TOPIC 5: UNSTEADY FLOW IN PIPES

OBJECTIVES

1. Recognise the potential for large **pressure transients** when pipe flow is stopped abruptly.

2. Predict pressure rise and the speed of **water-hammer waves** in rigid and non-rigid pipes.

3. Predict the **time series** of events at any point in a pipeline following sudden closure.

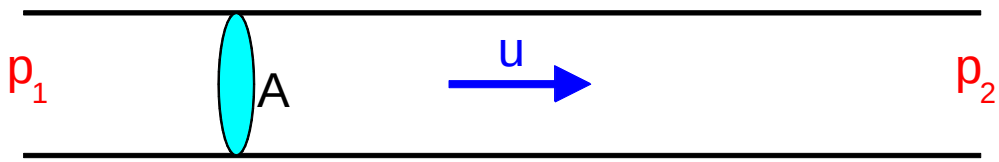
4. Derive and use the **unsteady incompressible pipe-flow equation** to analyse the pressure-relieving behaviour of **surge tanks** and **pump bypasses**.

VALVE CLOSURE

In **incompressible** flow all fluid in a pipe reacts **simultaneously** to valve opening or closing.

To accelerate or decelerate **all** the fluid together requires **very large pressures**.

These large, but short-lived, pressures are sometimes called **surge**.



force = mass × acceleration

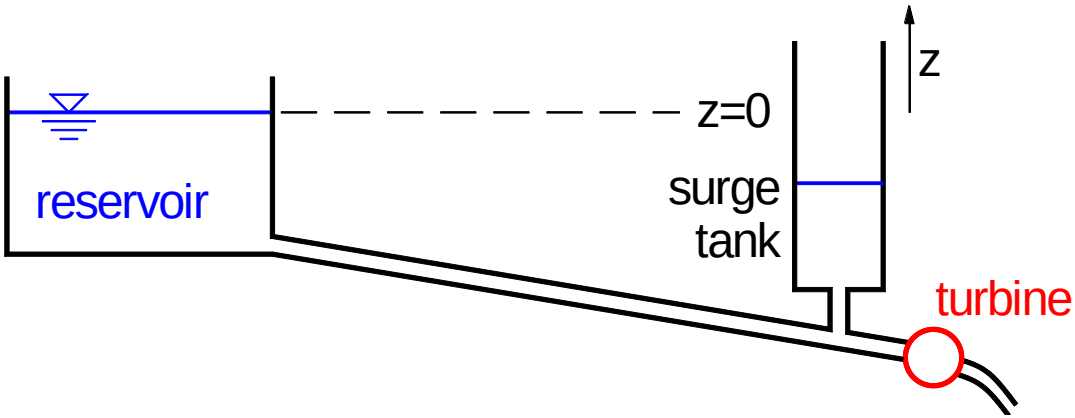
$$(p_1 - p_2)A = (\rho A L)\frac{0 - u}{t}$$

$$\Delta p = \frac{\rho L u}{t}$$

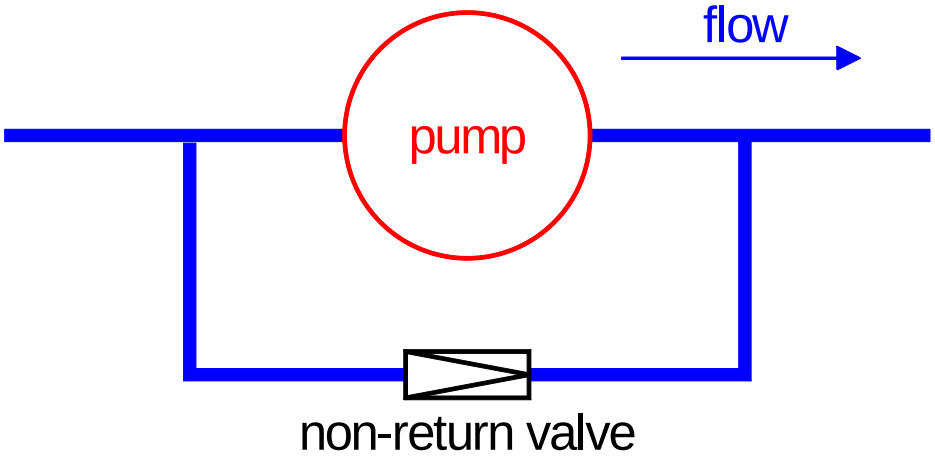
e.g. $\rho = 1000\ \mathrm{kg\ m^{-3}}$, $L = 1\ \mathrm{km}$, $u = 1\ \mathrm{m\ s^{-1}}$, $t = 1\ \mathrm{s}$ $\Rightarrow$ $\boldsymbol{\Delta p = 10^6\ \mathrm{Pa}\ (10\ \mathrm{atm})}$

METHODS FOR LIMITING PRESSURE TRANSIENTS

Surge tank



Pump bypass



EXAMPLE SHEET

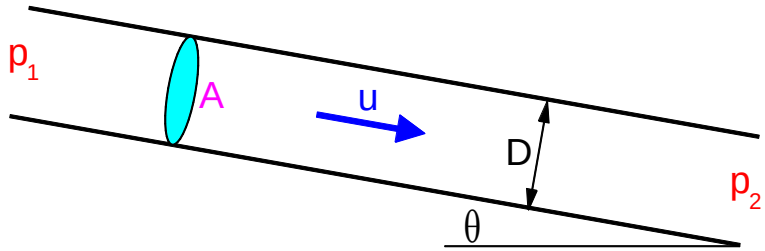
A horizontal pipe is 1500 m long and 0.15 m in diameter and has a friction factor $\lambda = 0.02$. The pipe discharges to atmosphere and its intake is 20 m below the water surface of the reservoir. The control valve at exit can be closed completely in 4 s and gives a uniform deceleration of the water in the pipe. Assuming incompressible flow, calculate:

(a) the velocity in the pipe prior to closure;

(b) the excess head due to surge just upstream of the valve;

(c) the excess head due to surge at 500 m upstream of the valve.

UNSTEADY INCOMPRESSIBLE FLOW



mass × acceleration = force

$$m\frac{\mathrm{d}u}{\mathrm{d}t} = \underbrace{p_1A - p_2A}_{\text{net pressure force}} + \underbrace{mg\sin\theta}_{\text{streamwise component of weight}} - \underbrace{\tau_w \times \pi DL}_{\text{wall friction}}$$

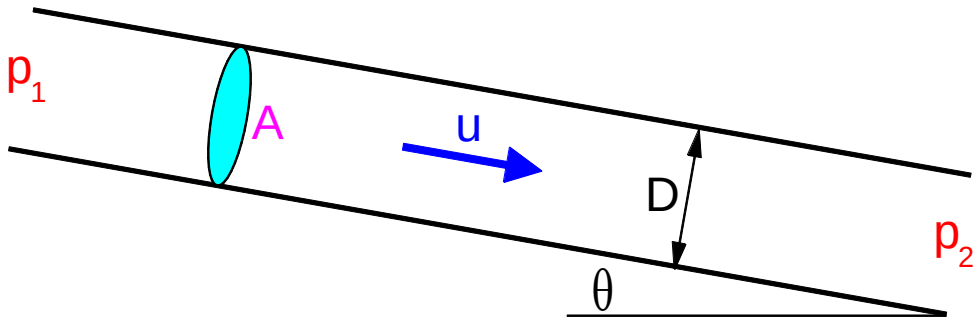
$$(\rho AL)\frac{\mathrm{d}u}{\mathrm{d}t} = (p_1 - p_2)A + (\rho AL)g\frac{z_1 - z_2}{L} - \underbrace{\frac{\lambda}{4} \times \frac{1}{2}\rho u|u| \times \pi DL}_{\text{accounting for flow direction}}$$

$$\frac{L}{g}\frac{\mathrm{d}u}{\mathrm{d}t} = \underbrace{\frac{p_1^* - p_2^*}{\rho g}}_{\text{piezometric head}} - \underbrace{\lambda\frac{L}{D}\frac{u|u|}{2g}}_{\text{frictional head loss}}$$

$$\frac{L}{g}\frac{\mathrm{d}u}{\mathrm{d}t} = \underbrace{H_1 - H_2}_{\text{head difference}} - \underbrace{\lambda\frac{L}{D}\frac{u|u|}{2g}}_{\text{frictional head loss}}$$

unsteady pipe-flow equation

UNSTEADY PIPE-FLOW EQUATION

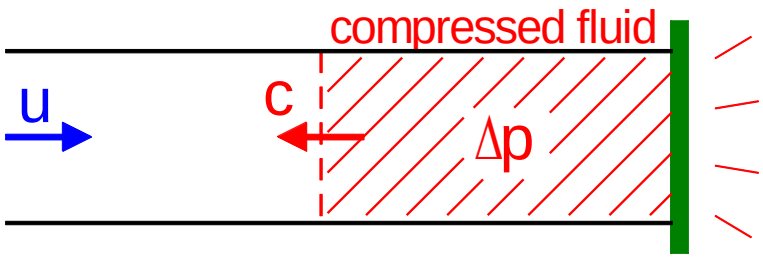


$$\frac{L}{g}\frac{\mathrm{d}u}{\mathrm{d}t} = \underbrace{H_1 - H_2}_{\text{head difference}} - \underbrace{\lambda\frac{L}{D}\frac{u|u|}{2g}}_{\text{frictional head loss}}$$

For **steady** flow ($\mathrm{d}u/\mathrm{d}t = 0$) this reduces to the familiar $h = \lambda\frac{L}{D}\frac{u^2}{2g}$

This equation only holds provided the closure is slow enough for the flow to be **incompressible** (pressure rises not sufficient to significantly change density)

UNSTEADY COMPRESSIBLE FLOW



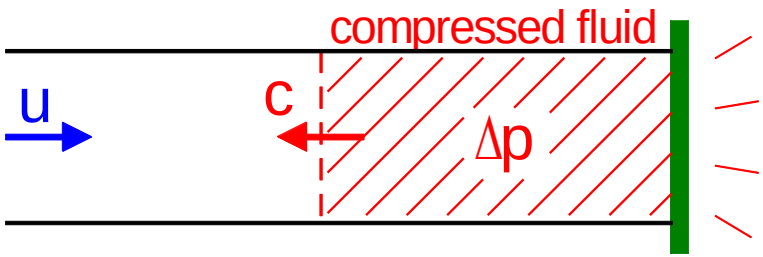
If the valve is closed very rapidly:

- there is a very large increase in pressure
- the fluid next to the valve is **compressed**;
- a large positive pressure pulse (**shock**) propagates back at speed c;
- the fluid upstream continues to move at speed u until the shock has passed.

Propagation of large-amplitude pressure waves in pipes is called **water hammer**.

A similar large **negative** pressure would occur if the valve was **opened** rapidly.

WATER HAMMER



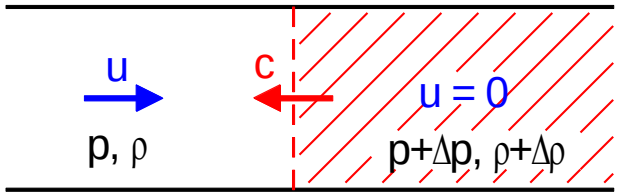
We need to know:

- **speed** of water-hammer waves, c;
- **pressure rise**, Δp;
- conditions for the **rapid-closure assumption** to be valid.

Remember:

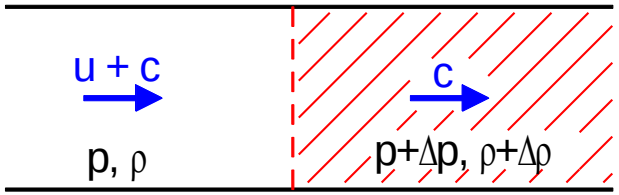
- Incompressible flow (**slow closure**): all the fluid in the pipe feels the pressure change immediately.
- Compressible flow (**rapid closure**): fluid feels the pressure change only after the front has passed.

WATER-HAMMER WAVES:
1. Rigid Pipes



1. Compressibility:

$$\Delta p = K\frac{\Delta\rho}{\rho}$$



2. Continuity:

$$\rho(c+u)A = (\rho+\Delta\rho)cA$$

$$(1+\frac{u}{c}) = (1+\frac{\Delta\rho}{\rho})$$

$$\frac{u}{c} = \frac{\Delta\rho}{\rho}$$

3. Momentum:

$$pA - (p+\Delta p)A = \rho(c+u)A \times (-u)$$

$$\Delta p = \rho(c+u)u$$

$$\Delta p = \rho c u$$

$$\rho c u = K\frac{u}{c}$$

$$c^2 = \frac{K}{\rho}$$

WATER-HAMMER WAVES:
1. Rigid Pipes

Pressure wave speed: $$c = \sqrt{\frac{K}{\rho}}$$

Pressure rise: $$\Delta p = \rho c u$$

EXAMPLE

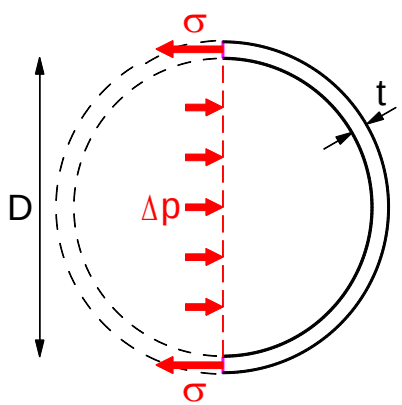
Water (density 1000 kg m^{-3}, bulk modulus 2.2 GPa) is flowing at 0.5 m s^{-1} in a pipe when the flow is suddenly halted. Assuming that the walls of the pipe are sufficiently thick for it to be approximated as rigid find:

(a) the speed of water hammer waves;

(b) the pressure rise.

WATER-HAMMER WAVES:
2. Non-Rigid Pipes

Expansion of the pipe absorbs some of the pressure rise.



Hoop stress balances internal pressure:

$$2\sigma t = \Delta p . D \qquad \sigma = E\frac{\Delta D}{D}$$

$$\frac{\Delta D}{D} = \frac{D}{2Et}\Delta p$$

$$A = \frac{\pi D^2}{4} \quad \Rightarrow \quad \Delta A = \frac{\mathrm{d}A}{\mathrm{d}D}\Delta D$$

$$\Rightarrow \quad \Delta A = \frac{2\pi D}{4}\Delta D$$

$$\Rightarrow \quad \frac{\Delta A}{A} = 2\frac{\Delta D}{D}$$

$$\frac{\Delta A}{A} = \frac{D}{Et}\Delta p$$

WATER-HAMMER WAVES:
2. Non-Rigid Pipes

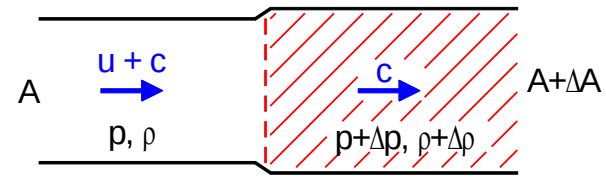
Continuity (frame of shock):

$$\rho(c+u)A = (\rho+\Delta\rho)c(A+\Delta A)$$

$$(1+\frac{u}{c}) = (1+\frac{\Delta\rho}{\rho})(1+\frac{\Delta A}{A})$$

$$= 1+\frac{\Delta\rho}{\rho}+\frac{\Delta A}{A}+\cdots$$

$$\frac{u}{c} = \frac{\Delta\rho}{\rho}+\frac{\Delta A}{A}$$



Momentum: $\Delta p = \rho c u$

Compressibility: $\Delta p = K\frac{\Delta\rho}{\rho}$

Elasticity: $\frac{\Delta A}{A} = \frac{D}{Et}\Delta p$

$$\frac{\Delta p}{\rho c^2} = \frac{\Delta p}{K}+\frac{D\Delta p}{Et}$$

$$\frac{1}{c^2} = \rho\left(\frac{1}{K}+\frac{D}{Et}\right)$$

$$\frac{1}{c^2} = \rho\left(\frac{1}{K'}\right)$$

WATER-HAMMER WAVES:
2. Non-Rigid Pipes

Pressure wave speed:

$$c = \sqrt{\frac{K'}{\rho}}$$

$$\frac{1}{K'} = \frac{1}{K} + \frac{D}{Et}$$

fluid pipe

elastic properties

Pressure rise:

$$\Delta p = \rho c u$$

EXAMPLE

Water (density 1000 kg m^{-3}, bulk modulus 2.2 GPa) is flowing at 0.5 m s^{-1} in a pipe when the flow is suddenly halted. Find the speed of water hammer waves and pressure rise for pipes of internal diameter $D = 200\ \mathrm{mm}$ and wall thickness 5 mm made of:

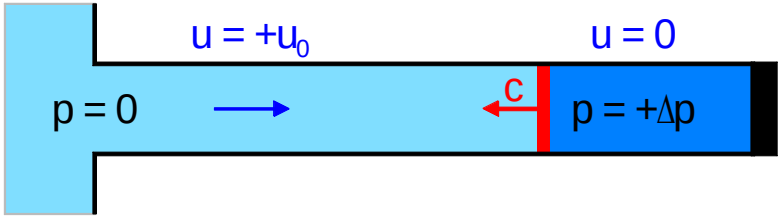
(a) steel ($E = 210\ \mathrm{GPa}$);

(b) PVC ($E = 2.6\ \mathrm{GPa}$).

TIME SERIES OF EVENTS FOLLOWING SUDDEN CLOSURE

$$\Delta t = \frac{L}{c}$$

$0 < t < \Delta t$

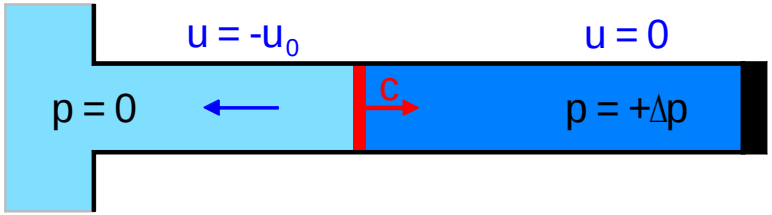


Compression

$t = \Delta t$; all fluid in pipe:

- at rest;
- at high pressure.

$\Delta t < t < 2\Delta t$

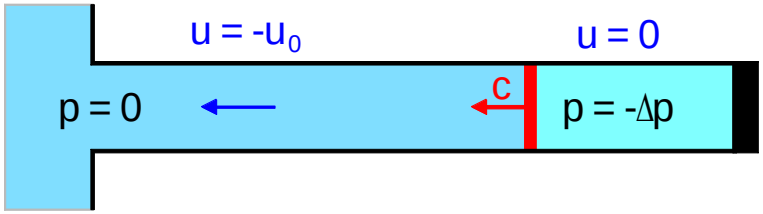


Decompression

$t = 2\Delta t$; all fluid in pipe:

- at velocity $-u_0$;
- at pressure 0.

$2\Delta t < t < 3\Delta t$

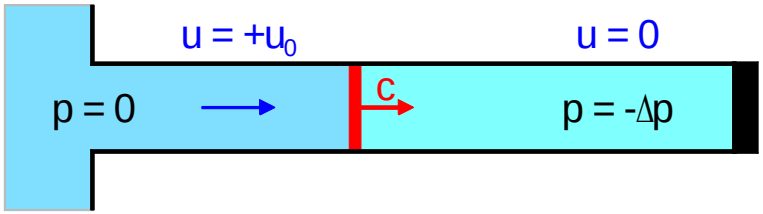


Expansion

$t = 3\Delta t$; all fluid in pipe:

- at rest;
- at low pressure.

$3\Delta t < t < 4\Delta t$

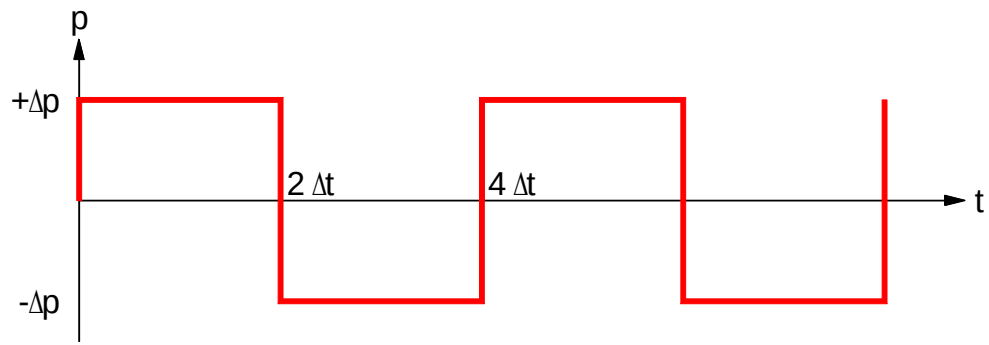


Recovery

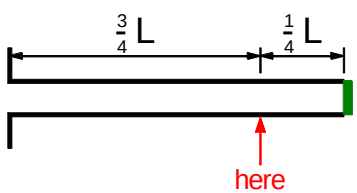
$$\text{Full cycle} = 4\Delta t = 4\frac{L}{c}$$

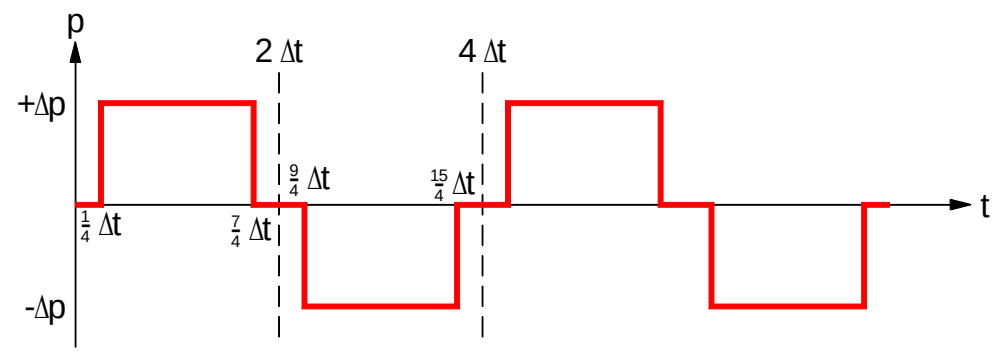
PRESSURE VARIATION

At the valve:



1/4 of the way back:





EXAMPLE

A steel pipeline of length 1155 m discharges water at velocity 2 m s^{-1} to atmosphere through a valve. The pipe has diameter 500 mm and wall thickness 10 mm. The bulk modulus of water is 2.0 GPa and the Young's modulus of the pipe material is 200 Gpa.

If a sudden closure of the valve occurs,

(a) determine the speed of water hammer waves;

(b) show pressure variations in time at the points immediately next to the valve and 866 m upstream of the valve.

Neglect friction in the pipe.

SLOW OR RAPID CLOSURE?

Use the rapid-closure assumption if the valve is closed in less time than it takes the pressure wave to travel along the pipe and back ($2L/c$).

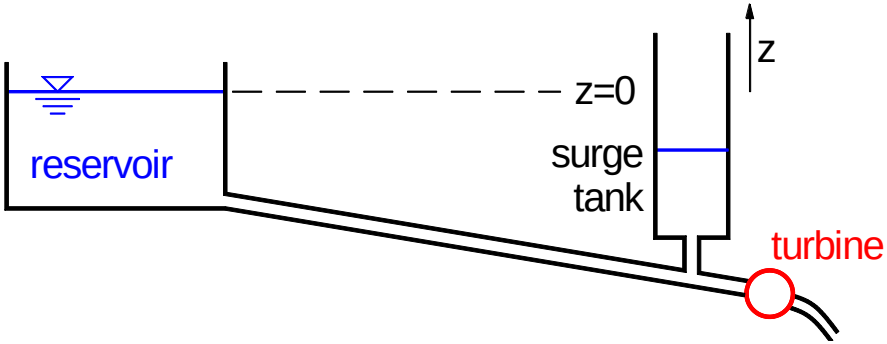
Rapid closure $\Leftrightarrow$ $t_{\text{closure}} \ll 2\frac{L}{c}$ (use **water-hammer** theory)

Slow closure $\Leftrightarrow$ $t_{\text{closure}} \gg 2\frac{L}{c}$ (use **incompressible-flow** theory)

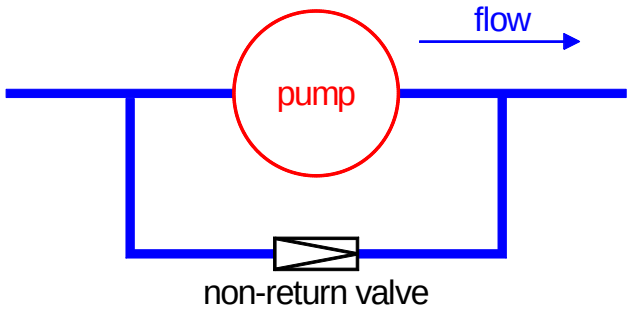
For closure times near $2L/c$, use the **method of characteristics**.

REDUCING PRESSURE TRANSIENTS

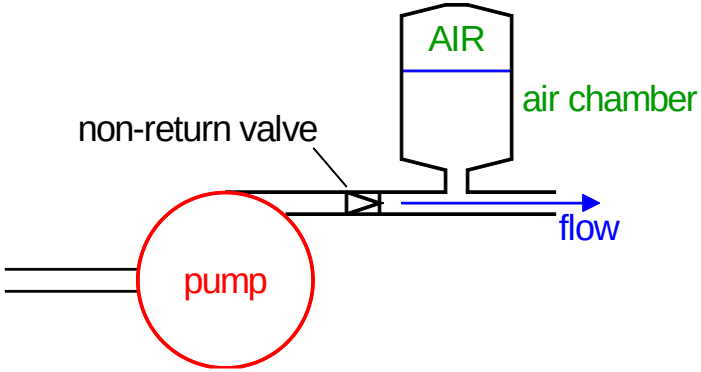
Surge tank / surge tower



Pump bypass



Pressure-relief valve



UNSTEADY PIPE-FLOW EQUATION

Where pressure change is moderate, use the unsteady pipe-flow equation:

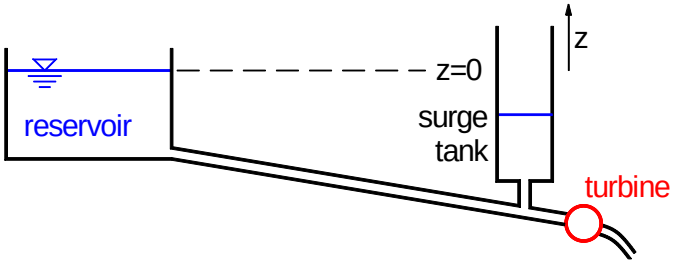
$$\frac{L}{g}\frac{\mathrm{d}u}{\mathrm{d}t} = H_1 - H_2 - \underbrace{c\frac{u|u|}{2g}}_{losses}$$

- For **variable** head difference, usually solve numerically
- For **fixed** head difference, write in the form

$$\frac{\mathrm{d}u}{\mathrm{d}t} = a - bu^2$$

and rearrange as a **standard integral**

SURGE TANK



Momentum: $$\frac{L}{g}\frac{\mathrm{d}u}{\mathrm{d}t} = -z - \lambda\frac{L}{D}\frac{u|u|}{2g}$$

Continuity: $$A_{ws}\frac{\mathrm{d}z}{\mathrm{d}t} = uA - Q$$

Initial boundary conditions (= steady-flow conditions): $$u_0 = \frac{Q_0}{A} \qquad z_0 = -\lambda\frac{L}{D}\frac{u_0^2}{2g}$$

Special case: **no friction** ($\lambda = 0$)

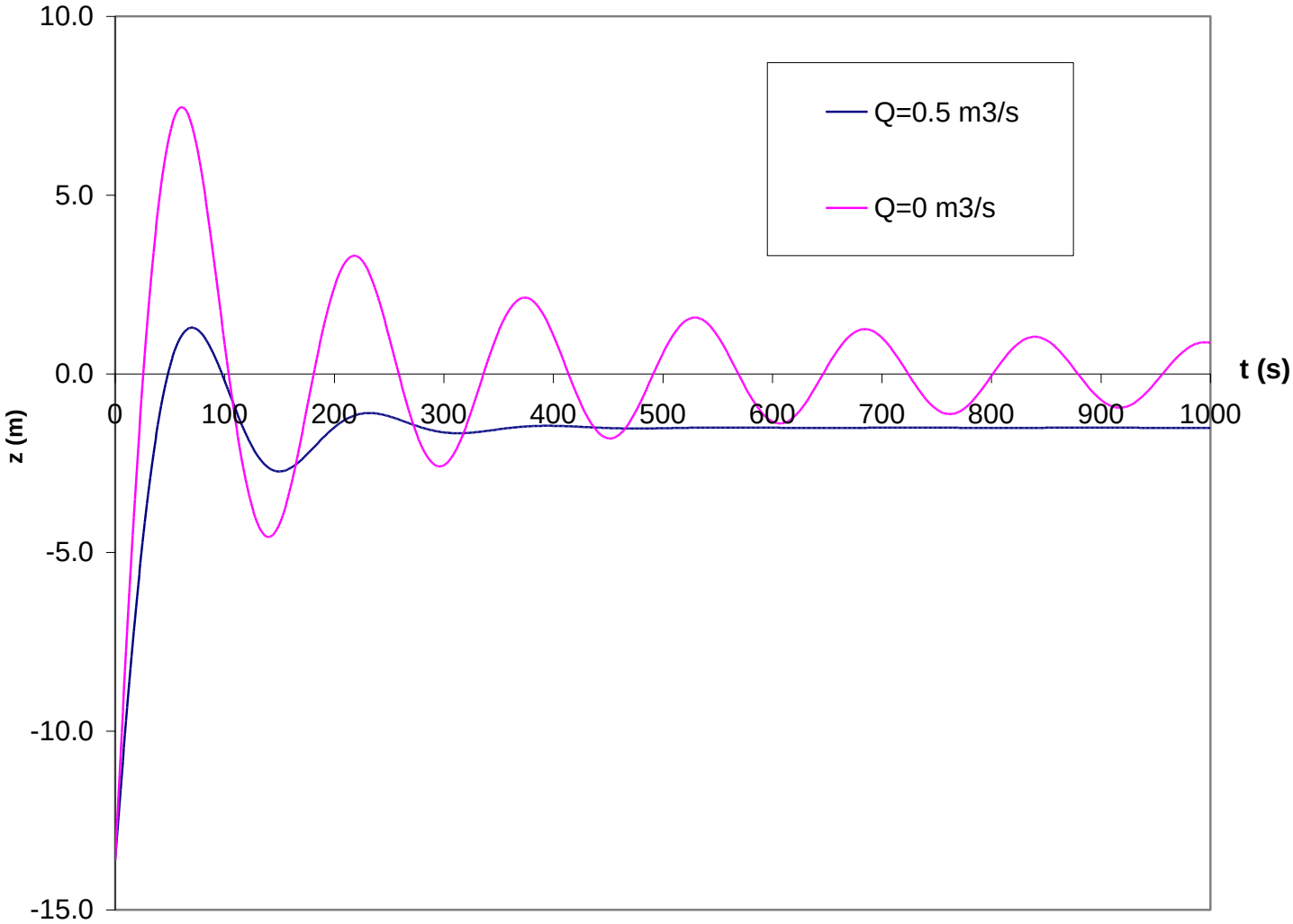
$$\left.\begin{aligned} \frac{L}{g}\frac{\mathrm{d}u}{\mathrm{d}t} &= -z \\ A_{ws}\frac{\mathrm{d}z}{\mathrm{d}t} &= uA - Q \end{aligned}\right\} \Rightarrow \frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -\left(\frac{g}{L}\frac{A}{A_s}\right)z$$

Simple harmonic motion: $$\frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -\omega^2 z \qquad \omega = \sqrt{\frac{g}{L}\frac{A}{A_s}}$$

Period: $$T = \frac{2\pi}{\omega}$$

Amplitude: $$z_{\max} = \frac{Q_0 - Q}{A_s\omega}$$

SURGE TANK: NUMERICAL SOLUTION



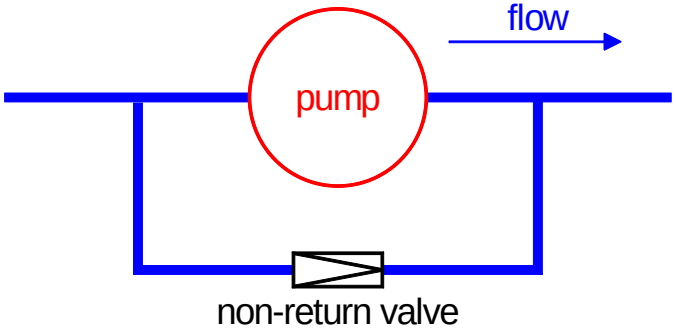
EXAMPLE

A pipe (length $L = 500\ \mathrm{m}$, diameter $D = 1.5\ \mathrm{m}$) is used to deliver water from a reservoir to a turbine at a volumetric flow rate of $2\ \mathrm{m}^3\ \mathrm{s}^{-1}$. The turbine is protected by a cylindrical surge tank of inside diameter 5 m. If friction losses can be neglected find the maximum surge in the surge tank and the period of oscillation if:

(a) the entire flow to the turbine is shut off;

(b) the flow to the turbine is halved.

PUMP BYPASS



Normal operation

Pressure on the delivery side of the pump is high and keeps the non-return valve closed.

Pump failure

Pressure on the delivery side drops (because the flow tries to keep moving, creating a void). The valve opens, allowing flow to bypass the pump and keep moving.

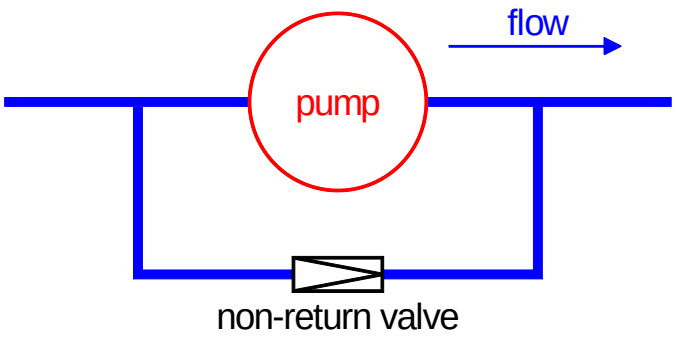
EXAMPLE

Sewage, the density and viscosity of which differ little from those of clean water, has to be pumped from a tank at the end of an interceptor sewer to the first tank of a treatment works by means of a pump situated at the upstream end. The difference in liquid levels is 32 m; the pipeline is 5.2 km long, 1.5 m diameter and has a friction factor of 0.014. The steady-flow discharge is 4.5 $m^3\ s^{-1}$.

(a) Calculate the steady flow velocity in the pipe and the pump power.

(b) Explain, briefly, what would happen if the inlet to the pipeline were suddenly completely blocked, assuming that no protective devices are installed.

(c) It is proposed to install a by-pass, incorporating a non-return valve to the pump, which prevents the pressure just downstream of it falling below atmospheric. Calculate the time taken for the water to come to rest after such a blockage.



EXAMPLE

Water is pumped from one reservoir to another in which the water level is 12 m above the pumping station, through a horizontal pipe 1500 m long and 0.5 m diameter at a rate of 0.4 $m^3 s^{-1}$. The head loss due to pipe friction at this velocity is 8 m.

(a) Calculate the bulk velocity during normal operating conditions and the friction factor in the pipe.

(b) To protect the pump an air inlet valve is fitted to the pipe just downstream of the pump. This valve is designed to allow air to flow into the pipe when the pressure falls to 5 m of water below atmospheric pressure. Assuming a constant friction factor, calculate the time for the water to come to rest if the pump intake is suddenly and completely blocked.