Objectives

(1) Understand the role of pumps and turbines as **energy-conversion devices** and use, appropriately, the terms **head**, **power** and **efficiency**.
(2) Be aware of the main types of pumps and turbines and the distinction between **impulse** and **reaction** turbines and between **radial**, **axial** and **mixed-flow** devices.
(3) Match **pump characteristics** and **system characteristics** to determine the **duty point**.
(4) Calculate characteristics for **pumps in series and parallel** and use the **hydraulic scaling laws** to calculate pump characteristics at **different speeds**.
(5) Select the type of pump or turbine on the basis of **specific speed**.
(6) Understand the **mechanics** of a centrifugal pump and an impulse turbine.
(7) Recognise the problem of **cavitation** and how it can be avoided.

References

White (2006) – Chapter 11
Hamill (2001) – Chapter 11
Chadwick and Morfett (2004) – Chapter 7
Massey (2005) – Chapter 12

1. Energy Conversion

1.1 Energy Transfer in Pumps and Turbines

Pumps and turbines are *energy conversion* devices:

pumps turn electrical or mechanical energy into fluid energy;
turbines turn fluid energy into electrical or mechanical energy.

The energy per unit weight is the *head*, *H*:

$$H = \frac{p}{\rho g} + z + \frac{V^2}{2g}$$

The first two terms on the RHS comprise the *piezometric head*. The last term is the *dynamic head*.

1.2 Power

Power = rate of conversion of energy.

If a mass *m* is raised through a height *H* it gains energy *mgH*. If it does so in time *t* then the rate of conversion is *mgH/t*.

For a fluid in motion the mass flow rate (*m*/*t*) is ρQ. The rate of conversion to or from fluid energy when the total head is changed by *H* is, therefore, $\rho Q \times gH$, or

$$power = \rho g Q H$$

1.3 Efficiency

Efficiency, η, is given by

$$\eta = \frac{power_{out}}{power_{in}}$$

where "$power_{out}$" refers to the **useful** power; i.e. excluding losses.

For **turbines**: $\eta = \frac{power_{out}}{\rho g Q H}$

For **pumps**: $\eta = \frac{\rho g Q H}{power_{in}}$

Example.
A pump lifts water from a large tank at a rate of 30 L s^{-1}. If the input power is 10 kW and the pump is operating at an efficiency of 40%, find:

(a) the head developed across the pump;
(b) the maximum height to which it can raise water if the delivery pipe is vertical, with diameter 100 mm and friction factor $\lambda = 0.015$.

Answer: (a) 13.6 m; (b) 12.2 m

2. Types of Pumps and Turbines

2.1 Impulse and reaction turbines

In a pump or turbine a change in fluid head ($H = \frac{p}{\rho g} + z + \frac{V^2}{2g}$) may be brought about by a change in pressure or velocity or both.

- An *impulse turbine* (e.g. Pelton wheel; water wheel) is one where the change in head is brought about primarily by a change in **velocity**. This usually involves unconfined free jets of water (at atmospheric pressure) impinging on moving vanes.
- A *reaction turbine* (e.g. Francis turbine; Kaplan turbine; windmill) is one where the change in head is brought about primarily by a change in **pressure**.

2.2 Positive-Displacement and Dynamic pumps

Postive-displacement pumps operate by a change in volume; energy conversion is intermittent. Examples in the human body include the heart (*diaphragm pump*) and the intestines (*peristaltic pump*). In a *reciprocating pump* (e.g. a bicycle pump) fluid is sucked in on one part of the cycle and expelled (at higher pressure) in another.

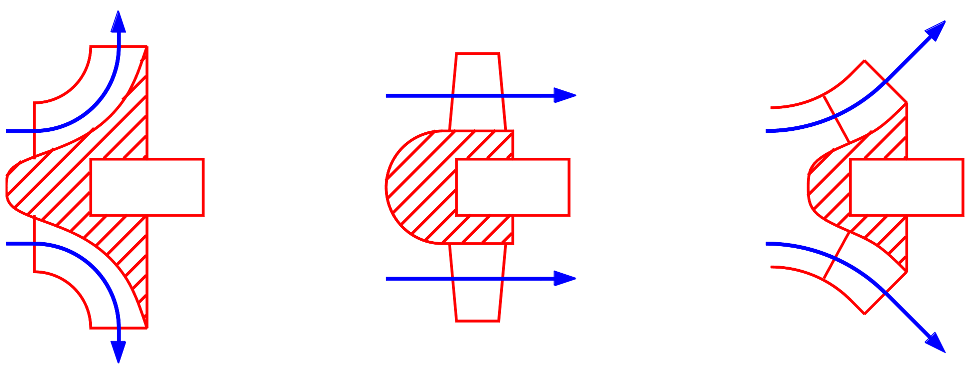
In *dynamic pumps* there is no change in volume and energy conversion is continuous. Most pumps are *rotodynamic* devices where fluid energy is exchanged with the mechanical energy of a rotating element (called a *runner* in turbines and an *impeller* in pumps), with a further conversion to or from electrical energy.

This course focuses entirely on rotary devices.

Note that, for gases, pumps are usually referred to as *fans* (for low pressures), *blowers* or *compressors* (for high pressures).

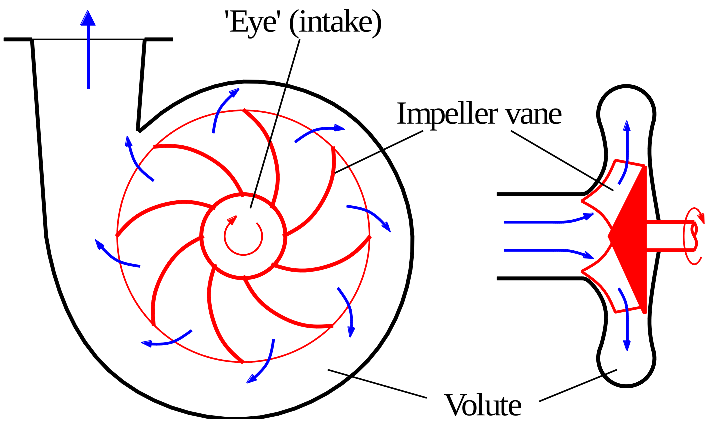
2.3 Radial, Axial and Mixed-Flow Devices

The terms *radial* and *axial* refer to the change in direction of flow through a rotodynamic device (pump or turbine):

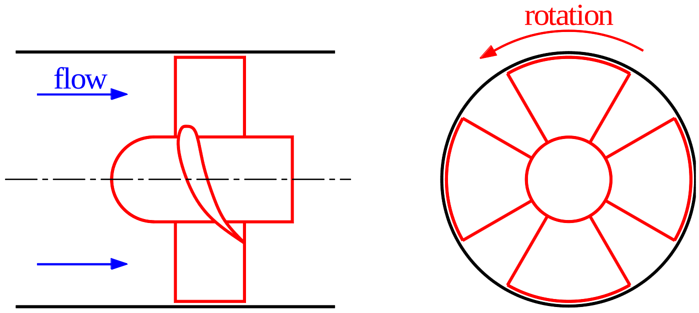


Radial Axial Mixed

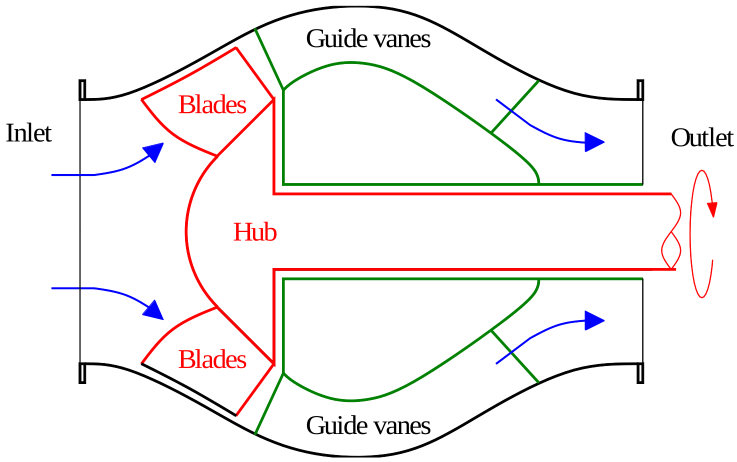
In a *centrifugal pump* flow enters along the axis and is expelled radially. (The reverse is true for a turbine.)



An *axial-flow* pump is like a propeller; the direction of the flow is unchanged after passing through the device.



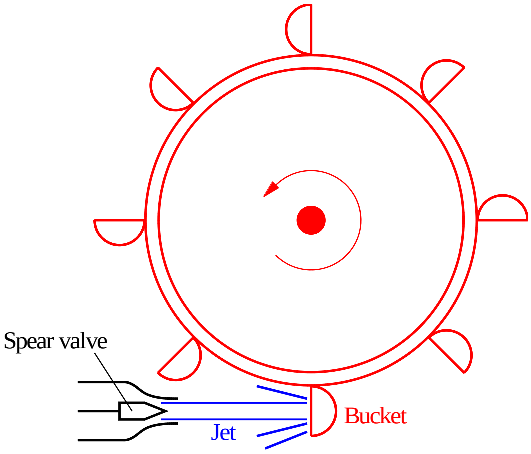
A *mixed-flow device* is a hybrid device, used for intermediate heads.



In many cases – notably in pumped-storage power stations – a device can be run as either a pump or a turbine.

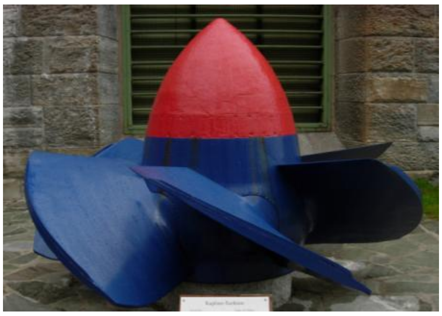
Inward-flow reaction turbine (e.g. *Francis turbine*)	↔	centrifugal pump (high head / low discharge)
Propeller turbine (e.g. *Kaplan turbine*; *windmill*)	↔	axial-flow pump (low head / high discharge)

2.4 Common Types of Turbine



Pelton wheels are impulse turbines used in hydroelectric plant where there is a very high head of water. Typically, 1 – 6 high-velocity jets of water impinge on buckets mounted around the circumference of a runner.

Francis turbines are used in many large hydropower projects (e.g the Hoover Dam), with an efficiency in excess of 90%. Such moderate- to high-head turbines are also used in *pumped-storage* power stations (e.g. Dinorwig and Ffestiniog in Wales; Foyers in Scotland), which pump water uphill during periods of low energy demand and then run the system in reverse to generate power during the day. This smooths the power demands on fossil-fuelled and nuclear power stations which are not easily brought in and out of operation. Francis turbines are like centrifugal pumps in reverse.



Kaplan turbines are axial-flow (propeller) turbines. In the Kaplan design the blade angles are adjustable to ensure efficient operation for a range of discharges.

Wells turbines were specifically developed for wave-energy applications. They have the property that they rotate in the same direction irrespective of the flow direction.

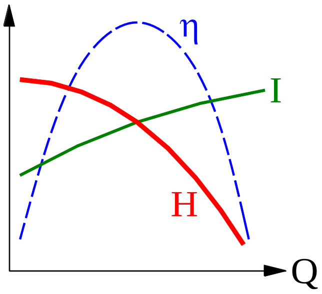
Bulb generators are large-diameter variants of the Kaplan propeller turbine, which are suitable for the low-head, high-discharge applications in tidal barrages (e.g. La Rance in France). Flow passes around the bulb, which contains the alternator.

The *Archimedes screw* has been used since ancient times to raise water. It is widely used in water treatment plants because it can accommodate submerged debris. Recently, several devices have been installed beside weirs in the north of England to run in reverse and generate power.

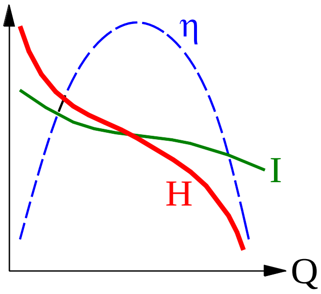
3. Pump and System Characteristics

3.1 Pump characteristics

Pump characteristics are the head (*H*), input power (*I*) and efficiency (η) as functions of discharge (*Q*). The most important is the *H* vs *Q* relationship. Typical shapes of these characteristics are sketched below for centrifugal and axial-flow pumps.



centrifugal pump



axial-flow pump

Given the pump characteristics at one rotation rate (*N*), those at different rotation rates may be determined using the hydraulic scaling laws (Section 4).

Ideally, one would like to operate the pump:
- as close as possible to the *design point* (point of maximum efficiency);
- in a region where the *H*-*Q* relationship is steep; (otherwise there are significant fluctuations in discharge for small changes in head).

3.2 System characteristics

In general the pump has to supply enough energy to:
- lift water through a certain height – the *static lift* H_s;
- overcome losses dependent on the discharge, *Q*.

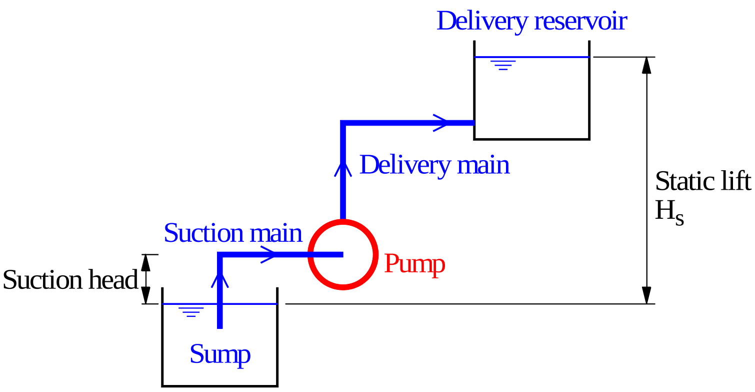
Thus the *system head* is

$$H = H_s + h_{losses}$$

Typically, losses (whether frictional or due to pipe fittings) are proportional to Q^2, so that the system characteristic is often quadratic:

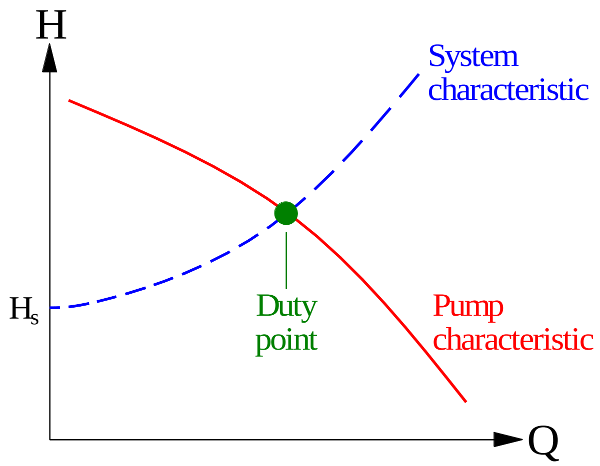
$$H = H_s + aQ^2$$

The static lift is often decomposed into the rise from sump to the level of the pump (the *suction head*, H_{s1}) and that between the pump and the delivery point (H_{s2}). The first of these is limited by the maximum suction height (approximately 10 m, corresponding to 1 atmosphere) and will be discussed later in the context of cavitation.



3.3 Finding the Duty Point

The pump operates at a *duty point* where the head supplied by the pump precisely matches the head requirements of the system at the same discharge; i.e. **where the pump and system characteristics intersect**.



Example. (Examination 2005 – part)
A water pump was tested at a rotation rate of 1500 rpm. The following data was obtained. (Q is quantity of flow, H is head of water, η is efficiency).

Q (L s^{-1})	0	10	20	30	40	50
H (m)	10.0	10.5	10.0	8.5	6.0	2.5
η	0.0	0.40	0.64	0.72	0.64	0.40

It is proposed to used this pump to draw water from an open sump to an elevation 5.5 m above. The delivery pipe is 20.0 m long and 100 mm diameter, and has a friction factor of 0.005.

If operating at 1500 rpm, find:
(a) the maximum discharge that the pump can provide;
(b) the pump efficiency at this discharge;
(c) the input power required.

Answer: (a) 37.5 L s^{-1}; (b) 0.67; (c) 3.7 kW

In practice, it is desirable to run the pump at a speed where the duty point is close to that of maximum efficiency. To do this we need to determine how the pump characteristic varies with rotation rate N – see below.

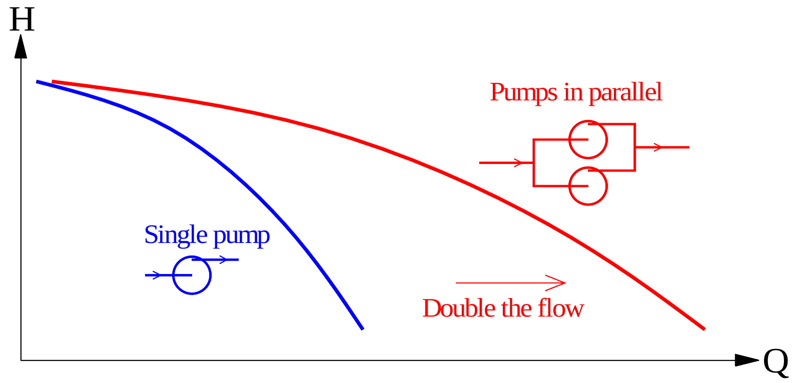
3.4 Pumps in Parallel and in Series

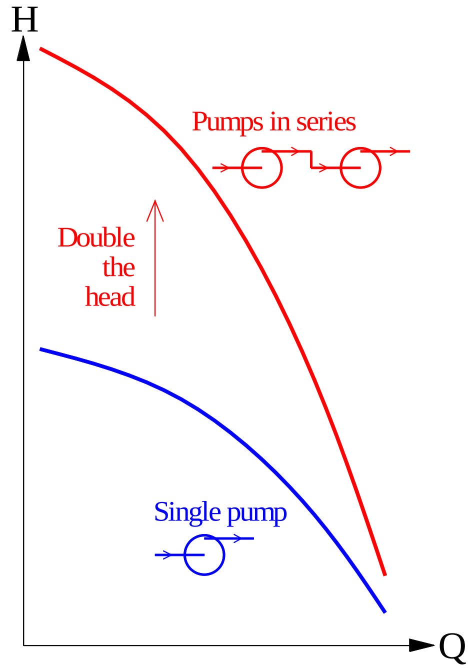
<u>Pumps in Parallel</u>

Same head: H
Add the discharges: $Q_1 + Q_2$

Advantages of pumps in parallel are:

- high capacity: permits a large total discharge;
- flexibility: pumps can be brought in and out of service if the required discharge varies;
- redundancy: pumping can continue if one is not operating due to failure or planned maintenance.





<u>Pumps in Series</u>

Same discharge: Q
Add the heads: $H_1 + H_2$

Pumps in series may be necessary to generate high heads, or provide regular "boosts" along long pipelines without large pressures at any particular point.

Example. (Examination, January 2004)
A rotodynamic pump, having the characteristics tabulated below, delivers water from a river at elevation 102 m to a reservoir with a water level of 135 m, through a 350 mm diameter cast-iron pipe. The frictional head loss in the pipeline is given by $h_f = 550\,Q^2$, where h_f is the head loss in m and Q is the discharge in m^3 s^{-1}. Minor head losses from valves and fittings amount to $50\,Q^2$ in the same units.

Q (m^3 s^{-1})	0	0.05	0.10	0.15	0.20
H (m)	60	58	52	41	25
η (%)	---	44	65	64	48

Pump characteristics: Q is discharge, H is head, η is efficiency.

(a) Calculate the discharge and head in the pipeline (at the duty point).

If the discharge is to be increased by the installation of a second identical pump:

(b) determine the unregulated discharge and head produced by connecting the pump:

(i) in parallel;

(ii) in series;

(c) determine the power demand at the duty point in the case of parallel operation.

Answer: (a) 0.137 m^3 s^{-1} and 44 m;
(b) (i) 0.185 m^3 s^{-1} and 53.5 m; (ii) 0.192 m^3 s^{-1} and 55.1 m; (c) 155 kW

4 Hydraulic Scaling

4.1 Dimensional Analysis

Provided that the mechanical efficiency is the same, the performance of a particular geometrically-similar family of pumps or turbines ("*homologous series*") may be expected to depend on:

discharge	Q	$[L^3T^{-1}]$	
pressure change	ρgH	$[ML^{-1}T^{-2}]$	
power	P	$[ML^2T^{-3}]$	(input for pumps; output for turbines)
rotor diameter	D	$[L]$	
rotation rate	N	$[T^{-1}]$	
fluid density	ρ	$[ML^{-3}]$	
fluid viscosity	μ	$[ML^{-1}T^{-1}]$	

(Rotor diameter may be replaced by any characteristic length, since geometric similarity implies that length ratios remain constant. Rotation rate is typically expressed in either rad s^{-1} or rpm.)

Since there are 7 variables and 3 independent dimensions, Buckingham's Pi Theorem yields a relationship between 4 independent groups, which may be taken as (exercise):

$$\Pi_1 = \frac{Q}{ND^3}, \qquad \Pi_2 = \frac{gH}{N^2D^2}, \qquad \Pi_3 = \frac{P}{\rho N^3 D^5}, \qquad \Pi_4 = \frac{\rho ND^2}{\mu} = \mathrm{Re}$$

For fully-turbulent flow the dependence on molecular viscosity μ and hence the Reynolds number (Π_4) vanishes. Then, for geometrically-similar pumps with different sizes (D) and rotation rates (N):

$$\left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2, \qquad \left(\frac{gH}{N^2D^2}\right)_1 = \left(\frac{gH}{N^2D^2}\right)_2, \qquad \left(\frac{P}{\rho N^3 D^5}\right)_1 = \left(\frac{P}{\rho N^3 D^5}\right)_2$$

For pumps (input power P, output power ρgQH), any one of Π_1, Π_2, Π_3 may be replaced by

$$\frac{\Pi_1\Pi_2}{\Pi_3} = \frac{\rho gQH}{P} = \eta \ (\textit{efficiency})$$

The reciprocal of this would be used for turbines.

Example.
A ¼-scale model centrifugal pump is tested under a head of 7.5 m at a speed of 500 rpm. It was found that 7.5 kW was needed to drive the model. Assuming similar mechanical efficiencies, calculate:

(a) the speed and power required by the prototype when pumping against a head of 44 m;
(b) the ratio of the discharges in the model to that in the prototype.

Answer: (a) 303 rpm and 1710 kW; (b) $Q_m/Q_p = 1/38.8 = 0.0258$

4.2 Change of Speed

For the **same** pump (i.e. same D) operating at different speeds N_1 and N_2:

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}, \qquad \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2, \qquad \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3$$

Thus,

$$Q \propto N, \qquad H \propto N^2, \qquad P \propto N^3$$

(This might be expected, since $Q \propto$ velocity, whilst $H \propto$ energy $\propto$ velocity^2).

These are called the *hydraulic scaling laws* or *affinity laws*.

Speed Scaling Laws For a Single Pump
$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}, \qquad \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2, \qquad \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3, \qquad \eta_1 = \eta_2$

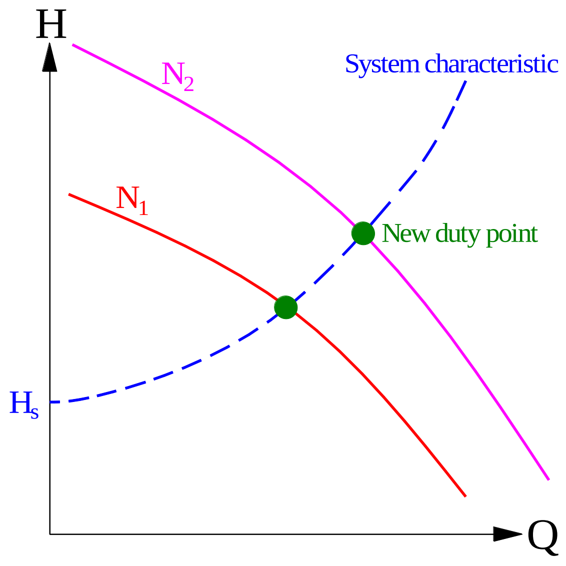
Given pump characteristics at one speed one can use the hydraulic scaling laws to deduce characteristics at a different speed.

4.2.1 Finding the Duty Point at a New Pump Speed

Scale each (Q,H) pair on the original characteristic at speed N_1 to get the new characteristic at speed N_2; i.e.

$$Q_2 = \left(\frac{N_2}{N_1}\right) Q_1, \qquad H_2 = \left(\frac{N_2}{N_1}\right)^2 H_1$$

Where this scaled characteristic intercepts the system curve gives the new duty point.

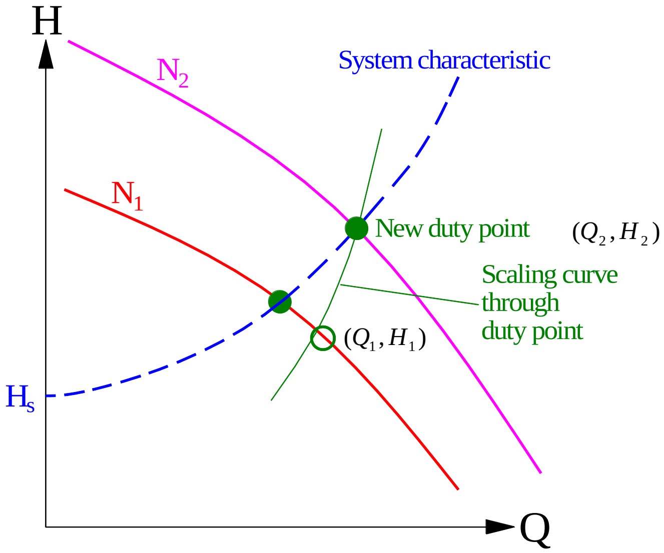


4.2.2 Finding the Pump Speed For a Given Duty Point (*Harder*)

To find the pump speed for a given discharge or head plot a hydraulic-scaling curve back from the required duty point (Q_2, H_2) on the system curve, at **unknown** speed N_2:

$$\frac{H}{H_2} = \left(\frac{Q}{Q_2}\right)^2$$

Very important: the hydraulic scaling curve is not the same as the system curve.



Where the hydraulic scaling curve cuts the original characteristic gives a scaled duty point (Q_1, H_1) and thence the ratio of pump speeds from either the ratio of discharges or the ratio of heads:

$$\frac{N_2}{N_1} = \frac{Q_2}{Q_1} \qquad \text{or} \qquad \left(\frac{N_2}{N_1}\right)^2 = \frac{H_2}{H_1}$$

Example.

Water from a well is pumped by a centrifugal pump which delivers water to a reservoir in which the water level is 15.0 m above that in the sump. When the pump speed is 1200 rpm its pipework has the following characteristics:

Pipework characteristics:

Discharge (L s^{-1}):	20	30	40	50	60
Head loss in pipework (m):	1.38	3.11	5.52	8.63	12.40

Pump characteristics:

Discharge (L s^{-1}):	0	10	20	30	40
Head (m):	22.0	21.5	20.4	19.0	17.4

A variable-speed motor drives the pump.

(a) Plot the graphs of the system and pump characteristics and determine the discharge at a speed of 1200 rpm.

(b) Find the pump speed in rpm if the discharge is increased to 40 L s^{-1}.

Answer: (a) 32 L s^{-1}; (b) 1290 rpm

4.3 Specific speed

The *specific speed* (or *type number*) is a guide to the **type** of pump or turbine required for a particular role.

4.3.1 Specific Speed for Pumps

The *specific speed*, N_s, is the rotational speed needed to discharge 1 unit of flow against 1 unit of head. (For what "unit" means in this instance, see below.)

For a given pump, the hydraulic scaling laws give

$$\Pi_1 = \frac{Q}{ND^3} = constant\,, \qquad \Pi_2 = \frac{gH}{N^2 D^2} = constant$$

Eliminating D (and choosing an exponent that will make the combination proportional to N):

$$\left(\frac{\Pi_1^2}{\Pi_2^3}\right)^{1/4} = \frac{Q^{1/2} N}{(gH)^{3/4}}$$

or, since g is constant, then at any given (e.g. maximum) efficiency:

$$\frac{Q^{1/2} N}{H^{3/4}} = (dimensional)\ constant$$

The constant is the *specific speed*, N_s, when Q and H in specified units (see below) are numerically equal to 1:

Specific speed (pump)

$$N_S = \frac{Q^{1/2} N}{H^{3/4}}$$

Notes.

- The specific speed is a single value calculated at the normal operating point (i.e. Q and H at the maximum efficiency point for the anticipated rotation rate N).

- With the commonest definition (in the UK and Europe), N is in rpm, Q in $\mathrm{m^3\ s^{-1}}$, H in m, but this is far from universal, so be careful.

- In principle, the units of N_s are the same as those of N, which doesn't look correct from the definition but only because that has been shortened from

$$\frac{(1\,\mathrm{m^3\ s^{-1}})^{1/2} \times N_s}{(1\,\mathrm{m})^{3/4}} = \frac{Q^{1/2} N}{H^{3/4}}$$

- Because of the omission of g the definition of N_s depends on the units of Q and H. A less-common (but, IMHO, more mathematically correct) quantity is the *dimensionless specific speed* K_n given by

$$K_n = \frac{Q^{1/2} N}{(gH)^{3/4}}$$

  If the time units are consistent then K_n has the same angular units as N (rev or rad).

- High specific speed ↔ large discharge / small head (axial-flow device).

Low specific speed ↔ small discharge / large head (centrifugal device).
Approximate ranges of N_s are (from Hamill, 2011):

Type	N_s	
Radial (centrifugal)	10 – 70	large head
Mixed flow	70 – 170	
Axial	> 110	small head

Example.
A pump is needed to operate at 3000 rpm (i.e. 50 Hz) with a head of 6 m and a discharge of 0.2 $m^3 s^{-1}$. By calculating the specific speed, determine what sort of pump is required.

Answer: N_s = 350; axial-flow pump

4.3.2 Specific Speed for Turbines

For turbines the output power, P, is more important than the discharge, Q. The relevant dimensionless groups are

$$\Pi_2 = \frac{gH}{N^2 D^2}, \qquad \Pi_3 = \frac{P}{\rho N^3 D^5}$$

Eliminating D,

$$\left(\frac{\Pi_3^2}{\Pi_2^5}\right)^{1/4} = \frac{(P/\rho)^{1/2} N}{(gH)^{5/4}}$$

or, since ρ and g are usually taken as constant (there does seem to be a presumption that turbines are always operating in fresh water) then at any given efficiency:

$$\frac{P^{1/2} N}{H^{5/4}} = (dimensional)\ constant$$

The *specific speed* of a **turbine**, N_s, is the rotational speed needed to develop 1 unit of power for a head of 1 unit. (For what "unit" means in this instance, see below.)

Specific speed (turbine)

$$N_S = \frac{P^{1/2} N}{H^{5/4}}$$

Notes.

- With the commonest definition (in the UK and Europe), N is in rpm, P in kW (**note**), H in m, but, again, this is not a universal convention. As with pumps, the units of N_s are the same as those of N.

- As with pumps, a less-commonly-used but mathematically more acceptable quantity is the *dimensionless specific speed* K_n, which retains the ρ and g dependence:

$$K_n = \frac{(P/\rho)^{1/2} N}{(gH)^{5/4}}$$

K_n has the angular units of N (revs or radians) – see Massey (2005).

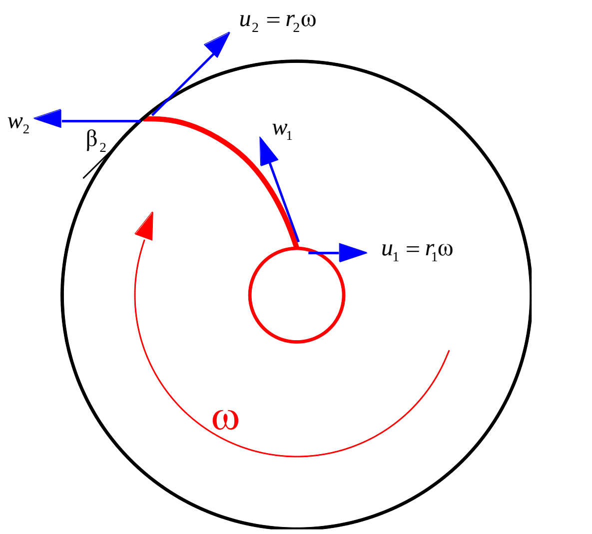
- High specific speed ↔ small head (axial-flow device)
  Low specific speed ↔ large head (centrifugal or impulse device).

  Approximate ranges are (from Hamill, 2001):

Type	N_s	
Pelton wheel (impulse)	12 – 60	very large head
Francis turbine (radial-flow)	60 – 500	large head
Kaplan turbine (axial-flow)	280 – 800	small head

5. Mechanics of Rotodynamic Devices

5.1 Centrifugal Pump



Fluid enters at the *eye* of the impeller and flows outward. As it does so it picks up the tangential velocity of the impeller *vanes* (or *blades*) which increases linearly with radius ($u = r\omega$). At exit the fluid is expelled nearly tangentially at high velocity (with kinetic energy subsequently converted to pressure energy in the expanding volute). At the same time fluid is sucked in through the inlet to take its place and maintain continuous flow.

The analysis makes use of rotational dynamics:

power = *torque* × *angular velocity*

torque = *rate of change of angular momentum*

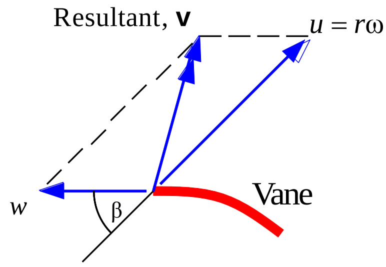
where angular momentum is given by "(tangential) momentum × radius".

The *absolute* velocity of the fluid is the vector sum of:

impeller velocity (tangential)

+

velocity relative to the impeller (parallel to the vanes)



Write:

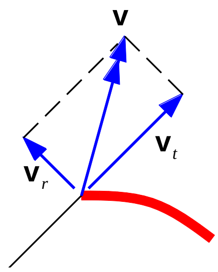
u for the impeller velocity ($u = r\omega$)

w for the fluid velocity relative to the impeller

v = **u** + **w** for the absolute velocity

The radial component of absolute velocity is determined primarily by the flow rate:

$$v_r = \frac{Q}{A}$$



where A is the effective outlet area. The tangential part (also called the *whirl* velocity) is a combination of impeller speed ($u = r\omega$) and tangential component relative to the vanes:

$$v_t = u - w\cos\beta$$

Only v_t contributes to the angular momentum.

With subscripts 1 and 2 denoting inlet and outlet respectively,

$$\textit{torque}\ T = \rho Q(v_{t2}r_2 - v_{t1}r_1)$$

$$\textit{power} = T\omega \quad = \rho Q(v_{t2}r_2\omega - v_{t1}r_1\omega)$$

But head $H = \dfrac{power}{\rho g Q}$, whilst $r\omega = u$. Hence:

Euler's turbomachinery equation:

$$H = \frac{1}{g}(v_{t2}u_2 - v_{t1}u_1)$$

The pump is usually designed so that the initial angular momentum is small; i.e. $v_{t1} \approx 0$. Then

$$H = \frac{1}{g} v_{t2} u_2$$

Effect of Blade Angle

Because in the frame of the impeller the fluid leaves the blades in a direction parallel to their surface, forward-facing blades would be expected to increase the whirl velocity v_t whilst backward-facing blades would diminish it.

Tangential and radial components of velocity:

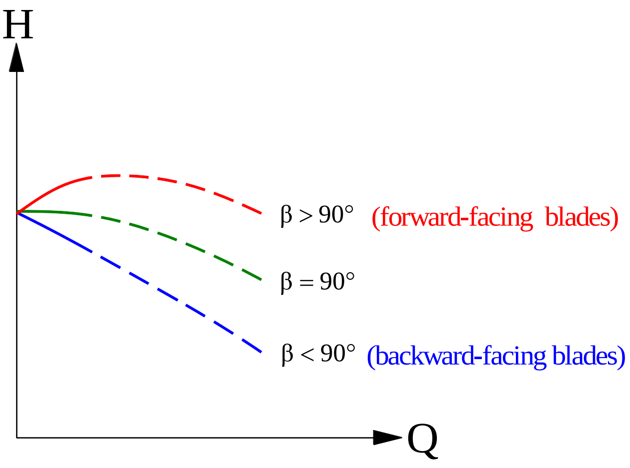
$$v_r = w \sin\beta = \frac{Q}{A}, \qquad v_t = u - w\cos\beta,$$

Eliminating w:

$$v_t = u - \frac{Q}{A}\cot\beta$$

Hence, if inlet whirl can be ignored,

$$H = \frac{u_2}{g}(u_2 - \frac{Q}{A}\cot\beta)$$



This is of the form $H = a - bQ$, where

H initially *decreases* with *Q* for *backward-facing blades* ($\beta < 90°$; $\cot\beta > 0$)

H initially *increases* with *Q* for *forward-facing blades* ($\beta > 90°$; $\cot\beta < 0$)

This gives rise to the typical pump characteristics shown. Of the two, the former (backward-facing blades) is usually preferred because, although forward-facing blades might be expected to increase whirl velocity and hence output head, the shape of the characteristic is such that small changes in head cause large changes in discharge and the pump tends to "hunt" for its operating point (*pump surge*).

Non-Ideal Behaviour

The above is a very ideal analysis. There are many sources of losses. These include:

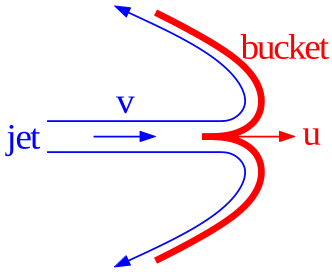
- leakage back from the high-pressure volute to the low-pressure impeller eye;
- frictional losses;
- "shock" or flow-separation losses at entry;
- non-uniform flow at inlet and outlet of the impeller;
- cavitation (when the inlet pressure is small).

Example.
A centrifugal pump is required to provide a head of 40 m. The impeller has outlet diameter 0.5 m and inlet diameter 0.25 m and rotates at 1500 rpm. The flow approaches the impeller radially at 10 m s^{-1} and the radial velocity falls off as the reciprocal of the radius. Calculate the required vane angle at the outlet of the impeller.

Answer: 9.7°

5.2 Pelton Wheel

A Pelton wheel is the most common type of impulse turbine. One or more jets of water impinge on buckets arranged around a turbine runner. The deflection of water changes its momentum and imparts a force to rotate the runner.



The power (per jet) P is given by:

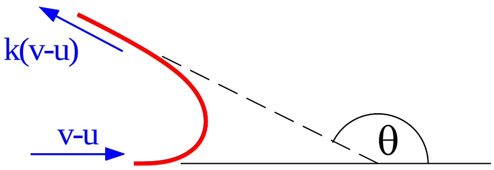
$$power = force\ (on\ bucket) \times velocity\ (of\ bucket)$$

Force F on the bucket is equal and opposite to that on the jet; by the momentum principle:

$$force\ (on\ fluid) = mass\ flux \times change\ in\ velocity$$

Because the absolute velocity of water leaving the bucket is the vector resultant of the runner velocity (u) and the velocity relative to the bucket, the *change* in velocity is most easily established in the frame of reference of the moving bucket.

Assuming that the relative velocity *leaving* the buckets is k times the relative velocity of approach, $v - u$ (where k is slightly less than 1.0 due to friction):



$$change\ in\ x\text{-}velocity = k(v-u)\cos\theta - (v-u)$$
$$= -(v-u)(1-k\cos\theta)$$

where θ is the total angle turned (here, greater than 90°). The maximum force would be obtained if the flow was turned through 180°, but the necessity of deflecting it clear of the next bucket means that θ is typically 165°.

From the momentum principle:

$$-F = -\rho Q(v-u)(1-k\cos\theta)$$

The power transferred in each jet is then

$$P = Fu = \rho Q(v-u)u(1-k\cos\theta)$$

The velocity part of the power may be written

$$(v-u)u = \tfrac{1}{4}v^2 - (\tfrac{1}{2}v - u)^2$$

Hence, for a given jet (Q and v), the power is a maximum when the runner speed u is such that $u = \tfrac{1}{2}v$, or the runner speed is half the jet speed. (At this point the *absolute* velocity leaving the runner at 180° would be 0 if $k = 1$, corresponding to a case where all the kinetic energy of the fluid is transferred to the runner.) In practice, the runner speed u is often fixed by the need to synchronise the generator to the electricity grid, so it is usually the jet which is controlled (by a spear valve). Because of other losses the speed ratio is usually slightly less than ½, a typical value being 0.46.

The jet velocity is given by Bernoulli's equation, with a correction for non-ideal flow:

$$v = c_v\sqrt{2gH}$$

where H is the head upstream of the nozzle (= gross head minus any losses in the pipeline) and c_v is an orifice coefficient with typical values in the range 0.97– 0.99.

Example.
In a Pelton wheel, 6 jets of water, each with a diameter of 75 mm and carrying a discharge of 0.15 m^3 s^{-1} impinge on buckets arranged around a 1.5 m diameter Pelton wheel rotating at 180 rpm. The water is turned through 165° by each bucket and leaves with 90% of the original relative velocity. Neglecting mechanical and electrical losses within the turbine, calculate the power output.

Answer: 471 kW

6. Cavitation

Cavitation is the formation, growth and rapid collapse of vapour bubbles in flowing liquids.

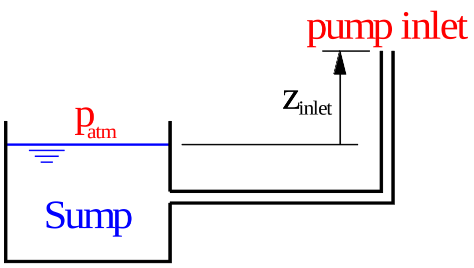
Bubbles form at low pressures when the absolute pressure drops to the vapour pressure and the liquid spontaneously boils. (Bubbles may also arise from dissolved gases coming out of solution.) When the bubbles are swept into higher-pressure regions they collapse very rapidly, with large radial velocities and enormous short-term pressures. The problem is particularly acute at solid surfaces.

Cavitation may cause performance loss, vibration, noise, surface pitting and, occasionally, major structural damage. Besides the inlet to pumps the phenomenon is prevalent in marine-current turbines, ship and submarine propellers and on reservoir spillways.

The best way of preventing cavitation in a pump is to ensure that the inlet (suction) pressure is not too low. The *net positive suction head* (NPSH) is the difference between the pressure head at inlet and that corresponding to the vapour pressure:

$$\text{NPSH} = \frac{p_{inlet} - p_{cav}}{\rho g}$$

The net positive suction head must be kept well above zero to allow for further pressure loss in the impeller.



The inlet pressure may be determined from Bernoulli:

$$H_{pump\,inlet} = H_{sump} - head\ loss$$

$$\Rightarrow \qquad \frac{p_{inlet}}{\rho g} + z_{inlet} + \frac{V^2}{2g} = \frac{p_{atm}}{\rho g} - h_f$$

Hence,

$$\frac{p_{inlet}}{\rho g} = \frac{p_{atm}}{\rho g} - z_{inlet} - \frac{V^2}{2g} - h_f$$

To avoid cavitation one should aim to keep p_{inlet} as large as possible by:
- keeping z_{inlet} small or, better still, negative (i.e. *below* the level of water in the sump;
- keeping V small (large-diameter pipes);
- keeping h_f small (short, large-diameter pipes).

The first also assists in pump *priming*.