Open-Channel-Flow Formulae

Friction formulae:

Chézy: $V = C\sqrt{R_h S}$

Manning: $V = \frac{1}{n} R_h^{2/3} S^{1/2}$

Sequent depths for a hydraulic jump:

$$h_B = \frac{h_A}{2}\left(-1 + \sqrt{1 + 8\,\mathrm{Fr}_A^2}\right)$$

Gradually-varied flow equation:

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_0 - S_f}{1 - \mathrm{Fr}^2}$$

Wave Formulae

Density of fresh water: 1000 kg m^{-3}; sea water, 1025 kg m^{-3}
Gravitational acceleration: g = 9.81 m s^{-2}

Linear Wave Theory

For a surface elevation: $\eta = A\cos(kx - \omega t)$

Dispersion relationship: $(\omega_a - kU_0)^2 = \omega_r^2 = gk\tanh kh$

Velocity potential: $\phi = \frac{Ag}{\omega}\frac{\cosh k(h+z)}{\cosh kh}\sin(kx - \omega t)$

Velocity: $u = \frac{\partial\phi}{\partial x} = \frac{Agk}{\omega}\frac{\cosh k(h+z)}{\cosh kh}\cos(kx - \omega t)$

$w = \frac{\partial\phi}{\partial z} = \frac{Agk}{\omega}\frac{\sinh k(h+z)}{\cosh kh}\sin(kx - \omega t)$

Pressure: $p = -\rho gz - \rho\frac{\partial\phi}{\partial t} = -\rho gz + \rho g\eta\frac{\cosh k(h+z)}{\cosh kh}$

Phase velocity: $c = \frac{\omega}{k}$

Group velocity: $c_g \equiv \frac{\mathrm{d}\omega}{\mathrm{d}k} = nc$, $n = \frac{1}{2}\left[1 + \frac{2kh}{\sinh 2kh}\right]$

Energy density: $$E = \frac{1}{2}\rho g A^2 \quad = \frac{1}{8}\rho g H^2$$

Wave power: $$P = E c_g \quad = \frac{1}{8}\rho g H^2 (nc)$$

Snell's Law: $$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2} \qquad \text{or} \qquad k_1 \sin\theta_1 = k_2 \sin\theta_2$$

Shoaling: $$(H^2 nc\cos\theta)_1 = (H^2 nc\cos\theta)_2$$

(In these formulae, θ is the angle between wave crests and the depth contours, or between the wave rays and the normals to the depth contours.)

Wave Breaking

Miche breaking criterion: $$\left(\frac{H}{L}\right)_b = 0.14\tanh(kh)_b$$

Breaker height index: $$\Omega_b \equiv \frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-1/5}$$

Breaker depth index: $$\gamma_b \equiv \frac{H_b}{h_b} = b - a\frac{H_b}{gT^2}, \quad a = 43.8(1 - \mathrm{e}^{-19m}), \quad b = \frac{1.56}{1 + \mathrm{e}^{-19.5m}}$$

Surf-similarity parameter: $$\xi_0 = \frac{m}{\sqrt{H_0/L_0}} \qquad \text{or} \qquad \xi_b = \frac{m}{\sqrt{H_b/L_0}}$$

Random Waves

Rayleigh distribution: $$P(\text{height} > H) = \exp\left[-\left(\frac{H}{H_{\text{rms}}}\right)^2\right]$$

Bretschneider spectrum: $$S(f) = \frac{5}{16}H_s^2\frac{f_p^4}{f^5}\exp\left(-\frac{5}{4}\frac{f_p^4}{f^4}\right)$$

JONSWAP equations: $$\frac{gH_s}{U^2} = 0.0016\left(\frac{gF}{U^2}\right)^{1/2} \quad < 0.2433$$

$$\frac{gT_p}{U} = 0.2857\left(\frac{gF}{U^2}\right)^{1/3} \quad < 8.134$$

$$\frac{gt_{\text{min}}}{U} = 68.8\left(\frac{gF}{U^2}\right)^{2/3} \quad < 7.15 \times 10^4$$