Objectives

(1) Understand the role of pumps and turbines as **energy-conversion devices** and use, appropriately, the terms **head**, **power** and **efficiency**.
(2) Be aware of the main types of pumps and turbines and the distinction between **impulse** and **reaction** turbines and between **radial**, **axial** and **mixed-flow** devices.
(3) Match **pump characteristics** and **system characteristics** to determine the **duty point**.
(4) Calculate characteristics for **pumps in series and parallel** and use the **hydraulic scaling laws** to calculate pump characteristics at **different speeds**.
(5) Select the type of pump or turbine on the basis of **specific speed**.
(6) Understand the **mechanics** of a centrifugal pump and an impulse turbine.
(7) Recognise the problem of **cavitation** and how it can be avoided.

1. Energy conversion
 1.1 Energy transfer in pumps and turbines
 1.2 Power
 1.3 Efficiency

2. Types of pumps and turbines
 2.1 Impulse and reaction turbines
 2.2 Positive-displacement and dynamic pumps
 2.3 Radial, axial and mixed-flow devices
 2.4 Common types of turbine

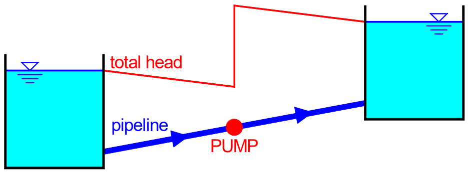
3. Pump and system characteristics
 3.1 Pump characteristics
 3.2 System characteristics
 3.3 Finding the duty point
 3.4 Pumps in parallel and in series

4. Hydraulic scaling
 4.1 Dimensional analysis
 4.2 Change of speed
 4.3 Specific speed

5. Mechanics of rotodynamic devices
 5.1 Centrifugal pump
 5.2 Pelton wheel

6. Cavitation

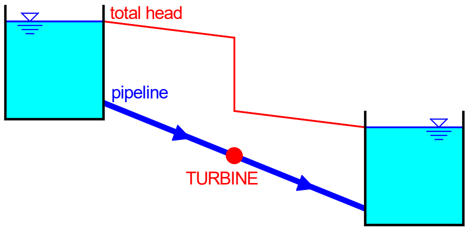
1. Energy Conversion



1.1 Energy Transfer in Pumps and Turbines

Pumps and turbines are *energy conversion* devices:

pumps turn electrical or mechanical energy into fluid energy;
turbines turn fluid energy into electrical or mechanical energy.



The energy per unit weight is the total *head*, H:

$$H = \frac{p}{\rho g} + z + \frac{V^2}{2g}$$

The first two terms on the RHS comprise the *piezometric head*. The last term is the *dynamic head*.

1.2 Power

Power = rate of conversion of energy.

If a mass m is raised through a height H it gains energy mgH. If it does so in time t then the rate of conversion is mgH/t.

For a fluid in motion the mass flow rate (m/t) is ρQ. The rate of conversion to or from fluid energy when the total head is changed by H is, therefore, $\rho Q \times gH$, or

$$\text{power} = \rho g Q H$$

1.3 Efficiency

Efficiency, η, is given by

$$\eta = \frac{(\text{power})_{\text{out}}}{(\text{power})_{\text{in}}}$$

where *power* refers to the rate of doing "useful" work (i.e. what the device was intended to do.)

For **turbines**: $$\eta = \frac{(\text{power})_{out}}{\rho g Q H}$$

For **pumps**: $$\eta = \frac{\rho g Q H}{(\text{power})_{in}}$$

Example.
A pump lifts water from a large tank at a rate of 30 L s^{-1}. If the input power is 10 kW and the pump is operating at an efficiency of 40%, find:
(a) the head developed across the pump;
(b) the maximum height to which it can raise water if the delivery pipe is vertical, with diameter 100 mm and friction factor $\lambda = 0.015$.

(a) Given:

$Q = 0.03 \text{ m}^3 \text{ s}^{-1}$
$I = 10000 \text{ W}$
$\eta = 0.4$

Then,

$$\eta = \frac{\text{output power}}{\text{input power}} \quad = \frac{\rho g Q H}{I}$$

$$\Rightarrow \qquad 0.4 = \frac{1000 \times 9.81 \times 0.03 \times H}{10000}$$

$$\Rightarrow \qquad H = 13.59 \text{ m}$$

Answer: 13.6 m

(b) Given:

$D = 0.1 \text{ m}$
$\lambda = 0.015$

The pump must lift water by height h_s and overcome pipe friction over a length L:

$$H = h_s + \frac{\lambda L}{D} \frac{V^2}{2g}$$

In this instance, $L = h_s$, as the pipe is vertical:

$$H = h_s \left(1 + \frac{\lambda}{D} \frac{V^2}{2g}\right)$$

The velocity V is given by

$$V = \frac{\text{flow rate}}{\text{area}} \quad = \frac{Q}{\pi D^2/4} \quad = 4 \times \frac{0.03}{\pi \times 0.1^2} \quad = 3.820 \text{ m s}^{-1}$$

Hence,

$$13.59 = h_s \left(1 + \frac{0.015}{0.1} \times \frac{3.820^2}{2 \times 9.81}\right)$$

$$\Rightarrow \qquad h_s = 12.23 \text{ m}$$

Answer: 12.2 m

2. Types of Pumps and Turbines

2.1 Impulse and reaction turbines

In a pump or turbine a change in fluid head

$$H = \frac{p}{\rho g} + z + \frac{V^2}{2g}$$

may be brought about by a change in pressure or velocity or both.

- An *impulse turbine* (e.g. Pelton wheel; water wheel) is one where the change in head is brought about primarily by a change in **velocity**. This usually involves unconfined free jets of water (at atmospheric pressure) impinging on moving vanes.
- A *reaction turbine* (e.g. Francis turbine; Kaplan turbine; windmill) is one where the change in head is brought about primarily by a change in **pressure**.

2.2 Positive-Displacement and Dynamic pumps

Positive-displacement pumps operate by a change in volume; energy conversion is intermittent. Examples in the human body include the heart (*diaphragm pump*) and the intestines (*peristaltic pump*). In a *reciprocating pump* (e.g. a bicycle pump) fluid is sucked in on one part of the cycle and expelled (at higher pressure) in another.

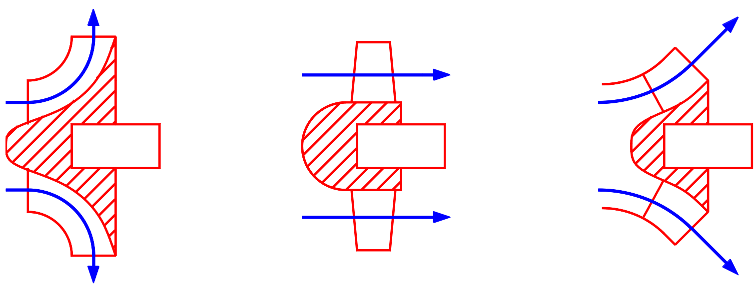
In *dynamic pumps* there is no change in volume and energy conversion is continuous. Most pumps are *rotodynamic* devices where fluid energy is exchanged with the mechanical energy of a rotating element (called a *runner* in turbines and an *impeller* in pumps), with a further conversion to or from electrical energy.

This course focuses entirely on rotary devices.

Note that, for gases, pumps are usually referred to as *fans* (for low pressures), *blowers* or *compressors* (for high pressures).

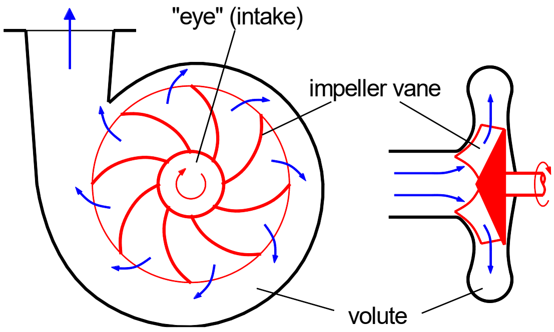
2.3 Radial, Axial and Mixed-Flow Devices

The terms *radial* and *axial* refer to the change in direction of flow through a rotodynamic device (pump or turbine):

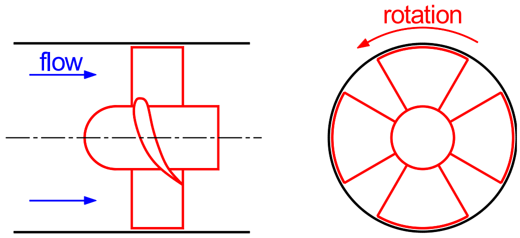


Radial Axial Mixed

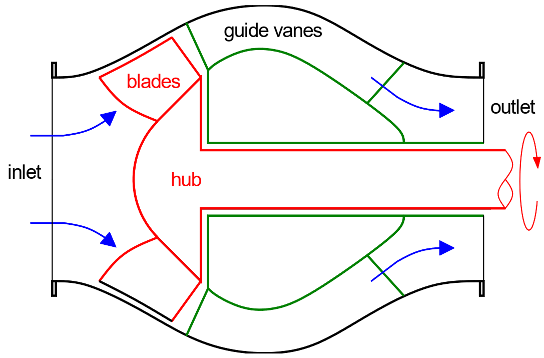
In a *centrifugal pump* flow enters along the axis and is expelled radially. (The reverse is true for a turbine.)



An *axial-flow* pump is like a propeller; the direction of the flow is unchanged after passing through the device.



A *mixed-flow device* is a hybrid device, used for intermediate heads.

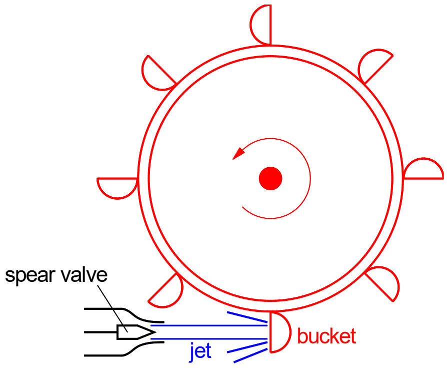


In many cases – notably in pumped-storage power stations – a device can be run as either a pump or a turbine.

Inward-flow reaction turbine ↔ centrifugal pump (high head / low discharge)
(e.g. *Francis turbine*)

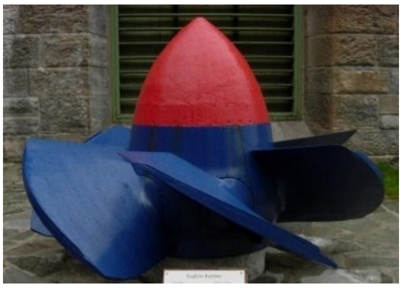
Propeller turbine ↔ axial-flow pump (low head / high discharge)
(e.g. *Kaplan turbine*; *windmill*)

2.4 Common Types of Turbine



Pelton wheels are impulse turbines used in hydroelectric plant where there is a very high head of water. Typically, 1 to 6 high-velocity jets of water impinge on buckets mounted around the circumference of a runner. A modern variant, which can handle a greater water flow and is popular in microhydropower is called a *Turgo turbine*.

Francis turbines are used in many large hydropower projects (e.g. the Hoover Dam), with an efficiency in excess of 90%. Such moderate- to high-head turbines are also used in *pumped-storage* power stations (e.g. Dinorwig and Ffestiniog in Wales; Foyers in Scotland), which pump water uphill during periods of low energy demand and then run the system in reverse to generate power during the day. This smooths the power demands on fossil-fuelled and nuclear power stations which are not easily brought in and out of operation. Francis turbines are like centrifugal pumps in reverse.



Kaplan turbines are axial-flow (propeller) turbines. In the Kaplan design the blade angles are adjustable to ensure efficient operation for a range of discharges.

Wells turbines were specifically developed for wave-energy applications. They have the property that they rotate in the same direction irrespective of the flow direction.

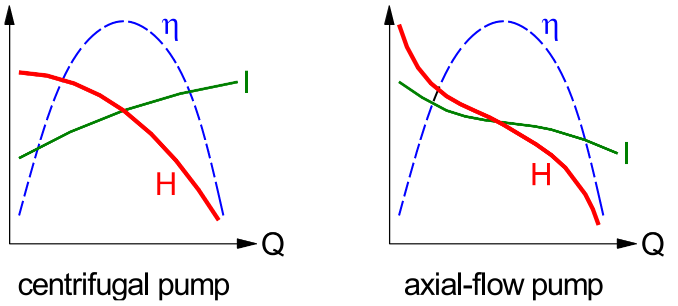
Bulb generators are large-diameter variants of the Kaplan propeller turbine, which are suitable for the low-head, high-discharge applications in tidal barrages (e.g. La Rance in France). Flow passes around the bulb, which contains the electrical generator.

The *Archimedes screw* has been used since ancient times to raise water. It is widely used in water treatment plants because it can accommodate submerged debris. Recently, several devices have been installed beside weirs in the north of England to run in reverse as a turbine and generate power. This one is a mile away from my home!

3. Pump and System Characteristics

3.1 Pump characteristics

Pump characteristics are the head, H, input power, I, and efficiency, η, as functions of discharge, Q. The most important is the H vs Q relationship. Typical shapes of these characteristics are sketched below for centrifugal and axial-flow pumps.



Many pumps are variable-speed devices. Given pump characteristics at one rotation rate, N, those at different rotation rates may be determined using hydraulic scaling laws (Section 4).

Ideally, one would like to operate the pump:
- as close as possible to the *design point* (point of maximum efficiency);
- in a region where the H vs Q relationship is steep; (otherwise there are significant fluctuations in discharge for small changes in head).

3.2 System characteristics

In general the pump has to supply enough energy to:
- lift water through a certain height – the *static lift*, h_s;
- overcome losses dependent on the discharge, Q.

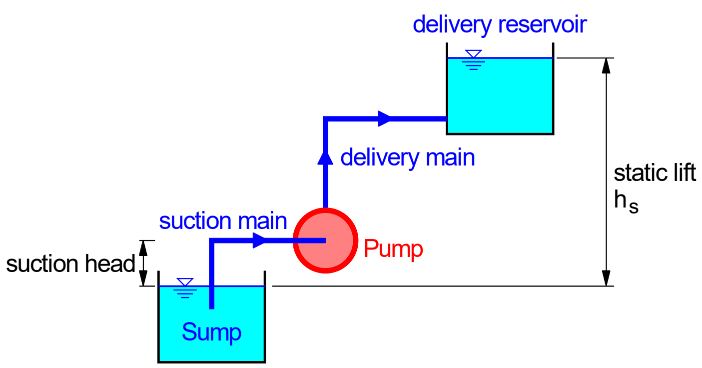
Thus the *system head* or *system characteristic* is

$$H = h_s + h_{\text{losses}}$$

Typically, losses (whether frictional or due to pipe fittings) are proportional to Q^2, so that the system characteristic is often quadratic:

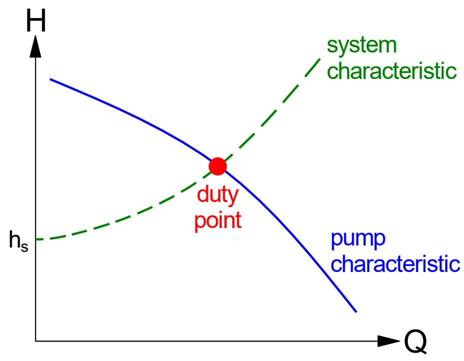
$$H = h_s + \alpha Q^2$$

The static lift is often decomposed into the rise from sump to the level of the pump (the *suction head*, h_{s1}) and that between the pump and the delivery point (h_{s2}). The first of these is limited by the maximum suction height (approximately 10 m, corresponding to 1 atmosphere) and will be discussed later in the context of cavitation.



3.3 Finding the Duty Point

The pump operates at a *duty point* where the head supplied by the pump precisely matches the head requirements of the system at the same discharge; i.e. where the **pump and system characteristics intersect**.



Example.
A water pump was tested at a rotation rate of 1500 rpm. The following data was obtained. (Q is quantity of flow; H is head of water; η is efficiency).

Q (L s^{-1})	0	10	20	30	40	50
H (m)	10.0	10.5	10.0	8.5	6.0	2.5
η	0.0	0.40	0.64	0.72	0.64	0.40

It is proposed to use this pump to draw water from an open sump to an elevation 5.5 m above. The delivery pipe is 20.0 m long and 100 mm diameter and has a friction factor of 0.005.

If operating at 1500 rpm, find:
(a) the maximum discharge that the pump can provide;
(b) the pump efficiency at this discharge;
(c) the input power required.

(a) Given:

$h_s = 5.5$ m
$L = 20$ m
$D = 0.1$ m
$\lambda = 0.005$

The pump characteristic (head available for a given flow) is given in the table; the system characteristic (head required for a given flow) must be found.

Since the pump has to raise water through height h_s and overcome pipe friction over length L:

$$H_{\text{sys}} = h_s + \frac{\lambda L}{D} \frac{V^2}{2g}, \qquad \text{where} \quad V = \frac{Q}{\pi D^2/4}$$

$$\Rightarrow \qquad H_{sys} = h_s + \frac{8\lambda L}{\pi^2 g D^5} Q^2$$

$$\Rightarrow \qquad H_{sys} = 5.5 + 826.3Q^2 \qquad (H \text{ in m}, Q \text{ in m}^3\text{ s}^{-1})$$

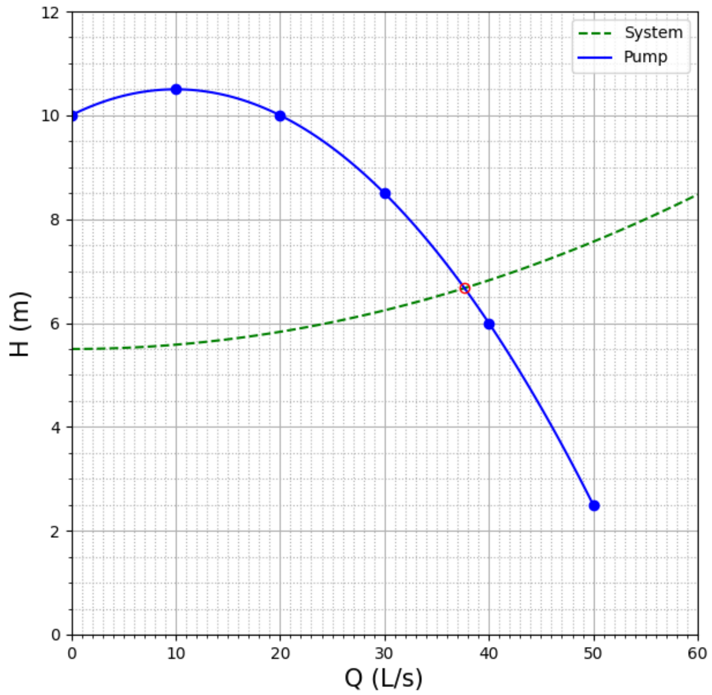
or, more conveniently here, if Q is in L s^{-1},

$$H_{sys} = 5.5 + 826.3(Q/1000)^2$$

$$\Rightarrow \qquad H_{sys} = 5.5 + 0.0008263Q^2 \qquad (H \text{ in m}, Q \text{ in L s}^{-1})$$

To find where the head available matches the head required, i.e. where the pump head matches the system head, it is convenient to add the system head to the table and then plot a graph of H vs Q for pump and system characteristics.

Q (L s^{-1})	0	10	20	30	40	50
H_{pump} (m)	10.0	10.5	10.0	8.5	6.0	2.5
η	0.0	0.40	0.64	0.72	0.64	0.40
H_{sys} (m)	5.5	5.58	5.83	6.24	6.82	7.57

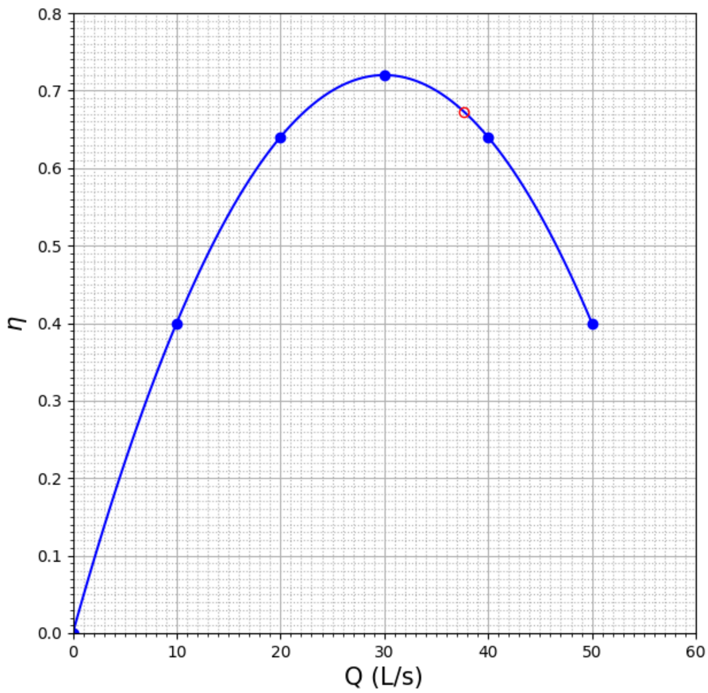


The graphs intersect at

$$Q = 37.7 \text{ L s}^{-1}$$

Answer: 37.7 L s^{-1}

(b) The efficiency can be read off a graph at this value of Q.



At $Q = 37.7 \text{ L s}^{-1}$ the graph gives $\eta = 0.673$.

Answer: 0.673

(c) Input and output power are related via the efficiency:

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\rho g Q H}{I}$$

To find H *either* read it directly from the intersection point on the H vs Q graph *or* substitute the already-found value of Q into the expression for the system characteristic. Both give

$$H = 6.67 \text{ m}$$

Then, (remembering that Q must be in consistent units, $37.7 \text{ L s}^{-1} = 0.0377 \text{ m}^3 \text{ s}^{-1}$),

$$I = \frac{\rho g Q H}{\eta} = \frac{1000 \times 9.81 \times 0.0377 \times 6.67}{0.673} = 3670 \text{ W}$$

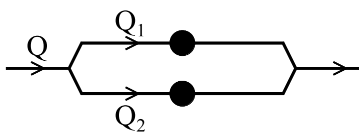
Answer: 3.67 kW

In practice, it is desirable to run the pump at a speed where the duty point is close to that of maximum efficiency. To do this we need to determine how the pump characteristic varies with rotation rate, *N* – see later.

3.4 Pumps in Parallel and in Series

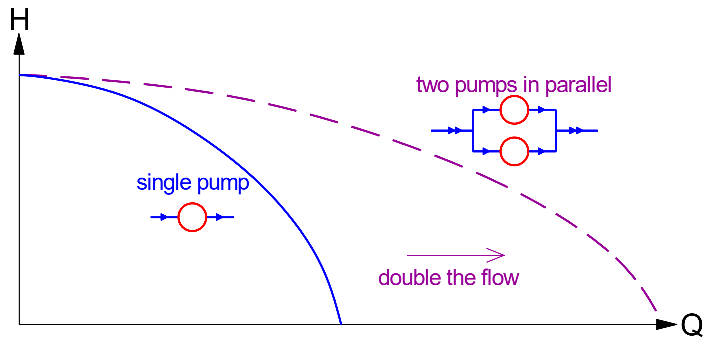
<u>Pumps in Parallel</u>

For the *combined pump complex*:
– same head: H
– total discharge: $Q = Q_1 + Q_2$



Advantages of pumps in parallel are:

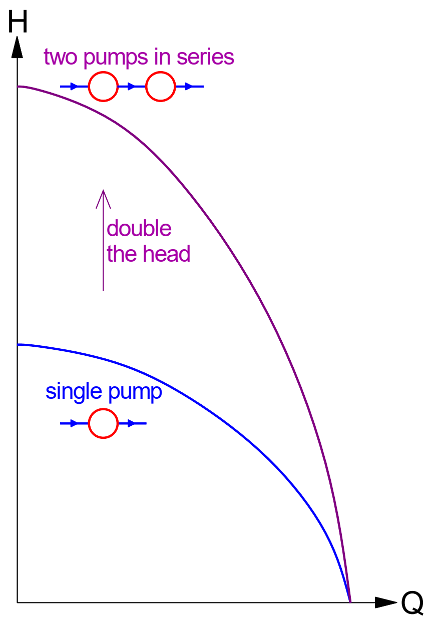
- high capacity: permits a large total discharge;
- flexibility: pumps can be brought in and out of service if the required discharge varies;
- redundancy: pumping can continue if one is not operating due to failure or planned maintenance.



<u>Pumps in Series</u>

For the *combined pump complex*:
– same discharge: Q
– total head: $H = H_1 + H_2$

Pumps in series may be used to generate high overall head, or to provide regular "boosts" along long pipelines without large pressures at any particular point.



Example.
A rotodynamic pump, having the characteristics tabulated below, delivers water from a river at elevation 102 m to a reservoir with a water level of 135 m, through a pipe of length 1 km and diameter 350 mm. The friction factor of the pipe may be taken as $\lambda = 0.035$ and minor losses from valves and fittings can be described by a loss coefficient $K = 9$.

Q (m^3 s^{-1})	0	0.05	0.10	0.15	0.20
H (m)	60	58	52	41	25
η (%)	0	44	65	64	48

(a) Calculate the discharge and head in the pipeline (at the duty point).

If the discharge is to be increased by the installation of a second identical pump:
(b) determine the unregulated discharge and head produced by connecting the pump:
  (i) in parallel;
  (ii) in series;

(c) determine the power demand at the duty point in the case of parallel operation;

(d) in the case of parallel operation, if the total flow is throttled by a valve to 0.12 m^3 s^{-1}, calculate the head lost across the valve.

(a) Given:

$h_s = 135 - 102 \quad = 33$ m
$L = 1000$ m
$D = 0.35$ m
$\lambda = 0.035$
$K = 9$

The discharge Q and head H are determined from where the pump head intersects the system head. For the system head,

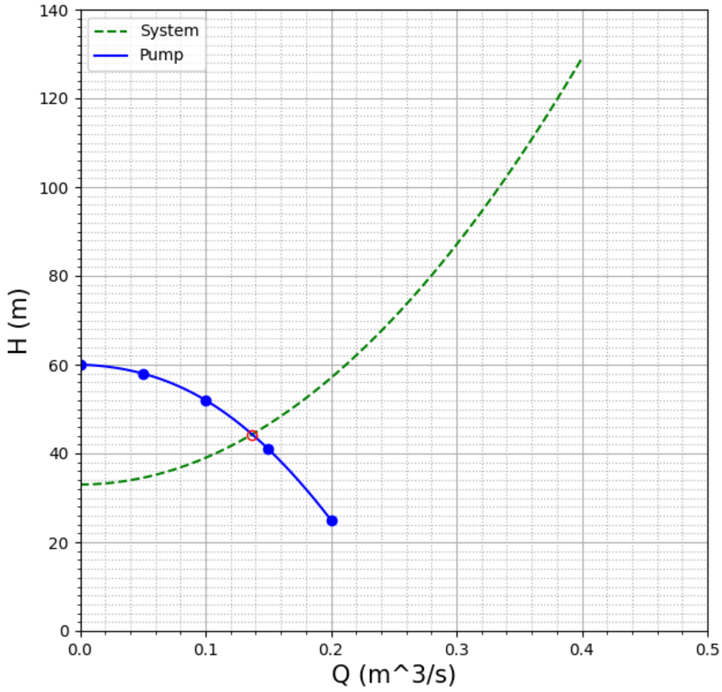
$$H_{\text{sys}} = h_s + \left(\frac{\lambda L}{D} + K\right)\frac{V^2}{2g}, \qquad \text{where} \quad V = \frac{Q}{\pi D^2/4}$$

$$\Rightarrow \qquad H_{\text{sys}} = h_s + \left(\frac{\lambda L}{D} + K\right)\frac{8}{\pi^2 g D^4} Q^2$$

$$\Rightarrow \qquad H_{\text{sys}} = 33 + 600.2 Q^2 \qquad (H \text{ in m}, Q \text{ in m}^3\text{ s}^{-1})$$

We need to find where this intersects the tabulated pump characteristic. We do so graphically, first, for convenience, adding system curve values to the given table.

Q (m^3 s^{-1})	0	0.05	0.10	0.15	0.20
H_{pump} (m)	60	58	52	41	25
H_{sys} (m)	33	34.5	39.0	46.5	57.0



From the intersection point,

$$Q = 0.137 \text{ m}^3 \text{ s}^{-1}$$

From either the intersection point or by simply substituting this value of Q into the system characteristic,

$$H = 44.3 \text{ m}$$

Answer: $Q = 0.137 \text{ m}^3 \text{ s}^{-1}; \quad H = 44.3 \text{ m}$

(b) For pumps in parallel, the combined assembly drives the same flow through each of two pipes and hence, for the same head, would pass twice the flow.

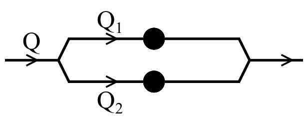
For pumps in series, one particular flow gets a boost in head twice and hence, for the same flow, would create twice the overall head.

These new pump-assembly characteristics are summarised in the tables below.

Single pump

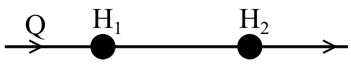
Q (m^3 s^{-1})	0	0.05	0.10	0.15	0.20
H (m)	60	58	52	41	25

Parallel pumps ($Q \to 2Q$)



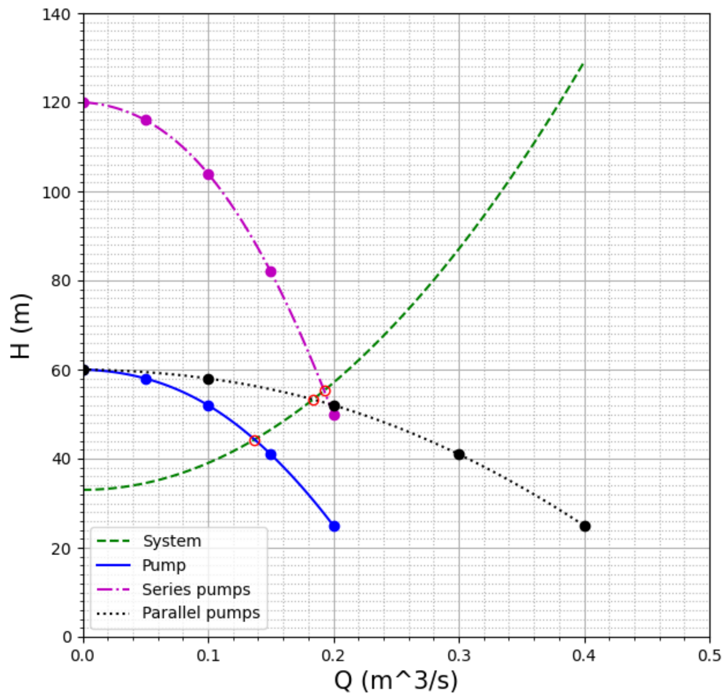
Q (m^3 s^{-1})	0	0.10	0.20	0.30	0.40
H (m)	60	58	52	41	25

Series pumps ($H \to 2H$)



Q ($\mathrm{m^3\ s^{-1}}$)	0	0.05	0.10	0.15	0.20
H (m)	120	116	104	82	50

The system characteristic, defining the head required from the pump assembly, is unchanged (assuming that the more complex pipework doesn't introduce significant extra minor losses). So, series and parallel pump characteristics can be plotted on a graph and the intersections with the system curve give new duty points.



(i) Parallel pumps:

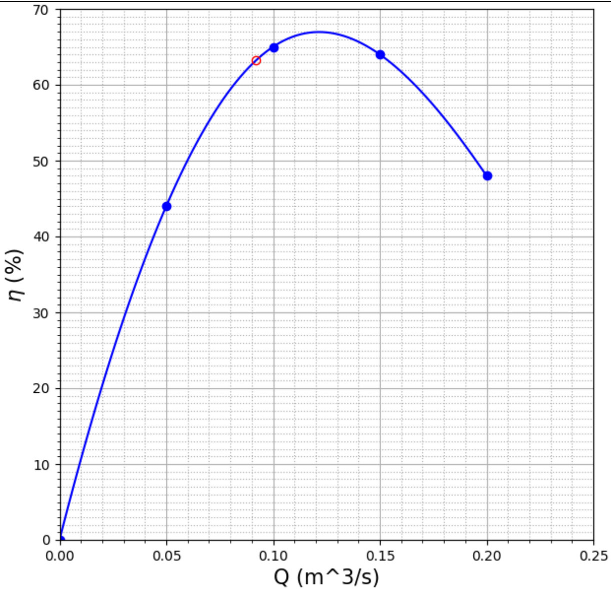
$$Q = 0.184\ \mathrm{m^3\ s^{-1}}$$
$$H = 53.3\ \mathrm{m}$$

(Note that Q here is the total over two pumps, equal to the actual flow in the main pipeline.)

(ii) Series pumps:

$$Q = 0.193\ \mathrm{m^3\ s^{-1}}$$
$$H = 55.4\ \mathrm{m}$$

(c) Input power can be obtained by dividing the output power $\rho g Q H$ by the efficiency η. For a single pump the tabulated data gives the following efficiency graph.



The efficiency of each pump depends on the flow rate through that pump, *not* the total for the combination. Here, both pumps carry the same flow $Q_1 = 0.184/2 = 0.092 \text{ m}^3 \text{ s}^{-1}$, which, from the graph above, gives efficiency – individual pumps or combination – of

$$\eta = 63.2\% \quad (= 0.632)$$

Thus we can get the input power for the combination of pumps as either that for both pumps (total flow $Q = 0.184 \text{ m}^3\text{s}^{-1}$, head $H = 53.3$ m):

$$\frac{\text{output power}}{\text{efficiency}} = \frac{\rho g Q H}{\eta} = \frac{1000 \times 9.81 \times 0.184 \times 53.3}{0.632} = 152200 \text{ W}$$

or doubling the input power for a single pump ($Q_1 = 0.092 \text{ m}^3\text{s}^{-1}$, head $H = 53.3$ m):

$$2 \times \frac{\rho g Q H}{\eta} = 2 \times \frac{1000 \times 9.81 \times 0.092 \times 53.3}{0.632} = 152200 \text{ W}$$

Both ways work here because both pumps happen to carry the same flow. In the general case it is safer to compute the input powers of individual pumps and add them up, because different pumps may be operating at different flow rates and, hence, different efficiencies.

Answer: 152 kW

(d) At $Q = 0.12 \text{ m}^3 \text{ s}^{-1}$ the total pump head for parallel operation is 57.1 m (see the graph), whilst the system head is only 41.6 m (from either the graph or the system curve). The difference between these,

$$57.1 - 41.6 = 15.5 \text{ m}$$

constitutes the head lost at the valve.

Answer: 15.5 m

4 Hydraulic Scaling

4.1 Dimensional Analysis

Provided that the mechanical efficiency is the same, the performance of a particular geometrically-similar family of pumps or turbines ("*homologous series*") may be expected to depend on:

discharge	Q	$[\mathrm{L^3T^{-1}}]$	
pressure change	$\rho g H$	$[\mathrm{ML^{-1}T^{-2}}]$	
power	P	$[\mathrm{ML^2T^{-3}}]$	(input for pumps; output for turbines)
rotor diameter	D	$[\mathrm{L}]$	
rotation rate	N	$[\mathrm{T^{-1}}]$	
fluid density	ρ	$[\mathrm{ML^{-3}}]$	
fluid viscosity	μ	$[\mathrm{ML^{-1}T^{-1}}]$	

(Rotor diameter may be replaced by any characteristic length, since geometric similarity implies that length ratios remain constant. Rotation rate is typically expressed in either rad $\mathrm{s^{-1}}$ or rpm.)

Since there are 7 variables and 3 independent dimensions, Buckingham's Pi Theorem yields a relationship between 4 independent groups, which may be rearranged as (exercise):

$$\Pi_1 = \frac{Q}{ND^3}, \qquad \Pi_2 = \frac{gH}{N^2D^2}, \qquad \Pi_3 = \frac{P}{\rho N^3 D^5}, \qquad \Pi_4 = \frac{\rho N D^2}{\mu} = \mathrm{Re}$$

For fully-turbulent flow the dependence on molecular viscosity, μ, and hence the Reynolds number, Π_4, vanishes. Then, for geometrically-similar pumps with different sizes, D, and rotation rates, N:

$$\left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2, \qquad \left(\frac{gH}{N^2D^2}\right)_1 = \left(\frac{gH}{N^2D^2}\right)_2, \qquad \left(\frac{P}{\rho N^3 D^5}\right)_1 = \left(\frac{P}{\rho N^3 D^5}\right)_2$$

For pumps (input power P, output power $\rho g Q H$), any one of Π_1, Π_2, Π_3 may be replaced by

$$\frac{\Pi_1 \Pi_2}{\Pi_3} = \frac{\rho g Q H}{P} = \eta \quad \text{(efficiency)}$$

The reciprocal of this would be used for turbines.

Example.
A ¼-scale model centrifugal pump is tested under a head of 7.5 m at a speed of 500 rpm. It was found that 7.5 kW was needed to drive the model. Assuming similar mechanical efficiencies, calculate:

(a) the speed and power required by the prototype when pumping against a head of 44 m;

(b) the ratio of the discharge in the model to that in the prototype.

(a) Given:

$L_{\text{model }(m)}/L_{\text{prototype }(p)} = 1/4$

$H_m = 7.5$ m

$Q_m = 500$ rpm

$P_m = 7.5$ kW

From the head scaling:

$$\left(\frac{gH}{N^2D^2}\right)_m = \left(\frac{gH}{N^2D^2}\right)_p$$

As g is the same at both scales,

$$\left(\frac{N_p}{N_m}\right)^2 = \left(\frac{H_p}{H_m}\right)\left(\frac{D_m}{D_p}\right)^2 \quad = \frac{44}{7.5}\times\left(\frac{1}{4}\right)^2 \quad = 0.3667$$

$$\Rightarrow \quad \frac{N_p}{N_m} = 0.6056$$

$$\Rightarrow \quad N_p = 0.6056N_m \quad = 0.6056\times(500\text{ rpm}) \quad = 302.8\text{ rpm}$$

From the power scaling:

$$\left(\frac{P}{\rho N^3D^5}\right)_p = \left(\frac{P}{\rho N^3D^5}\right)_m$$

Assuming the same working fluid, ρ is the same at both scales. Hence,

$$\frac{P_p}{P_m} = \left(\frac{N_p}{N_m}\right)^3\left(\frac{D_p}{D_m}\right)^5 \quad = 0.6056^3\times 4^5 \quad = 227.4$$

$$\Rightarrow \quad P_p = 227.4P_m \quad = 227.4\times(7.5\text{ kW}) \quad = 1706\text{ kW}$$

Answer: $N_p = 303$ rpm; $P_p = 1710$ kW

(b) From the discharge scaling,

$$\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p$$

$$\Rightarrow \qquad \frac{Q_m}{Q_p} = \left(\frac{N_m}{N_p}\right)\left(\frac{D_m}{D_p}\right)^3 \quad = \frac{1}{0.6056} \times \left(\frac{1}{4}\right)^3 \quad = 0.02580$$

Answer: 0.0258

4.2 Change of Speed

For the **same** pump (i.e. same D) operating at different speeds N_1 and N_2 the constancy of the dimensionless groups

$$\frac{Q}{ND^3}, \qquad \frac{gH}{N^2D^2}, \qquad \frac{P}{\rho N^3 D^5}, \qquad \eta$$

gives

$$Q \propto N, \qquad H \propto N^2, \qquad P \propto N^3, \qquad \eta \text{ constant}$$

(As an aid to memory, this might be expected, since $Q \propto$ velocity, whilst $H \propto$ energy $\propto$ velocity^2).

These are called the *hydraulic scaling laws* or *affinity laws*.

Speed-scaling laws for a single pump:

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}, \qquad \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2, \qquad \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3, \qquad \eta_1 = \eta_2$$

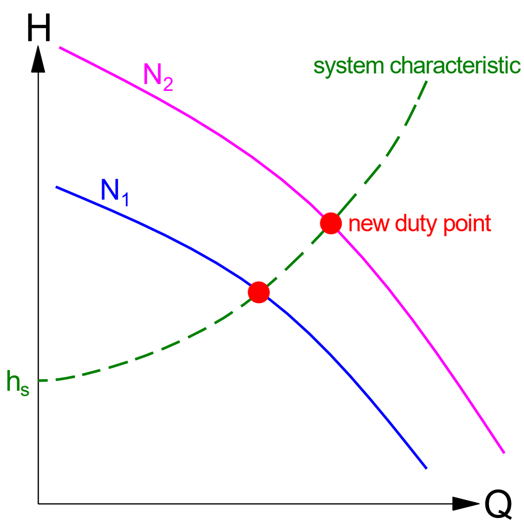
Given pump characteristics at one speed one can use the hydraulic scaling laws to deduce characteristics at a different speed.

4.2.1 Finding the Duty Point at a New Pump Speed

Scale each (Q, H) pair on the original characteristic at speed N_1 to get the new characteristic at speed N_2; i.e.

$$Q_2 = \left(\frac{N_2}{N_1}\right) Q_1, \qquad H_2 = \left(\frac{N_2}{N_1}\right)^2 H_1$$

Where this scaled characteristic intercepts the system curve gives the duty point at speed N_2.

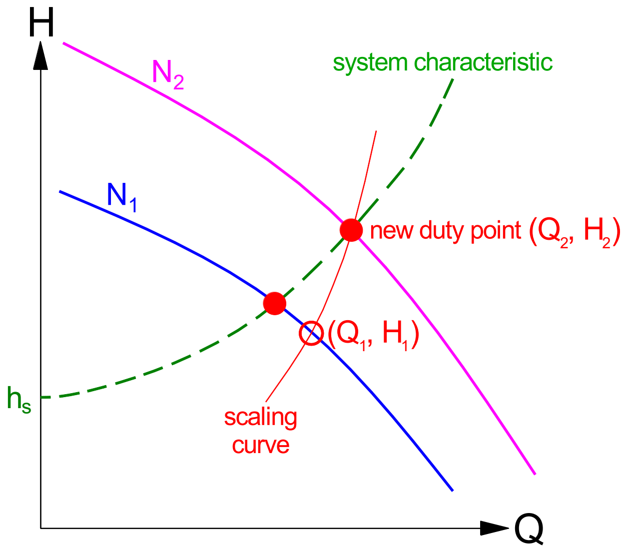


4.2.2 Finding the Pump Speed For a Given Duty Point (*Harder*)

To find the pump speed for a given discharge or head plot a hydraulic-scaling curve back from the required duty point (Q_2, H_2) on the system curve, at **unknown** speed N_2:

$$\frac{H}{H_2} = \left(\frac{Q}{Q_2}\right)^2$$

Very important: the hydraulic scaling curve is not the same as the system curve, although, since they are both quadratic, they can be quite close.



Where the hydraulic scaling curve cuts the original characteristic gives a scaled duty point (Q_1, H_1) and thence the ratio of pump speeds from *either* the ratio of discharges or the ratio of heads:

$$\frac{N_2}{N_1} = \frac{Q_2}{Q_1} \qquad \text{or} \qquad \left(\frac{N_2}{N_1}\right)^2 = \frac{H_2}{H_1}$$

Example.
Water from a well is pumped by a centrifugal pump which delivers water to a reservoir in which the water level is 15.0 m above that in the sump. When the pump speed is 1200 rpm its pipework has the following characteristics:

Pipework characteristics:

Discharge (L s^{-1}):	20	30	40	50	60
Head loss in pipework (m):	1.38	3.11	5.52	8.63	12.40

Pump characteristics:

Discharge (L s^{-1}):	0	10	20	30	40
Head (m):	22.0	21.5	20.4	19.0	17.4

A variable-speed motor drives the pump.

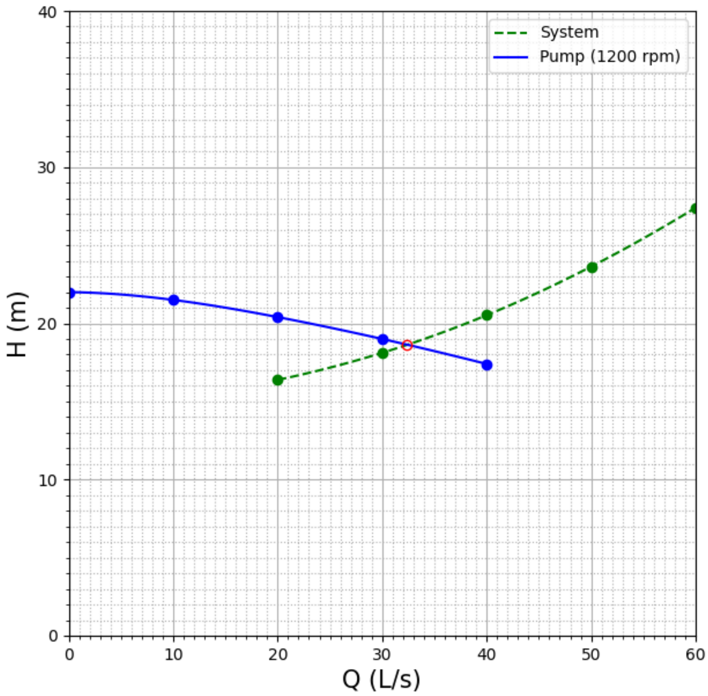
(a) Plot the graphs of the system and pump characteristics and determine the discharge at a speed of 1200 rpm.

(b) Find the pump speed in rpm if the discharge is increased to 40 L s^{-1}.

(a) The system head is (static lift) + (pipe head loss). Hence, including a static lift of 15 m, the system head is given in the following table.

Discharge (L s^{-1}):	20	30	40	50	60
System head (m):	16.38	18.11	20.52	23.63	27.40

Pump and system heads can then be plotted on the same graph:



These cross at $Q = 32.4 \text{ L s}^{-1}$.

Answer: 32.4 L s^{-1}

(b) The original speed is $N_1 = 1200$ rpm.

At some new (unknown) speed N_2 the discharge is

$$Q_2 = 40 \text{ L s}^{-1}$$

The system curve is unchanged, so we can read off the graph (or, in this case, directly from the table) the head at that speed:

$$H_2 = 20.52 \text{ m}$$

So, given point (Q_2, H_2) what point (Q_1, H_1) on the original characteristic did it scale from?

To find this we derive the *scaling curve* through (Q_2, H_2) and follow it back until it crosses the N_1 pump characteristic. (See the graph below).

The equation of the scaling curve is found as follows. As the speed N changes,

$$\frac{Q}{Q_2} = \frac{N}{N_2}, \qquad \frac{H}{H_2} = \left(\frac{N}{N_2}\right)^2$$

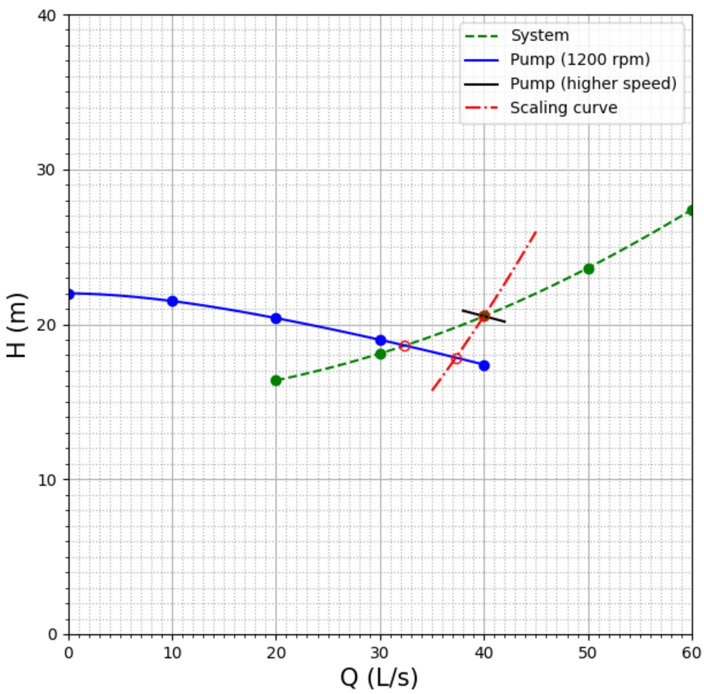
Eliminating N/N_2,

$$\frac{H}{H_2} = \left(\frac{Q}{Q_2}\right)^2$$

$$\Rightarrow \qquad H = \frac{H_2}{Q_2^2} Q^2$$

$$\Rightarrow \qquad H = 0.0128 Q^2$$

Taking a few values of Q this curve can be plotted until it crosses the N_1 characteristic. It is shown in red below.



At the crossing point on the N_1 characteristic,

$$Q_1 = 37.3 \text{ L s}^{-1}$$

Then,

$$\frac{N_2}{N_1} = \frac{Q_2}{Q_1} = \frac{40}{37.3} = 1.072$$

$$\Rightarrow \qquad N_2 = 1.072 N_1 = 1.072 \times (1200 \text{ rpm}) = 1286 \text{ rpm}$$

Answer: 1290 rpm

Example. (Exam 2020)
A variable-speed pump draws water from a reservoir to an elevated tank. The difference in water levels between the reservoir and the tank is 10 m. The pipe between them has length $L = 150$ m, diameter $D = 150$ mm and friction factor $\lambda = 0.02$. Minor losses can be lumped into an overall minor loss coefficient, K, which is unknown. The characteristics of the pump at the operational speed are given in the table below.

Pump characteristics at 2400 rpm

Discharge (L s^{-1})	16	26	36	47	57
Head (m)	39.0	36.4	31.3	22.9	12.1
Efficiency (%)	50.8	65.9	70.0	60.5	38.0

(a) Determine the head loss due to friction as a function of discharge, giving numerical values of the function coefficients and stating the units used for head and discharge.

The discharge at the duty point is 46 L s^{-1}.
(b) Find the pump head and power consumption at the duty point.

(c) Determine the overall minor loss coefficient, K.

After a rearrangement of facilities, the elevated tank is raised by 15 m and the pipe lengthened by 70 m. Through careful engineering, minor losses have been significantly reduced and can be assumed to be negligible ($K \approx 0$).

(d) If the same discharge is to be maintained, find the new rotation speed of the pump.

$h_s = 10$ m
$L = 150$ m
$D = 0.15$ m
$\lambda = 0.02$
$K = ?$

(a) The head loss due to friction is

$$h_f = \lambda \frac{L}{D}\left(\frac{V^2}{2g}\right) \qquad \text{where} \qquad V = \frac{Q}{\pi D^2/4}$$
$$= \frac{8\lambda L}{\pi^2 g D^5} Q^2$$

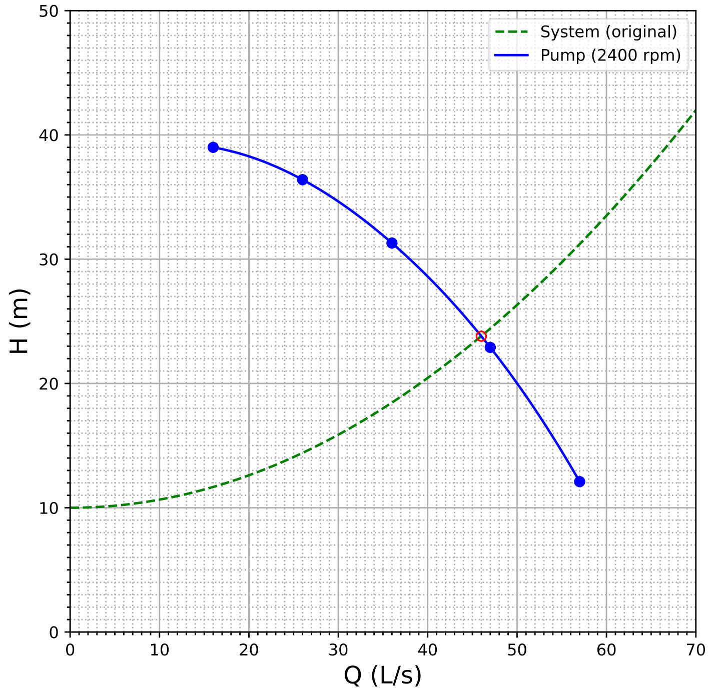
With the data above, this gives

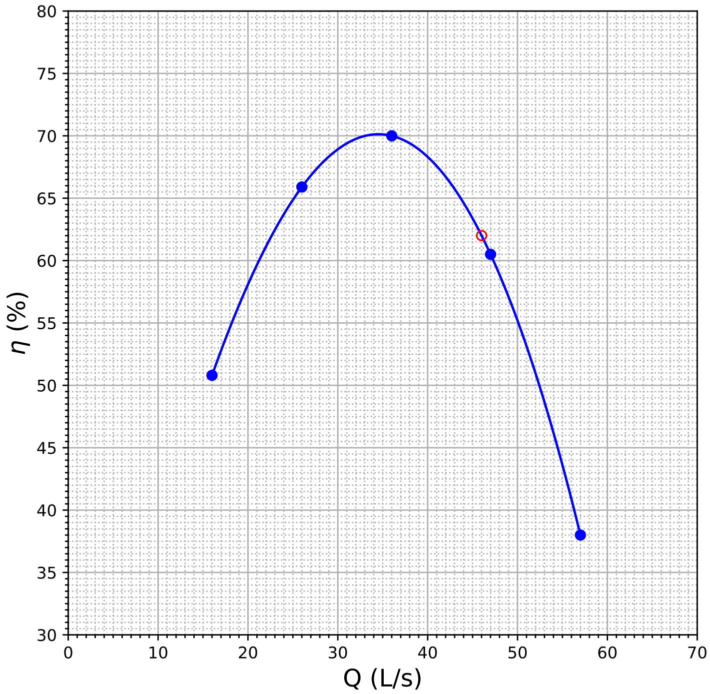
$$h_f = 3264 Q^2 \qquad (h_f \text{ in m}, Q \text{ in m}^3\text{ s}^{-1})$$

or

$$h_f = 0.003264 Q^2 \qquad (h_f \text{ in m}, Q \text{ in L s}^{-1})$$

(b) The discharge $Q = 46$ L s^{-1} ($= 0.046$ m^3 s^{-1}). Draw graphs of H vs Q and η vs Q to determine pump head and efficiency at the duty point.





At the duty point,

$$H = 23.8 \text{ m}$$

$$\eta = 62.0\% \quad (= 0.62)$$

The power consumption is then

$$\text{input power} = \frac{\text{output power}}{\text{efficiency}}$$
$$= \frac{\rho g Q H}{\eta}$$
$$= \frac{1000 \times 9.81 \times 0.0460 \times 23.8}{0.62}$$
$$= 17320 \text{ W}$$

Answer: $H = 23.8$ m; $P = 17.3$ kW

(c) At the duty point, pump head equals the system requirement (static lift + frictional losses + minor losses):

$$H = h_s + h_f + K\left(\frac{V^2}{2g}\right) \qquad \text{where} \qquad V = \frac{Q}{\pi D^2/4}$$

$\Rightarrow$ $\quad 23.8 = 10 + 6.907 + 0.3454K$

$\Rightarrow$ $\quad K = 19.96$

Answer: 20.0

(d) In the new arrangement,

$h_s = 25$ m
$L = 220$ m
$D = 0.15$ m
$\lambda = 0.02$
$K = 0$

The system head is

$$H_{\text{sys}} = h_s + \lambda\frac{L}{D}\left(\frac{V^2}{2g}\right) \qquad \text{where} \qquad V = \frac{Q}{\pi D^2/4}$$
$$= h_s + \frac{8\lambda L}{\pi^2 g D^5}Q^2$$

With the data above and $Q = 0.046 \text{ m}^3 \text{ s}^{-1}$, this gives

$$H_{\text{sys}} = 35.1 \text{ m}$$

This must match the pump head. Hence, at an unknown speed N_2 the pump parameters are

$$Q_2 = 46.0 \text{ L s}^{-1}$$
$$H_2 = 35.1 \text{ m}$$

Plot a speed-scaling curve through this point until it reaches the characteristic for the original speed $N_1 = 2400$ rpm. This scaling curve is

$$\frac{H}{H_2} = \left(\frac{N}{N_2}\right)^2 = \left(\frac{Q}{Q_2}\right)^2$$

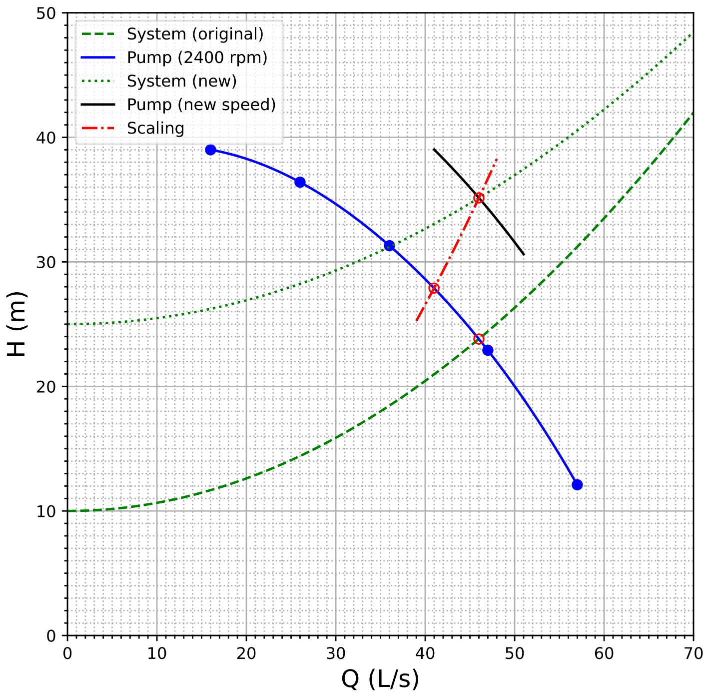
i.e., for H in m and Q in L s^{-1},

$$\frac{H}{35.1} = \left(\frac{N}{N_2}\right)^2 = \left(\frac{Q}{46.0}\right)^2$$

or

$$H = 0.0166Q^2$$

This is illustrated below.



The scaling curve meets the N_1 characteristic at $(Q_1, H_1) = (41.0, 27.9)$ and, hence, using the ratio of discharges (say),

$$\frac{N_2}{N_1} = \frac{Q_2}{Q_1}$$

$$\Rightarrow \qquad \frac{N_2}{2400} = \frac{46.0}{41.0}$$

$$\Rightarrow \qquad N_2 = 2693 \text{ rpm}$$

Answer: 2693 rpm

4.3 Specific speed

The *specific speed* (or *type number*) is a guide to the **type** of pump or turbine required for a particular role.

4.3.1 Specific Speed for Pumps

The *specific speed*, N_s, is the rotational speed needed to discharge 1 unit of flow against 1 unit of head. (For what "unit" means in this instance, see below.)

For a given pump, the hydraulic scaling laws give

$$\Pi_1 \equiv \frac{Q}{ND^3} = \text{constant}\,, \qquad \Pi_2 = \frac{gH}{N^2D^2} = \text{constant}$$

Eliminating D, and choosing an exponent that will make the combination proportional to N:

$$\left(\frac{\Pi_1^2}{\Pi_2^3}\right)^{1/4} = \frac{Q^{1/2}N}{(gH)^{3/4}}$$

or, since g is constant, then at any given (e.g. maximum) efficiency:

$$\frac{Q^{1/2}N}{H^{3/4}} = (\text{dimensional})\ \text{constant}$$

This constant is the *specific speed*, N_s, occurring when Q and H in specified units (see below) are numerically equal to 1.0:

Specific speed (pump):

$$N_S = \frac{Q^{1/2}N}{H^{3/4}}$$

Notes.

- The specific speed is a single value calculated at the "normal" operating point (usually Q and H at the maximum efficiency point for the anticipated rotation rate N).

- With the commonest definition (in the UK and Europe), N is in rpm, Q in m^3 s^{-1}, H in m, but this is far from universal, so be careful.

- In principle, the units of N_s are the same as those of N, which doesn't look correct from the definition but only because that has been shortened from

$$\frac{(1\ \text{m}^3\ \text{s}^{-1})^{1/2} \times N_s}{(1\ \text{m})^{3/4}} = \frac{Q^{1/2}N}{H^{3/4}}$$

- Because of the omission of g the definition of N_s depends on the units of Q and H. A less-common (but, IMHO, more mathematically correct) quantity is the *dimensionless specific speed* K_n given by

$$K_n = \frac{Q^{1/2}N}{(gH)^{3/4}}$$

If the time units are consistent then K_n has the same angular units as N (rev or rad).

- High specific speed ↔ large discharge / small head (axial-flow device).
  Low specific speed ↔ small discharge / large head (centrifugal device).
  Approximate ranges of N_s are (from Hamill, 2011):

Type	N_s (rpm)	
Radial (centrifugal)	10 – 70	large head
Mixed flow	70 – 170	
Axial	> 110	small head

Example.
A pump is needed to operate at 3000 rpm (i.e. 50 Hz) with a head of 6 m and a discharge of 0.2 $m^3 s^{-1}$. By calculating the specific speed, determine what sort of pump is required.

Given (in the required units):

$N = 3000$ rpm
$H = 6$ m
$Q = 0.2\ m^3\ s^{-1}$

Specific speed:

$$N_s = \frac{Q^{1/2}N}{H^{3/4}} \quad = \frac{0.2^{1/2} \times 3000}{6^{3/4}} \quad = 350.0 \text{ (rpm)}$$

This is substantially larger than 110 rpm. Hence, choose an *axial-flow* pump. (In terms of typical applications, 200 L s^{-1} is quite a high flow and 6 m a fairly low head.)

Answer: 350 rpm; axial-flow pump

4.3.2 Specific Speed for Turbines

For turbines the output power, P, is more important than the discharge, Q. The relevant dimensionless groups are

$$\Pi_2 \equiv \frac{gH}{N^2D^2}, \qquad \Pi_3 \equiv \frac{P}{\rho N^3 D^5}$$

Eliminating D,

$$\left(\frac{\Pi_3^2}{\Pi_2^5}\right)^{1/4} = \frac{(P/\rho)^{1/2}N}{(gH)^{5/4}}$$

or, since ρ and g are usually taken as constant (there does seem to be a presumption that turbines are always operating in fresh water) then at any given efficiency:

$$\frac{P^{1/2}N}{H^{5/4}} = \text{(dimensional) constant}$$

The *specific speed* of a **turbine**, N_s, is the rotational speed needed to develop 1 unit of power for a head of 1 unit. (For what "unit" means in this instance, see below.)

Specific speed (turbine):

$$N_s = \frac{P^{1/2}N}{H^{5/4}}$$

Notes.

- With the commonest definition (in the UK and Europe), N is in rpm, P in kW (**note**), H in m, but, again, this is not a universal convention. As with pumps, the units of N_s are the same as those of N.

- As with pumps, a less commonly used, but mathematically more acceptable, quantity is the *dimensionless specific speed* K_n, which retains the ρ and g dependence:

  $$K_n = \frac{(P/\rho)^{1/2}N}{(gH)^{5/4}}$$

  K_n has the angular units of N (revs or radians) – see Massey (2011).

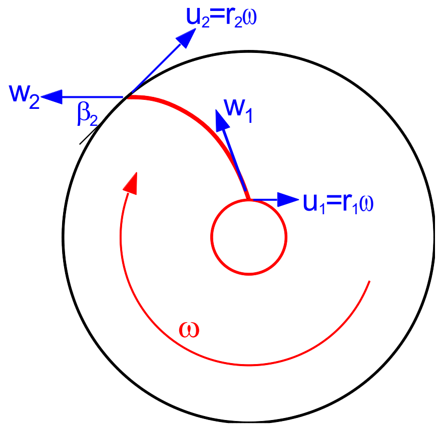
- High specific speed ↔ small head (axial-flow device)
  Low specific speed ↔ large head (centrifugal or impulse device).

  Approximate ranges are (from Hamill, 2011):

Type	N_s (rpm)	
Pelton wheel (impulse)	12 – 60	very large head
Francis turbine (radial-flow)	60 – 500	large head
Kaplan turbine (axial-flow)	280 – 800	small head

5. Mechanics of Rotodynamic Devices

5.1 Centrifugal Pump



Fluid enters at the *eye* of the impeller and flows outward. As it does so it picks up the tangential velocity of the impeller *vanes* (or *blades*) which increases linearly with radius ($u = r\omega$). At exit the fluid is expelled nearly tangentially at high velocity. Kinetic energy is subsequently converted to pressure energy in the expanding *volute*.

The analysis makes use of rotational dynamics:

$$\text{power} = \text{torque} \times \text{angular velocity}$$

$$\text{torque} = \text{rate of change of angular momentum}$$

where angular momentum is given by "tangential component of momentum × radius".

The *absolute* velocity of the fluid is the vector sum of:

impeller velocity (tangential)

+

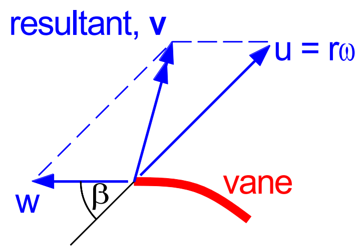
velocity relative to the impeller (parallel to the vanes)

Write:

u	for the impeller velocity ($u = r\omega$)
w	for the fluid velocity relative to the impeller
v = **u** + **w**	for the absolute velocity

The radial component of absolute velocity is determined primarily by the flow rate:

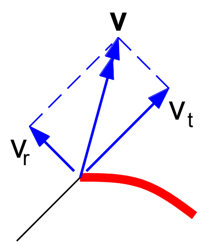
$$v_r = \frac{Q}{A}$$



where A is the effective outlet area. The tangential part (also called the *whirl* velocity) is a combination of impeller speed ($u = r\omega$) and tangential component relative to the vanes:

$$v_t = u - w\cos\beta$$

Only v_t contributes to the angular momentum.



With subscripts 1 and 2 denoting inlet and outlet respectively,

$$\text{torque, } T = \rho Q(v_{t2}r_2 - v_{t1}r_1)$$

$$\text{power} = T\omega = \rho Q(v_{t2}r_2\omega - v_{t1}r_1\omega)$$

But head $H = \text{power}/(\rho g Q)$, whilst $r\omega = u$.

Hence, we have:

Euler's turbomachinery equation:

$$H = \frac{1}{g}(v_{t2}u_2 - v_{t1}u_1)$$

The pump is usually designed so that the initial angular momentum is small; i.e. $v_{t1} \approx 0$. Then

$$H = \frac{1}{g}v_{t2}u_2$$

Effect of Blade Angle

Because in the frame of the impeller the fluid leaves the blades in a direction parallel to their surface, forward-facing blades would be expected to increase the whirl velocity v_t whilst backward-facing blades would diminish it.

Radial and tangential components of velocity:

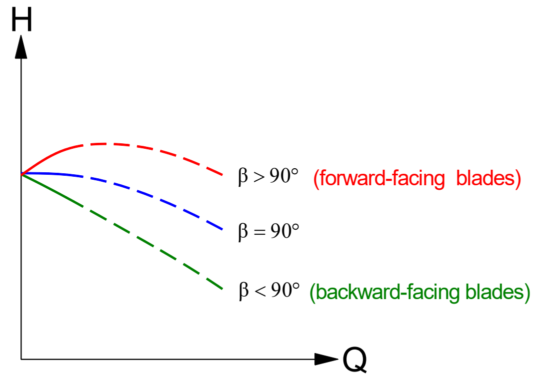
$$v_r = w\sin\beta\,, \qquad v_t = u - w\cos\beta$$

Eliminating w:

$$v_t = u - v_r\cot\beta$$

Hence, if inlet whirl can be ignored,

$$H = \frac{u_2}{g}(u_2 - v_{r2}\cot\beta)$$



Since v_r is also Q/A (where A is exit area of the impeller):

$$H = \frac{r_2\omega}{g}(r_2\omega - \frac{Q}{A}\cot\beta)$$

This is of the form $H = a - bQ$, where

H initially *decreases* with Q for *backward-facing blades* ($\beta < 90°$; $\cot\beta > 0$)

H initially *increases* with Q for *forward-facing blades* ($\beta > 90°$; $\cot\beta < 0$)

This gives rise to the pump characteristics shown. Backward-facing blades are usually preferred because, although forward-facing blades might be expected to increase whirl velocity and hence output head, the shape of the characteristic is such that small changes in head cause large changes in discharge and the pump tends to "hunt" for its operating point (*pump surge*).

Non-Ideal Behaviour

The above is a very ideal analysis. There are many sources of losses. These include:

- leakage back from the high-pressure volute to the low-pressure impeller eye;
- frictional losses;
- "shock" or flow-separation losses at entry;
- non-uniform flow at inlet and outlet of the impeller;
- cavitation (when the inlet pressure is small).

Example.
A centrifugal pump is required to provide a head of 40 m. The impeller has outlet diameter 0.5 m and inlet diameter 0.25 m and rotates at 1500 rpm. The flow approaches the impeller radially at 10 m s^{-1} and the radial velocity falls off as the reciprocal of the radius. Calculate the required vane angle at the outlet of the impeller.

The head is given by Euler's formula

$$H = \frac{u_2}{g}(u_2 - v_{r2}\cot\beta)$$

where $u_2 = r_2\omega$ is the outlet blade velocity and v_{r2} is the outlet radial velocity.

Here,

$$H = 40 \text{ m}$$

$$\omega = 1500\frac{\text{rev}}{\text{min}} \quad = 1500 \times \frac{2\pi \text{ rad}}{60 \text{ s}} \quad = 157.1 \text{ rad s}^{-1}$$

$$u_2 = r_2\omega \quad = \frac{0.5}{2} \times 157.1 \quad = 39.28 \text{ m s}^{-1}$$

The radial velocity is stated to fall off as the reciprocal of (i.e. in inverse proportion to) the radius. As the outlet radius is twice the inlet radius the outlet (radial) velocity is half the inlet velocity; i.e.

$$v_{r2} = 5 \text{ m s}^{-1}$$

Hence, in Euler's formula,

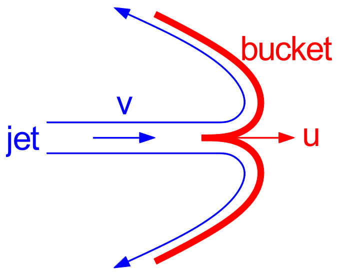
$$40 = \frac{39.28}{9.81}(39.28 - 5\cot\beta)$$

$\Rightarrow \qquad \cot\beta = 5.858$

$\Rightarrow \qquad \beta = 9.687°$

Answer: 9.69°

5.2 Pelton Wheel



A Pelton wheel is the most common type of impulse turbine. One or more jets of water impinge on buckets arranged around a turbine runner. The deflection of water changes its momentum and imparts a force to rotate the runner.

The power (per jet) P is given by:

$$\text{power} = \text{force (on bucket)} \times \text{velocity (of bucket)}$$

Force F on the bucket is equal and opposite to that on the jet; by the momentum principle:

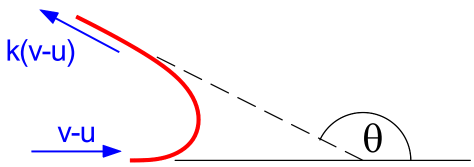
$$\text{force (on fluid)} = \text{mass flux} \times \text{change in velocity}$$

Because the absolute velocity of water leaving the bucket is the vector resultant of the runner velocity (u) and the velocity relative to the bucket, the *change* in velocity is most easily established in the frame of reference of the moving bucket.

Assuming that the relative velocity *leaving* the buckets is k times the relative velocity of approach, $v - u$ (where k is slightly less than 1 due to friction):

$$\begin{aligned} \text{change in } x - \text{velocity} \quad &= -k(v-u)\cos(180° - \theta) - (v-u) \\ &= -(v-u)(1 - k\cos\theta) \end{aligned}$$

where θ is the total angle turned (here, greater than 90°). The maximum force would be obtained if the flow was turned through 180°, but the necessity of deflecting it clear of the next bucket means that θ is typically about 165°.



From the momentum principle:

$$-F = -\rho Q(v-u)(1 - k\cos\theta)$$

The power transferred in each jet is then

$$P = Fu = \rho Q(v-u)u(1 - k\cos\theta)$$

The velocity part of the power may be written

$$(v-u)u = \frac{1}{4}v^2 - (\frac{1}{2}v - u)^2$$

Hence, for a given jet (Q and v), the power is a maximum when the runner speed u is such that $u = \frac{1}{2}v$, or the runner speed is half the jet speed. (At this point the *absolute* velocity leaving the runner at 180° would be 0 if $k = 1$, corresponding to the case where all kinetic energy of the fluid is transferred to the runner.) In practice, the runner speed u is often fixed by the need to synchronise the generator to the electricity grid, so it is usually the jet which is controlled (by a spear valve). Because of other losses the speed ratio is usually slightly less than ½, a typical value being 0.46.

The jet velocity is given by Bernoulli's equation, with a correction for non-ideal flow:

$$v = c_v\sqrt{2gH}$$

where H is the head upstream of the nozzle (= original head minus any losses in the pipeline) and c_v is an orifice coefficient with typical values in the range 0.97–0.99.

Example.
In a Pelton wheel, 6 jets of water, each with a diameter of 75 mm and carrying a discharge of 0.15 m^3 s^{-1} impinge on buckets arranged around a 1.5 m diameter Pelton wheel rotating at 180 rpm. The water is turned through 165° by each bucket and leaves with 90% of the original relative velocity. Neglecting mechanical and electrical losses within the turbine, calculate the power output.

Consider individual jets first, multiplying by 6 later to get total power.

In each jet, calculate transferred power as

$$\text{force (on the bucket)} \times \text{(velocity of bucket)}$$

with the force on the bucket inferred from the change of momentum (in the frame of the bucket).

Bucket absolute velocity:

$$u = R\omega \quad = (1.5/2) \times \left(180 \times \frac{2\pi}{60}\right) \quad = 14.14 \text{ m s}^{-1}$$

Jet absolute velocity:

$$v = \frac{Q}{\pi d^2/4} \quad = \frac{4 \times 0.15}{\pi \times 0.075^2} \quad = 33.95 \text{ m s}^{-1}$$

Hence, in the frame of the moving bucket (i.e. subtract u from all x-velocity components) we have, accounting for the speed reduced to 90% and angle turned on reflection leaving water flowing at $180 - 165 = 15°$ to the negative x axis:

$$v_{x1} = v - u \quad = 19.81 \text{ m s}^{-1}$$

$$v_{x2} = -0.9 \times (v - u) \times \cos 15° \quad = -17.22 \text{ m s}^{-1}$$

Hence, using "force = rate of change of momentum" (on the fluid):

$$-F = \rho Q(v_{x2} - v_{x1}) \quad = 1000 \times 0.15 \times (-17.22 - 19.81)$$

$$\Rightarrow \qquad F = 5555 \text{ N}$$

Then,

$$\text{power (per jet)} = \text{force} \times \text{velocity} \quad = Fu \quad = 5555 \times 14.14 \quad = 78550 \text{ W}$$

The total power (six jet/bucket combinations) is

$$6 \times 78550 = 471300 \text{ W}$$

Answer: 471 kW

6. Cavitation

Cavitation is the formation, growth and rapid collapse of vapour bubbles in flowing liquids.

Bubbles form at low (sub-atmospheric) pressures when the absolute pressure drops to the vapour pressure and the liquid spontaneously boils. (Bubbles may also arise from dissolved gases coming out of solution.) When the bubbles are subsequently swept into higher-pressure regions they collapse very rapidly, with large radial velocities and enormous transient pressures, potentially leading to surface damage.

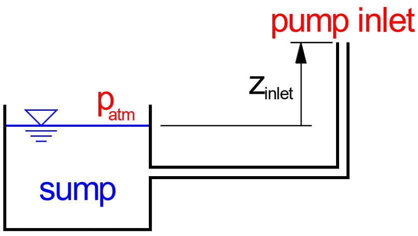
Cavitation may cause performance loss, vibration, noise, surface pitting and, occasionally, major structural damage. Besides the inlet to pumps, the phenomenon is prevalent in marine-current turbines, ship and submarine propellers and on reservoir spillways.

The best way of preventing cavitation in a pump is to ensure that the inlet (suction) pressure is not too low. The *net positive suction head* (NPSH) is defined as the difference between the (stagnation) pressure head and that corresponding to the vapour (or cavitation) pressure:

$$\text{NPSH} = \frac{p_0 - p_{\text{cav}}}{\rho g} \quad = \frac{\left(p + \frac{1}{2}\rho V^2\right) - p_{\text{cav}}}{\rho g}$$

It is, in length units, the margin by which the stagnation pressure p_0 (the pressure when the fluid is brought to rest) exceeds that at which cavitation may occur.

The net positive suction head must be kept well above zero to allow for further pressure loss in the impeller. A key measure is the *available* net positive suction head, NPSH_a, which is the NPSH at pump inlet. This should be compared with the *required* net positive suction head, NPSH_r, which is a manufacturer-specified quantity with a margin of safety to prevent cavitation, including an allowance for further pressure loss within the pump and any dissolved gases.



The inlet pressure may be determined from Bernoulli's equation, measuring z relative to the level in the sump:

$$H_{\text{pump inlet}} = H_{\text{sump}} - \text{head loss}$$

$$\Rightarrow \qquad \left(\frac{p + \rho g z + \frac{1}{2}\rho V^2}{\rho g}\right)_{\text{inlet}} = \frac{p_{\text{atm}}}{\rho g} - h_f$$

Hence,

$$\left(\frac{p_0}{\rho g}\right)_{\text{inlet}} = \frac{p_{\text{atm}}}{\rho g} - z_{\text{inlet}} - h_f$$

whence

$$\text{NPSH}_a \equiv \left(\frac{p_0 - p_{\text{cav}}}{\rho g}\right)_{\text{inlet}} \quad = \frac{p_{\text{atm}} - p_{\text{cav}}}{\rho g} - z_{\text{inlet}} - h_f$$

To avoid cavitation one should aim to keep $NPSH_a$ as large as possible by:

- keeping z_{inlet} small or, better still, negative (i.e. below the level of water in the sump);
- keeping h_f small (short, large-diameter pipes).

The first also assists in pump *priming*.

Example. (Exam 2022)
A variable-speed pump is used to supply water from a reservoir to an elevated tank. The difference in water levels between the reservoir and the tank is 7 m. The pipe between them has length $L = 300$ m, diameter $D = 200$ mm and friction factor $\lambda = 0.03$. Minor losses can be lumped into an overall minor loss coefficient $K = 25$. The characteristics of the pump at the operational speed are given in the table below.

Pump characteristics at 2900 rpm:

Discharge, Q (L s^{-1})	10	25	40	55	70	85
Head, H (m)	37.6	35.3	30.9	24.4	15.5	4.2
Efficiency, η (%)	22	54	78	86	71	25

(a) Find the system characteristic (head as a function of discharge), giving numerical values and the units that you have chosen to use for head and discharge.

The rotational speed of the pump is adjusted to supply a discharge of 45 L s^{-1}.

(b) Determine the rotational speed and power consumption of the pump.

(c) Explain what is meant by cavitation and why it can cause damage in hydraulic systems.

The pump inlet is located 5 m above the reservoir level and the suction pipe has length 10 m. The diameter and friction factor of the suction pipe are the same as those provided above. Minor losses on suction side can be accommodated by a loss coefficient $K_u = 4.5$.

(d) Assuming that atmospheric pressure is 101.2 kPa and the vapour pressure of water at the operating temperature 1.7 kPa, determine the Net Positive Suction Head available at pump inlet ($NPSH_a$) for the given discharge.

(e) Is the pump expected to cavitate if the Net Positive Suction Head required ($NPSH_r$) for the given discharge is 2.7 m?

Given:

$h_s = 7$ m
$L = 300$ m
$D = 0.2$ m
$\lambda = 0.03$
$K = 25$

(a)

$$\text{head} = \text{static lift} + \text{frictional (and other) losses}$$

$$H = h_s + \left(\lambda \frac{L}{D} + K\right)\frac{V^2}{2g} \qquad \text{where} \qquad V = \frac{Q}{\pi D^2/4}$$

whence

$$H = h_s + \left(\lambda \frac{L}{D} + K\right)\frac{8}{\pi^2 g D^4} Q^2$$

With the data above, this gives

$$H = 7 + 3615Q^2 \qquad (H \text{ in m}, Q \text{ in m}^3\ \text{s}^{-1})$$

or

$$H = 7 + 0.003615Q^2 \qquad (H \text{ in m}, Q \text{ in L s}^{-1})$$

(b) The pump characteristics are given in the question at a speed $N_1 = 2900$ rpm. The pump speed is to be changed to N_2, which is unknown. The duty point here is

$$Q_2 = 45 \text{ L s}^{-1}$$

This must still lie on the system curve, so that the head is, using the formula from part (a),

$$H_2 = 14.32 \text{ m}$$

We can now plot a speed-scaling curve ($Q \propto N$, $H \propto N^2$) back to meet the original pump characteristic:

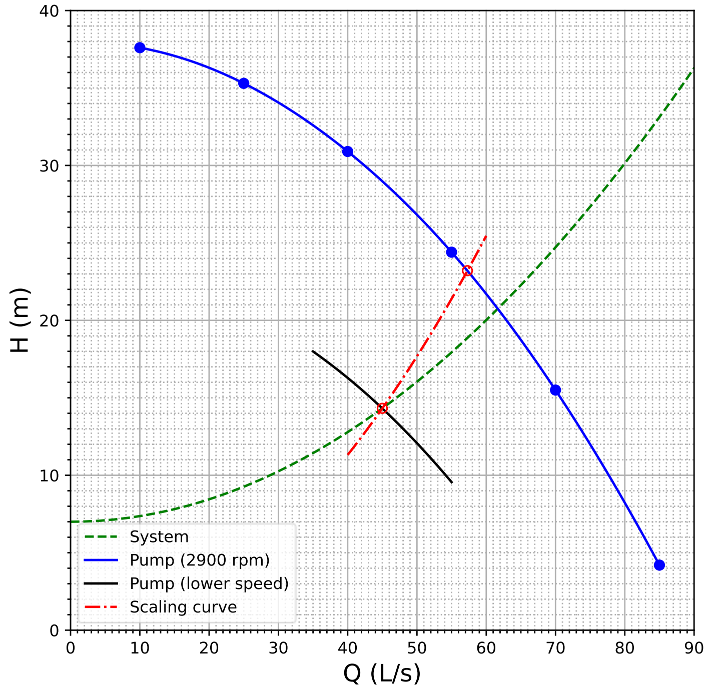
$$\frac{H}{H_2} = \left(\frac{N}{N_2}\right)^2 = \left(\frac{Q}{Q_2}\right)^2$$

$$\Rightarrow \qquad H = \frac{H_2}{Q_2^2} Q^2$$

$$\Rightarrow \qquad H = 0.007072Q^2 \qquad (H \text{ in m}, Q \text{ in L s}^{-1})$$

(Obviously, you may work in different units for Q if you prefer.)

Plot this on an H vs Q graph, along with original pump characteristic and system curve.



This scales to the following point on the N_1 characteristic (note that N_2 is the *lower* speed in this instance):

$$Q_1 = 57.3 \text{ L s}^{-1} \quad (= 0.0573 \text{ m}^3 \text{ s}^{-1})$$

$$H_1 = 23.2 \text{ m}$$

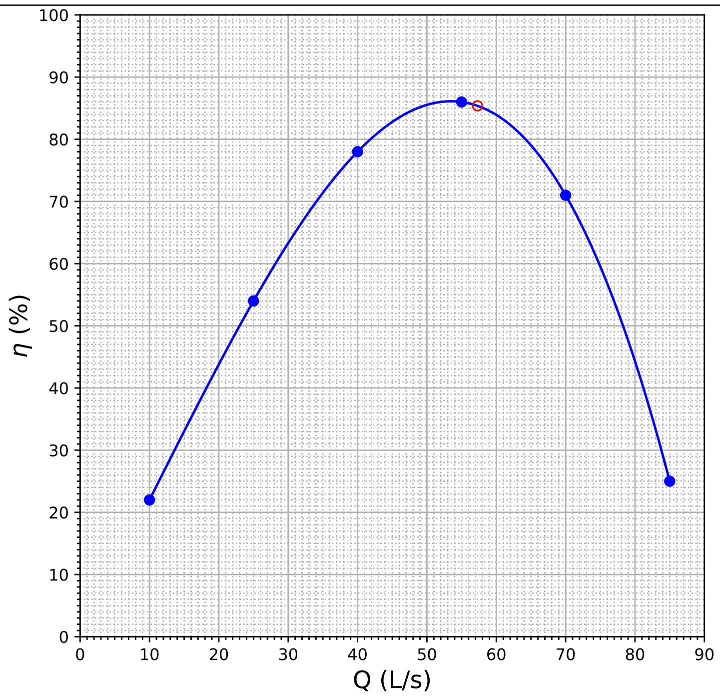
The ratio of Q values (or H values, if you prefer) then gives the ratio of speeds:

$$\frac{N_2}{N_1} = \frac{Q_2}{Q_1} = \frac{45.0}{57.3} = 0.7853$$

Then

$$N_2 = 0.7853 \times (2900 \text{ rpm}) = 2277 \text{ rpm}$$

The efficiency (which is unchanged by speed scaling) can be read off the original pump characteristic at $Q_1 = 57.3 \text{ L s}^{-1}$.



This gives

$$\eta_2 = \eta_1 = 85.4\% \quad (= 0.854)$$

The power consumption(at the new, N_2, speed) is then

$$\begin{aligned} \text{input power} &= \frac{\text{output power}}{\text{efficiency}} \\ &= \frac{\rho g Q_2 H_2}{\eta_2} \\ &= \frac{1000 \times 9.81 \times 0.0450 \times 14.32}{0.854} \\ &= 7402 \text{ W} \end{aligned}$$

Answer: $N = 2280$ rpm; $P = 7.40$ kW

(c) Cavitation is the formation of vapour bubbles at very low (sub-atmospheric) pressures. The problem occurs when the bubbles are subsequently swept into a higher-pressure zone, where they collapse rapidly, causing high local shock pressures that can lead to surface pitting.

(d) The net positive suction head available is the amount by which the inlet stagnation pressure exceeds the cavitation pressure, re-expressed in head units; i.e.

$$\text{NPSH}_\text{a} = \frac{\left(p + \frac{1}{2}\rho V^2\right)_\text{inlet} - p_\text{cav}}{\rho g}$$

By Bernoulli,

$$\left(p+\frac{1}{2}\rho V^2+\rho g z\right)_{\text{inlet}}=\left(p+\frac{1}{2}\rho V^2+\rho g z\right)_{\text{reservoir}}-\text{upstream losses}$$

and hence

$$\left(p+\frac{1}{2}\rho V^2\right)_{\text{inlet}}=p_{\text{atm}}+\rho g(z_{\text{reservoir}}-z_{\text{inlet}})-\left(\lambda\frac{L_u}{D}+K_u\right)(\frac{1}{2}\rho V^2)$$

Here, $P_{\text{atm}} = 101200$ Pa (absolute), $z_{\text{reservoir}} - z_{\text{inlet}} = -5$ m, $L_u = 10$ m, $K_u = 4.5$ and the pipe bulk velocity is

$$V=\frac{Q}{\pi D^2/4}\quad=\frac{4\times 0.045}{\pi\times 0.2^2}\quad=1.432\text{ m s}^{-1}$$

Hence, as an absolute pressure,

$$\left(p+\frac{1}{2}\rho V^2\right)_{\text{inlet}}=46000\text{ Pa}$$

Then, in terms of head rather than pressure,

$$\text{NPSH}_\text{a}=\frac{46000-1700}{1000\times 9.81}\quad=4.516\text{ m}$$

Answer: 4.52 m

(e) $\text{NPSH}_\text{a} > \text{NPSH}_\text{r}$, so the pump is safe from cavitation