table.

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.4818	0			
			0.5201	763.8	58.47
1	0.55835	58.47			
			0.5966	699.0	53.51
2	0.6349	**112.0**			

Answer: 112 m

Q34.
$b = 7$ m
$S = 0.005$
$n = 0.035\ \mathrm{m}^{-1/3}$ s

(a) For normal flow:

$$Q = VA, \qquad \text{where} \qquad V = \frac{1}{n}R_h^{2/3}S^{1/2}, \quad A = bh, \qquad R_h = \frac{bh}{b+2h}$$

Hence,

$$Q = \frac{1}{n}\left(\frac{bh}{b+2h}\right)^{2/3} S^{1/2}bh \; = \frac{1}{0.035}\left(\frac{7\times 1.6}{7+2\times 1.6}\right)^{2/3} \times \sqrt{0.005}\times 7\times 1.6 = 24.08\ \mathrm{m^3\ s^{-1}}$$

Answer: 24.1 $\mathrm{m^3\ s^{-1}}$

(b) Flow rate per unit width:

$$q = \frac{Q}{b} \quad = \frac{24.08}{7} \quad = 3.44\ \mathrm{m^2\ s^{-1}}$$

Critical depth:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{3.44^2}{9.81}\right)^{1/3} \quad = 1.065\ \mathrm{m}$$

The normal depth (1.6 m) is deeper than the critical depth (1.06 m) and so the normal flow is subcritical. The channel slope is therefore “mild” at this flow rate.

Answer: 1.06 m; slope is mild

(c) The specific energy in the approach flow (normal flow, since the channel is “long”) is

$$E_a = h_n + \frac{V_n^2}{2g} \quad = h_n + \frac{Q^2}{2gb^2h_n^2} \quad = 1.6 + \frac{24.08^2}{2\times 9.81\times 7^2\times 1.6^2} \quad = 1.836\ \mathrm{m}$$

Hence, relative to the undisturbed bed in the vicinity of the weir:

$$H_a = E_a \quad = 1.836\ \mathrm{m}$$

The critical specific energy is

$$E_c = \frac{3}{2}h_c \quad = \frac{3}{2}\times 1.065 \quad = 1.598\ \mathrm{m}$$

Under critical conditions the total head over the weir would be

$$H_c = z_{\mathrm{weir}} + E_c \quad = 1.8 + 1.598 \quad = 3.398\ \mathrm{m}$$

This is the minimum head for this flow rate over the weir and exceeds the head available in the approach flow. Hence, the flow backs up, the water level rises upstream of the weir and the flow over the top is critical. The total head in the vicinity of the weir is determined by critical conditions: $H = H_c = 3.398$ m.

Just outside the weir,

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2gb^2h^2}$$

$$\Rightarrow \qquad 3.398 = h + \frac{0.6031}{h^2}$$

Upstream, rearrange for the deep (subcritical) solution:

$$h = 3.398 - \frac{0.6031}{h^2}$$

Iteration (from, e.g., $h = 3.398$) gives

$$h = 3.344 \text{ m}$$

Downstream, rearrange for the shallow (supercritical) solution:

$$h = \sqrt{\frac{0.6031}{3.398 - h}}$$

Iteration (from, e.g., $h = 0$) gives

$$h = 0.4525 \text{ m}$$

Answer: upstream depth = 3.34 m;   downstream depth = 0.452

(d) Normal flow is subcritical (part (b)). Since flow just downstream of the weir is supercritical, a downstream hydraulic jump is necessary to revert to normal flow.

There is GVF between the end of the weir and the upstream side of the hydraulic jump. The flow on the downstream side of the jump is normal, $h_n = 1.6$ m and we must use the sequent-depth relation to find the depth on the upstream side of the jump:

$$V_n = \frac{Q}{bh_n} \quad = \frac{24.08}{7 \times 1.6} \quad = 2.150 \text{ m s}^{-1}$$

$$\text{Fr}_n = \frac{V_n}{\sqrt{gh_n}} \quad = \frac{2.15}{\sqrt{9.81 \times 1.6}} \quad = 0.5427$$

$$h_J = \frac{h_n}{2}(-1 + \sqrt{1 + 8\text{Fr}_n^2}) \quad = \frac{1.6}{2}(-1 + \sqrt{1 + 8 \times 0.5427^2}) \quad = 0.6656 \text{ m}$$

A GVF calculation is carried out from just downstream of the weir ($h = 0.4525$ m) to just upstream of the hydraulic jump ($h = 0.6656$ m). Using one step the depth increment is

$$\Delta h = 0.6656 - 0.4525 \quad = 0.2131 \text{ m}$$

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

For the direct-step method, invert the GVF equation

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right) \Delta h$$

For the working, write the derivative as a function of h; (all lengths in metres).

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{Q}{b\sqrt{gh^3}} \qquad \Rightarrow \qquad \mathrm{Fr}^2 = \frac{Q^2}{b^2 g h^3} \quad = \frac{1.206}{h^3}$$

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 (1 + 2h/b)^{4/3} \quad = 14.50 \times 10^{-3} \frac{(1 + 2h/7)^{4/3}}{h^{10/3}}$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{1.206}{h^3}}{\left[5 - 14.50 \dfrac{(1 + 2h/7)^{4/3}}{h^{10/3}}\right] \times 10^{-3}}, \qquad \Delta h = 0.2131$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.4525	0			
			0.5591	50.13	10.68
1	0.6656	10.68			

Answer: 10.7 m

Q35.
$b = 2.5$ m
$h_1 = 1.8$ m
$h_2 = 0.3$ m
$n = 0.012$ m$^{-1/3}$ s
$S_0 = 0.002$

(a) Assuming the same total head on either side of the gate:

$$z_{s1} + \frac{V_1^2}{2g} = z_{s2} + \frac{V_2^2}{2g}$$

$$\Rightarrow \qquad h_1 + \frac{Q^2}{2gb^2h_1^2} = h_2 + \frac{Q^2}{2gb^2h_2^2}$$

$$\Rightarrow \qquad h_1 - h_2 = \frac{Q^2}{2gb^2}\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)$$

Substituting values:

$$1.5 = 0.08809Q^2$$

Hence,

$$Q = 4.127 \text{ m}^3 \text{ s}^{-1}$$

Answer: 4.13 m^3 s^{-1}

(b)
Normal depth

$$Q = VA$$

where, in normal flow:

$$V = \frac{1}{n}R_h^{2/3}S^{1/2} \qquad A = bh \qquad R_h = \frac{bh}{b+2h} \quad = \frac{h}{1+2h/b}$$

Hence,

$$Q = \frac{1}{n}\frac{bh^{5/3}}{(1+2h/b)^{2/3}}S_0^{1/2}$$

or, rearranging as an iterative formula for h:

$$h = \left(\frac{nQ}{b\sqrt{S_0}}\right)^{3/5}(1+2h/b)^{2/5}$$

Substitution of numerical values yields iterative formula

$$h = 0.6135(1+0.8h)^{2/5}$$

Iteration (from, e.g., $h = 0.6135$) gives

$h_n = 0.7387$ m

Critical depth

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = \left(\frac{4.127^2}{2.5^2 \times 9.81}\right)^{1/3} = 0.6525 \text{ m}$$

Answer: normal depth = 0.739 m; critical depth = 0.652 m

(c) The depth just upstream of the jump is the sequent depth to the normal depth:

$$h_n = 0.7387 \text{ m}$$

$$V_n = \frac{Q}{bh_n} = \frac{4.127}{2.5 \times 0.7387} = 2.235 \text{ m s}^{-1}$$

$$\mathrm{Fr}_n = \frac{V_n}{\sqrt{gh_n}} = \frac{2.235}{\sqrt{9.81 \times 0.7387}} = 0.8303$$

so that the depth just upstream of the jump (call it h_J) is

$$h_J = \frac{h_n}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_n^2}\right) = \frac{0.7387}{2}\left(-1 + \sqrt{1 + 8 \times 0.8303^2}\right) = 0.5734 \text{ m}$$

We must therefore do a GVF calculation from just downstream of the sluice (where $h = 0.3$ m) to just upstream of the hydraulic jump (where $h = 0.5734$ m).

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

For the direct-step method rewrite the GVF equation "the other way up":

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)\Delta h$$

For the working, write the derivative as a function of *h*; (all lengths in metres).

$$\mathrm{Fr} = \frac{V}{\sqrt{gh}} = \frac{Q}{bh\sqrt{gh}} \qquad \Rightarrow \qquad \mathrm{Fr}^2 = \frac{(Q/b)^2}{gh^3} = \frac{0.2778}{h^3}$$

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 (1 + 2h/b)^{4/3} = 3.924 \times 10^{-4}\,\frac{(1 + 0.8h)^{4/3}}{h^{10/3}}$$

$$\Delta h = \frac{0.5734 - 0.3}{2} = 0.1367$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.2778}{h^3}}{\left[20 - 3.924 \times \dfrac{(1+0.8h)^{4/3}}{h^{10/3}}\right] \times 10^{-4}}, \qquad \Delta h = 0.1367$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.3	0			
			0.3684	338.8	46.31
1	0.4367	46.31			
			0.5051	288.1	39.38
2	0.5734	**85.69**			

Answer: 85.7 m

Q36.
(a) A hydraulic transition takes place, so the parallel-flow depth in the constriction is critical:

$$h_c = \left(\frac{q_m^2}{g}\right)^{1/3}$$

whence the flow rate (per unit width *of the constricted section*) is

$$q_m = \sqrt{g h_c^3} \quad = \sqrt{9.81 \times 2.5^3} \quad = 12.38\ \mathrm{m^2\ s^{-1}}$$

and the overall flow rate is

$$Q = q_m b_m \quad = 12.38 \times 2.0 \quad = 24.76\ \mathrm{m^3\ s^{-1}}$$

As the constricted flow is critical the head there is

$$H \quad = z_b + E_c \quad = 0 + \frac{3}{2} h_c \quad = \frac{3}{2} \times 2.5 \quad = 3.75\ \mathrm{m}$$

and, as this is RVF, there is the same total head throughout the venturi:

$$H = z_s + \frac{V^2}{2g} \quad = h + \frac{Q^2}{2 g b^2 h^2}$$

Just upstream or downstream, where $b = 4$ m, we need the sub- and supercritical solutions of

$$3.75 = h + \frac{24.76^2}{2 \times 9.81 \times 4^2 \times h^2} \quad = h + \frac{1.953}{h^2}$$

Upstream (subcritical):

$$h = 3.75 - \frac{1.953}{h^2}$$

Iteration (from, e.g., $h = 3.75$) gives $h = 3.599$ m.

Downstream (supercritical):

$$h = \sqrt{\frac{1.953}{3.75 - h}}$$

Iteration (from, e.g., $h = 0$) gives $h = 0.8158$ m.

Answer: flow rate = 24.8 $\mathrm{m^3\ s^{-1}}$; depth upstream = 3.60 m; depth downstream = 0.816 m

(b) With a long undisturbed fetch, the depth downstream of the jump must be normal flow.

$$Q = VA, \qquad \text{where} \qquad V = \frac{1}{n} R_h^{2/3} S^{1/2}, \quad R_h = \frac{bh}{b + 2h} \quad = \frac{h}{1 + 2h/b}$$

Hence,

$$Q = \frac{1}{n} \left(\frac{h}{1 + 2h/b}\right)^{2/3} S^{1/2} b h$$

$$\Rightarrow \qquad \frac{nQ}{b\sqrt{S}} = \frac{h^{5/3}}{(1+2h/b)^{2/3}}$$

$$\Rightarrow \qquad h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} (1+2h/b)^{2/5}$$

Here, with lengths in metres,

$$h = 1.584(1+0.5h)^{2/5}$$

Iterate (from, e.g., $h = 1.584$) to get

$$h = 2.114 \text{ m}$$

Take, therefore, downstream depth $h_B = 2.114$ m, whence

$$V_B = \frac{Q}{bh_B} \quad = \frac{24.76}{4 \times 2.114} \quad = 2.928 \text{ m s}^{-1}$$

$$\text{Fr}_b = \frac{V_B}{\sqrt{gh_B}} \quad = \frac{2.928}{\sqrt{9.81 \times 2.114}} \quad = 0.6430$$

and, by formula, the upstream depth is

$$h_A = \frac{2.114}{2}\left(-1 + \sqrt{1 + 8 \times 0.6430^2}\right) \quad = 1.1368 \text{ m}$$

Answer: depths (upstream, downstream) = (1.14, 2.11) m

(c) The flow between venturi and jump is supercritical, so integrate the GVF equation downstream from $h = 0.8158$ m to $h = 1.1368$ m using 1 step.

GVF equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \text{Fr}^2}$$

For the direct-step method invert the GVF equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \text{Fr}^2}{S_0 - S_f} \qquad \text{and} \qquad \Delta x \approx \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)\Delta h$$

For the working, write the derivative as a function of h; (all lengths in metres).

$$\text{Fr} = \frac{V}{\sqrt{gh}} \quad = \frac{Q}{b\sqrt{gh^3}} \qquad \Rightarrow \qquad \text{Fr}^2 = \frac{Q^2}{b^2gh^3} \quad = \frac{3.906}{h^3}$$

$$S_f = \left(\frac{nQ}{bh^{5/3}}\right)^2 (1+2h/b)^{4/3} \quad = 18.54 \times 10^{-3}\,\frac{(1+0.5h)^{4/3}}{h^{10/3}}$$

$$\Delta h = 1.1368 - 0.8158 \quad = 0.3210 \text{ m}$$

Working formulae:

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{3.906}{h^3}}{\left[4 - 18.54 \times \dfrac{(1+0.5h)^{4/3}}{h^{10/3}}\right] \times 10^{-3}} \qquad \Delta h = 0.3210$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	0.8158	0			
			0.9763	106.2	34.09
1	1.1368	34.09			

Answer: 34 m

Q37.

(a) A hydraulic transition occurs over the weir. Hence, the depth over the crest must be critical:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \quad = \left(\frac{0.8^2}{9.81}\right)^{1/3} \quad = 0.4026 \text{ m}$$

The total head is

$$H = z_{\text{weir}} + E_c \quad = z_{\text{weir}} + \frac{3}{2}h_c \quad = 1.8 + \frac{3}{2} \times 0.4026 \quad = 2.404 \text{ m}$$

Depths just upstream and downstream of the weir have the same total head:

$$H = h + \frac{V^2}{2g} \quad = h + \frac{q^2}{2gh^2}$$

For h in metres:

$$2.404 = h + \frac{0.03262}{h^2}$$

Rearranging for the deep (subcritical) solution:

$$h = 2.404 - \frac{0.03262}{h^2}$$

Iteration (from, e.g., $h = 2.404$) gives

$$h_{\text{upstream}} = 2.398 \text{ m}$$

Answer: depth over weir = 0.403 m;   depth just upstream = 2.40 m

(b) Inverting the gradually-varied-flow equation:

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \mathrm{Fr}^2}{S_0 - S_f}$$

or, in discrete form:

$$\Delta x = \left(\frac{1 - \mathrm{Fr}^2}{S_0 - S_f}\right)_{\text{mid}} \Delta h$$

Find expressions for Fr and S_f as functions of h (in m):

$$\mathrm{Fr}^2 = \frac{V^2}{gh} \quad = \frac{q^2}{gh^3} \quad = \frac{0.06524}{h^3}$$

The expression for S_f comes from assuming quasi-normal flow. In normal flow,

$$q = Vh \quad = \frac{1}{n}R_h^{2/3}S^{1/2}h, \qquad \text{where} \qquad R_h = h \qquad \text{(wide channel)}$$

$$\Rightarrow \qquad q = \frac{1}{n}h^{5/3}S^{1/2}$$

Rearranging for S and taking $S_f = S$ (the quasi-normal-flow assumption):

$$S_f = \left(\frac{nq}{h^{5/3}}\right)^2 \quad = \frac{4 \times 10^{-4}}{h^{10/3}}$$

Integration is carried out (in the *upstream* direction, because the flow here is subcritical) from $h = 2.398$ m to $h = 1.800$ m with 2 steps; i.e. $\Delta h = -0.299$ m.

Working formulae (all lengths in m):

$$\Delta x = \left(\frac{\mathrm{d}x}{\mathrm{d}h}\right)_{\mathrm{mid}} \Delta h$$

where

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{1 - \dfrac{0.06524}{h^3}}{10^{-4}\left(10 - \dfrac{4}{h^{10/3}}\right)} \qquad \Delta h = -0.299$$

i	h_i	x_i	h_{mid}	$(\mathrm{d}x/\mathrm{d}h)_{\mathrm{mid}}$	Δx
0	2.398	0			
			2.249	1022	–305.6
1	2.099	–305.6			
			1.950	1036	–309.8
2	1.800	**–615.4**			

Answer: 615 m

(c) If the weir fails catastrophically then the depth is just the normal depth. From the relationship between flow rate and depth already derived in part (b), but using the geometric slope S_0,

$$q = \frac{1}{n} h^{5/3} S_0^{1/2}$$

$$\Rightarrow \qquad h = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} = \left(\frac{0.025 \times 0.8}{\sqrt{0.001}}\right)^{3/5} = 0.7597 \text{ m}$$

Answer: 0.760 m

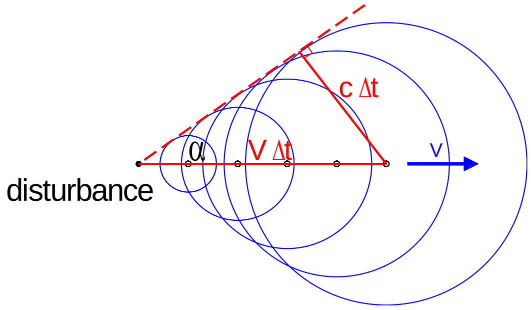
Q38.
(a)

$$\text{Fr} \equiv \frac{V}{\sqrt{gh}}$$

where V is velocity, g is gravitational acceleration and h is depth.

The Froude number is the ratio of current speed V to long-wave speed $c = \sqrt{gh}$.

(b) The wedge is the envelope of all waves that have spread out from points advected through the disturbance. In time Δt the centre of a wave travels downstream a distance $V\Delta t$ and its front has spread out a distance $c\Delta t$. From the diagram,



$$\sin\alpha = \frac{c\Delta t}{V\Delta t} \quad = \frac{c}{V} \quad = \frac{\sqrt{gh}}{V} \quad = \frac{1}{\text{Fr}}$$

(c)

$$\text{Fr} = \frac{1}{\sin 20°} \quad = 2.924$$

But

$$\text{Fr} \equiv \frac{V}{\sqrt{gh}} \qquad \text{and} \qquad V = \frac{q}{h}$$

$$\Rightarrow \qquad \text{Fr} = \frac{q}{\sqrt{gh^3}}$$

Hence,

$$h = \left(\frac{q^2}{g} \times \frac{1}{Fr^2}\right)^{1/3} \quad = \left(\frac{2.5^2}{9.81} \times \frac{1}{2.924^2}\right)^{1/3} \quad = 0.4208 \text{ m}$$

$$V = \frac{q}{h} \quad = \frac{2.5}{0.4208} \quad = 5.941 \text{ m s}^{-1}$$

Answer: $h = 0.421$ m; $V = 5.94$ m s^{-1}

(d) A hydraulic jump will be provoked if the approach flow does not have sufficient energy to provide the minimum head to pass over the bed rise ($H_{crit} = z_{bed} + E_{crit}$).

Approach flow:

$$H_a = E_a = h + \frac{V^2}{2g} \quad = 0.4208 + \frac{5.941^2}{2 \times 9.81} \quad = 2.220 \text{ m}$$

Critical conditions:

$$E_c = \frac{3}{2}h_c \quad = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} \quad = \frac{3}{2}\left(\frac{2.5^2}{9.81}\right)^{1/3} \quad = 1.291 \text{ m}$$

$$H_c = z_{\text{bed}} + E_c \quad = z_{\text{bed}} + 1.291$$

If a hydraulic jump just occurs:

$$H_c = H_a$$

$$\Rightarrow \qquad z_{\text{bed}} + 1.291 = 2.220$$

$$\Rightarrow \qquad z_{\text{bed}} = 0.929 \text{ m}$$

Answer: 0.929 m

(e) Otherwise,

$$z_{\text{bed}} + E = H_a$$

where $H_a = 2.220$, $z_{\text{bed}} = 0.4645$ m and

$$E = h + \frac{V^2}{2g} \quad = h + \frac{q^2}{2gh^2} \quad = h + \frac{0.3186}{h^2}$$

Hence,

$$0.4645 + h + \frac{0.3186}{h^2} = 2.220$$

$$\Rightarrow \qquad h + \frac{0.3186}{h^2} = 1.756$$

Since no hydraulic transition takes place and the approach flow is supercritical, the required solution is the shallow, supercritical one, so rearranging for this:

$$h = \sqrt{\frac{0.3186}{1.756 - h}}$$

Iterating (from, e.g. $h = 0$) gives

$$h = 0.5046 \text{ m}$$

Answer: 0.505 m