TOPIC 2: FLOW IN PIPES AND CHANNELS
OBJECTIVES

1. Calculate the friction factor for a pipe using the Colebrook-White equation.

2. Undertake head loss, discharge and sizing calculations for single pipelines.

3. Use head-loss vs discharge relationships to calculate flow in pipe networks.

4. Relate normal depth to discharge for uniform flow in open channels.
FLOW REGIMES

\[ \text{Re} \equiv \frac{VD}{v} \]

For a pipe, \( \text{Re}_{\text{crit}} \approx 2300 \)

\( V \) = average (or bulk) velocity
\( D \) = diameter
\[
\frac{L_{\text{dev}}}{D} = \begin{cases} 
0.06\text{Re} & \text{(laminar)} \\
4.4\text{Re}^{1/6} & \text{(turbulent)}
\end{cases}
\]
PIPE FLOW: BALANCE OF FORCES

- pressure
- gravity
- friction

\[ p(\pi r^2) - (p + \Delta p)(\pi r^2) + \frac{mg}{\text{weight}} \sin \theta + \frac{\tau(2\pi r\Delta l)}{\text{friction}} = 0 \]

\[ m = \rho \pi r^2 \Delta l \]
\[ \sin \theta = -\frac{\Delta z}{\Delta l} \]
\[ \Rightarrow -\Delta p(\pi r^2) - \rho \pi r^2 g \Delta z + \tau(2\pi r\Delta l) = 0 \]
\[ -\Delta(p + \rho g z)(\pi r^2) + \tau(2\pi r\Delta l) = 0 \]
\[ -\frac{\Delta(p + \rho g z)}{\Delta l} + 2\frac{\tau}{r} = 0 \]

\[ p^* = p + \rho g z \]
\[ \tau = \frac{1}{2} r \frac{\Delta p^*}{\Delta l} \]

(Streamwise) pressure gradient:
\[ G = -\frac{\Delta p^*}{\Delta l} = -\frac{dp^*}{dl} \]

\[ \tau = -\frac{1}{2} Gr \]
LAMINAR PIPE FLOW

Balance of forces: \( \tau = -\frac{1}{2} Gr \) (stress \( \propto \) pressure gradient)

Viscous stress: \( \tau = \mu \frac{du}{dr} \) (stress \( \propto \) velocity gradient)

\[
\frac{du}{dr} = -\frac{1}{2} \frac{G}{\mu} r
\]

\[
u = -\frac{Gr^2}{4\mu} + \text{constant}
\]

\( u = 0 \text{ on } r = R \)

\[
u = \frac{G}{4\mu} (R^2 - r^2)
\]

\[
u = U_0 \left( 1 - \frac{r^2}{R^2} \right)
\]

\[
U_0 = \frac{GR^2}{4\mu}
\]
Find, from the velocity distribution given above:

(a) the centreline velocity, \( U_0 \);

(b) the average velocity, \( V \);

(c) the volumetric flow rate, \( Q \), in terms of head loss and pipe diameter;

(d) the friction factor, \( \lambda \), defined by \( h_f = \lambda \frac{L}{D} \left( \frac{V^2}{2g} \right) \),

as a function of Reynolds number, \( Re \).
QUESTIONS

Which forces are in balance in steady pipe flow?

pressure, gravity, friction

How can one combine the effects of pressure and weight?

via piezometric pressure  \( p^* = p + \rho gz \)

How do we convert between pressure and head?

\[ p = \rho gh \quad h = \frac{p}{\rho g} \]

How do we define (a) dynamic pressure; (b) dynamic head?

\[ \frac{1}{2} \rho V^2 \quad \frac{V^2}{2g} \]

How do we define the skin-friction coefficient?

\[ c_f = \frac{\tau_w}{\frac{1}{2} \rho V^2} \]
PIPE FLOW: BALANCE OF FORCES

- pressure
- gravity
- friction

\[
-\Delta p \times \frac{\pi D^2}{4} + mg \sin \theta - \tau_w \times \pi DL = 0
\]

\[
m = \rho \frac{\pi D^2}{4} L \quad \Rightarrow \quad -\Delta p \times \frac{\pi D^2}{4} - \rho g\Delta z \times \frac{\pi D^2}{4} = \tau_w \times \pi DL
\]

\[
-\Delta(p + \rho gz) \times \frac{\pi D^2}{4} = \tau_w \times \pi DL
\]

\[
p^* = p + \rho gz \quad -\Delta p^* = 4 \frac{L}{D} \tau_w
\]

Definition of skin-friction coefficient: \( \tau_w = c_f \left( \frac{1}{2} \rho V^2 \right) \)

\[
|\Delta p^*| = 4c_f \frac{L}{D} \left( \frac{1}{2} \rho V^2 \right)
\]
DARCY-WEISBACH EQUATION

\[ \lambda = 4c_f \]

Pressure loss due to friction = \( \lambda \frac{L}{D} \times \text{dynamic pressure} \)

\[ |\Delta p^*| = \lambda \frac{L}{D} \left( \frac{1}{2} \rho V^2 \right) \]

Head loss due to friction = \( \lambda \frac{L}{D} \times \text{dynamic head} \)

\[ h_f = \lambda \frac{L}{D} \left( \frac{V^2}{2g} \right) \]

\[ h_f \quad = \quad \text{frictional head loss} \]
\[ \lambda \quad = \quad \text{friction factor} \]
\[ L \quad = \quad \text{length of pipe} \]
\[ D \quad = \quad \text{diameter} \]
\[ V \quad = \quad \text{average velocity (\( \propto Q \))} \]
A 0.75 m diameter pipe carries 0.6 cumec.

At point A, elevation 40 m, a Bourdon gauge fitted to the pipe records 1.75 bar, while at point B, elevation 34 m and 1.5 km along the pipe, a similar gauge reads 2.1 bar.

Determine the flow direction and calculate the friction factor.
CALCULATING THE FRICTION FACTOR, $\lambda$

Defined by:

$$h_f = \lambda \frac{L}{D} \left( \frac{V^2}{2g} \right)$$

Laminar flow:

$$\lambda = \frac{64}{Re}$$

Turbulent flow – two limits:

Smooth:

$$\frac{1}{\sqrt{\lambda}} = 2.0 \log_{10} \frac{Re \sqrt{\lambda}}{2.51}$$

Rough:

$$\frac{1}{\sqrt{\lambda}} = 2.0 \log_{10} \frac{3.7D}{k_s}$$

Colebrooke-White Equation:

$$\frac{1}{\sqrt{\lambda}} = -2.0 \log_{10} \left( \frac{k_s}{3.7D} + \frac{2.51}{Re \sqrt{\lambda}} \right)$$
<table>
<thead>
<tr>
<th>Material</th>
<th>$k_s$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riveted steel</td>
<td>0.9 – 9.0</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.3 – 3.0</td>
</tr>
<tr>
<td>Wood stave</td>
<td>0.18 – 0.9</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.26</td>
</tr>
<tr>
<td>Galvanised iron</td>
<td>0.15</td>
</tr>
<tr>
<td>Asphalted cast iron</td>
<td>0.12</td>
</tr>
<tr>
<td>Commercial steel or wrought iron</td>
<td>0.046</td>
</tr>
<tr>
<td>Drawn tubing</td>
<td>0.0015</td>
</tr>
<tr>
<td>Glass</td>
<td>0 (smooth)</td>
</tr>
</tbody>
</table>
OTHER LOSSES

Loss coefficient, $K$

\[
\text{head loss} = K \times (\text{dynamic head}) = K \frac{V^2}{2g}
\]

Commercial pipe fittings

<table>
<thead>
<tr>
<th>Fitting</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Globe valve</td>
<td>10</td>
</tr>
<tr>
<td>Gate valve – wide open</td>
<td>0.2</td>
</tr>
<tr>
<td>Gate valve – ½ open</td>
<td>5.6</td>
</tr>
<tr>
<td>90° elbow</td>
<td>0.9</td>
</tr>
<tr>
<td>Side outlet of T-junction</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Entry/exit losses

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bell-mouhthed entry</td>
<td>0</td>
</tr>
<tr>
<td>Abrupt entry</td>
<td>0.5</td>
</tr>
<tr>
<td>Protruding entry</td>
<td>1.0</td>
</tr>
<tr>
<td>Bell-mouhthed exit</td>
<td>0.2</td>
</tr>
<tr>
<td>Abrupt enlargement</td>
<td>0.5</td>
</tr>
</tbody>
</table>
**Main Design Parameters:**
- Head loss: \( h \)
- Quantity of flow: \( Q \)
- Diameter: \( D \)

**Other Parameters:**
- Length: \( L \)
- Roughness: \( k_s \)
- Kinematic viscosity: \( \nu \)
- Minor loss coefficient: \( K \)

**Method:** head difference  =  sum of head losses along the pipe
1. Head Losses

\[ h = \left( \lambda \frac{L}{D} + K \right) \left( \frac{V^2}{2g} \right) \]

2. Loss coefficients

e.g. friction factor (Colebrook-White):

\[ \frac{1}{\sqrt{\lambda}} = -2.0 \log_{10} \left( \frac{k_s}{3.7D} + \frac{2.51}{\text{Re} \sqrt{\lambda}} \right) \]
HEADS AT THE ENDS OF PIPES

Smooth exit to a downstream reservoir:

\[ H_1 = z_1 \]
\[ H_2 = z_2 \]

No residual dynamic head at exit.

Free jet to atmosphere (or abrupt exit to a tank):

\[ H_1 = z_1 \]
\[ H_2 = z_2 + \frac{V_2^2}{2g} \]

Dynamic head must be included at exit.
TYPICAL PIPELINE CALCULATIONS

Type 1 – flow
Know: diameter, \( D \), and head, \( h \)
Find: discharge, \( Q \)

Easy!

Type 2 – head
Know: diameter, \( D \), and discharge, \( Q \)
Find: head, \( h \)

Solve Colebrook-White equation (iteratively)

Type 3 – size
Know: discharge, \( Q \), head, \( h \)
Find: diameter, \( D \)

Solve Colebrook-White and head-loss equations simultaneously and iteratively
Crude oil (specific gravity 0.86, kinematic viscosity $9.0 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$) is to be pumped from a barge to a large storage tank. The pipeline is horizontal and of diameter 250 mm, length 400 m and roughness 0.1 mm. It enters the tank 8 m below the level of oil in the tank. When the control valve is fully open the static pressure at pump delivery is $3 \times 10^5 \text{ Pa}$ gauge. Ignore minor losses due to pipe fittings, entrance/exit losses etc.

Find:
(a) (using hydrostatics) the gauge pressure where the pipe enters the tank;
(b) (from the pressures at the two ends) the head loss along the pipeline;
(c) the volumetric flow rate in the pipeline.

If the pump delivery pressure remains the same but a valve reduces the flow by half, find:
(d) the head loss at the valve;
(e) the power loss at the valve.
(a) A pipeline is to be constructed to bring water from an upland storage reservoir to a town 30 km away, at an elevation 150 m below the water level of the reservoir. In summer the pipeline must be able to convey up to 5000 cubic metres per day. If the pipe is fabricated from material of roughness 0.3 mm, find the required diameter.

(b) During the winter, water requirements fall to only 3000 cubic metres per day and the excess head available can be used to drive a small turbine. If the turbine has an efficiency of 75%, find the maximum power output.

The Colebrook-White equation is

\[
\frac{1}{\sqrt{\lambda}} = -2.0 \log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{Re \sqrt{\lambda}}\right)
\]

where \( \lambda \) is the friction factor, \( k_s \) is the roughness, \( D \) is the pipe diameter, \( V \) is the average velocity and \( Re \equiv VD/\nu \) is the Reynolds number.

For water, take density \( \rho = 1000 \, \text{kg m}^{-3} \) and kinematic viscosity \( \nu = 1.0 \times 10^{-6} \, \text{m}^2 \text{s}^{-1} \).
A reservoir is to be used to supply water to a factory 5 km away. The water level in the reservoir is 60 m above the factory. The pipe lining has roughness 0.5 mm. Minor losses due to valves and pipe fittings can be accommodated by a loss coefficient $K = 80$. Calculate the minimum diameter of pipe required to convey a discharge of $0.3 \text{ m}^3 \text{ s}^{-1}$. 
GRAPHICAL REPRESENTATION OF HEAD

Energy Grade Line (EGL) \[ \frac{p}{\rho g} + z + \frac{V^2}{2g} \] total head

Hydraulic Grade Line (HGL) \[ \frac{p}{\rho g} + z \] piezometric head
Pipe friction only

Pipe friction with minor losses (exaggerated), including change in pipe diameter.

Pumped system
The two reservoirs illustrated are used for water storage and supply. The water levels in the reservoirs are constant and equal to 70 m AOD in the lower reservoir (Reservoir A) and 82 m AOD in the upper reservoir (Reservoir B). The reservoirs are connected by a 1.2 km long pipe with diameter $D = 200$ mm and wall roughness $k_s = 0.2$ mm. A pump is installed in the pipe as illustrated in the figure.

Neglecting minor losses,
(a) sketch the qualitative behaviour of the energy and hydraulic grade lines between Reservoir A and Reservoir B if the system operates under gravity alone (i.e., without the pump);

(b) sketch the qualitative behaviour of the energy and hydraulic grade lines between Reservoir A and Reservoir B when the pump is operating and the flow direction is from Reservoir A to Reservoir B;

(c) find the pump head required to deliver a discharge of 0.025 m$^3$ s$^{-1}$ to reservoir B.
Which way does the flow go in pipe BD?
What are the voltages at B and D?

Which way does the current go in BD?
PIPE NETWORKS: BASIC RULES

1. **Continuity**: at any *junction*,

\[ \sum Q_{\text{in}} = \sum Q_{\text{out}} \]

total flow in = total flow out

2. Each *point* has a unique *head*, \( H \)

3. Each *pipe* has a *head-loss vs discharge* (resistance) relation:

\[ h = \alpha Q^2 \]
ELECTRICAL ANALOGY

Continuity; unique head ↔ Kirchoff’s Laws

- head, $H$ ↔ potential, $V$
- discharge, $Q$ ↔ current, $I$

Resistance law:
- head loss, $\Delta H \propto Q^2$ ↔ potential difference, $\Delta V \propto I$

What are the hydraulic analogues of:
- a resistor?
- a capacitor?
- an inductor?
- a transistor?
PIPES IN SERIES AND PARALLEL

Pipes in series

\[ Q = Q_1 = Q_2 \]

same flow

\[ \Delta H = \Delta H_1 + \Delta H_2 \]

add head changes

\[ \alpha = \alpha_1 + \alpha_2 \]

\[ R = R_1 + R_2 \]

Pipes in parallel

\[ \Delta H = \Delta H_1 = \Delta H_2 \]

same head change

\[ Q = Q_1 + Q_2 \]

add flows

\[ \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{\alpha_1}} + \frac{1}{\sqrt{\alpha_2}} \]

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \]
JUNCTION PROBLEMS: GENERAL METHOD

Method: Adjust $H_J$ until net flow out of $J = 0$

(0) Establish the **head vs discharge** relations for all pipes: $|H_J - H_A| = \alpha Q_{JA}^2$ etc.

(1) Guess an initial value of **head** at the junction, $H_J$.

(2) Calculate **flow rates** in all pipes $Q_{JA}, Q_{JB}, Q_{JC}$

(3) Calculate **net flow** out of junction, $Q_{JA} + Q_{JB} + Q_{JC}$

(4) **Adjust** the head at the junction, $H_J$, until net flow out of junction = 0
In a water-storage scheme three reservoirs A, B and C are connected by a single junction J as shown. The water levels in A, B and C are 300 m, 200 m and 140 m respectively. The pipeline properties are given below. Friction factors may be assumed constant and minor losses may be neglected.

<table>
<thead>
<tr>
<th>Pipeline</th>
<th>JA</th>
<th>JB</th>
<th>JC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length L (m)</td>
<td>5000</td>
<td>3000</td>
<td>4000</td>
</tr>
<tr>
<td>Diameter D (m)</td>
<td>0.4</td>
<td>0.25</td>
<td>0.3</td>
</tr>
<tr>
<td>Friction factor $\lambda$</td>
<td>0.015</td>
<td>0.03</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Calculate the total flow in each pipe and the direction of flow in pipe JB if:

(a) there is a valve-regulated flow of 50 L s$^{-1}$ to reservoir C but water flows freely under gravity in the other pipes;

(b) water flows freely under gravity in all pipes.
# FLOW IN PIPES AND OPEN CHANNELS

<table>
<thead>
<tr>
<th></th>
<th>PIPE FLOW</th>
<th>OPEN-CHANNEL FLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fluid:</strong></td>
<td>LIQUIDS or GASES</td>
<td>LIQUIDS (free surface)</td>
</tr>
<tr>
<td><strong>Driven by:</strong></td>
<td>PRESSURE, GRAVITY or BOTH</td>
<td>GRAVITY (down slope)</td>
</tr>
<tr>
<td><strong>Size:</strong></td>
<td>DIAMETER</td>
<td>HYDRAULIC RADIUS</td>
</tr>
<tr>
<td><strong>Volume:</strong></td>
<td>FILLS pipe</td>
<td>Depends on DEPTH</td>
</tr>
<tr>
<td><strong>Equations:</strong></td>
<td>DARCY-WEISBACH (head loss)</td>
<td>MANNING’S FORMULA</td>
</tr>
<tr>
<td></td>
<td>COLEBROOK-WHITE (friction factor)</td>
<td></td>
</tr>
</tbody>
</table>
• **Normal flow** = steady, uniform flow (constant-depth flow under gravity)

• At best, an approximation for rivers / natural channels

• For any given $Q$ there is a particular **normal depth**
In normal flow:

- Equal hydrostatic pressure forces at any cross section
- Downslope component of weight balances bed friction
- Channel bed, free surface (= HGL) and EGL are parallel; i.e. loss of fluid head equals drop in height
- Usual to assume small slopes
PART 1: BALANCE OF FORCES

\[ A = \text{area of fluid cross-section} \]
\[ P = \text{wetted perimeter} \]

(downslope) component of weight = friction on sides

\[ mg \sin \theta = \tau_b \times \text{wetted surface area} \]

\[ \rho ALg \sin \theta = \tau_b PL \]
\[ \rho g \frac{A}{P} \sin \theta = \tau_b \]

Hydraulic radius (*** depends on depth ***):

\[ R_h \equiv \frac{\text{cross-sectional area}}{\text{wetted perimeter}} = \frac{A}{P} \]

Normal-flow relationship:

\[ \tau_b = \rho g R_h S \]
PART 2: EXPRESSION FOR FRICTION

\[ \tau_b = \rho g R_h S \]  

*\( R_h \) is the **hydraulic radius**

\[ c_f \left( \frac{1}{2} \rho V^2 \right) = \rho g R_h S \]  

definition of the **skin-friction coefficient**

\[ V^2 = \frac{2g}{c_f} R_h S \]

**Chézy’s Formula:**  

\[ V = C \sqrt{R_h S} \]

Robert Manning (compilation of experimental data):

\[ C = R_h^{1/6} \times \text{function of roughness} \]

\[ = \frac{R_h^{1/6}}{n} \]

**Manning’s Formula:**  

\[ V = \frac{1}{n} R_h^{2/3} S^{1/2} \]
# MANNING’S ROUGHNESS COEFFICIENT

<table>
<thead>
<tr>
<th>Channel type</th>
<th>Surface</th>
<th>n (m⁻¹/³ s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Artificial lined channels</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>Brass</td>
<td></td>
<td>0.011</td>
</tr>
<tr>
<td>Steel, smooth</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>painted</td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>riveted</td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td>Cast iron</td>
<td></td>
<td>0.013</td>
</tr>
<tr>
<td>Concrete, finished</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>unfinished</td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>Planed wood</td>
<td></td>
<td>0.012</td>
</tr>
<tr>
<td>Clay tile</td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>Brickwork</td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td>Asphalt</td>
<td></td>
<td>0.016</td>
</tr>
<tr>
<td>Corrugated metal</td>
<td></td>
<td>0.022</td>
</tr>
<tr>
<td>Rubble masonry</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td><strong>Excavated earth channels</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean</td>
<td></td>
<td>0.022</td>
</tr>
<tr>
<td>Gravelly</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td>Weedy</td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td>Stony, cobbles</td>
<td></td>
<td>0.035</td>
</tr>
<tr>
<td><strong>Natural channels</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean and straight</td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td>Sluggish, deep pools</td>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td>Major rivers</td>
<td></td>
<td>0.035</td>
</tr>
<tr>
<td><strong>Floodplains</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pasture, farmland</td>
<td></td>
<td>0.035</td>
</tr>
<tr>
<td>Light brush</td>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>Heavy brush</td>
<td></td>
<td>0.075</td>
</tr>
<tr>
<td>Trees</td>
<td></td>
<td>0.15</td>
</tr>
</tbody>
</table>
CALCULATION FORMULAE (SUMMARY)

Manning’s Formula: \[ V = \frac{1}{n} R_h^{2/3} S^{1/2} \]

- \( V \) = average velocity
- \( n \) = Manning’s roughness parameter
- \( S \) = slope (gradient)
- \( R_h \) = hydraulic radius

\[ R_h = \frac{A}{P} = \frac{\text{cross – sectional area}}{\text{wetted perimeter}} \]

Method

For a given channel:

1. Write area \( A \) and perimeter \( P \) as functions of a parameter (often depth, \( h \))
2. Calculate hydraulic radius
3. Calculate average velocity
4. Calculate quantity of flow

Two Main Types of Problem

- Given \( h \) find \( Q \)
- Given \( Q \) find \( h \)
A V-shaped channel with sides sloping at 30° to the horizontal has a gradient of 1 in 100 and an estimated Manning’s $n$ of 0.012 m$^{-1/3}$ s. Calculate:

(a) the discharge for a depth of 0.5 m;

(b) the depth when the discharge is 2 m$^3$ s$^{-1}$. 
CONVEYANCE

(a) Manning’s formula: \[ V = \frac{1}{n} R_h^{2/3} S^{1/2} \quad R_h = \frac{A}{P} \]

(b) Discharge: \[ Q = VA \]

\[ Q = \frac{1}{n} \left( \frac{A}{P} \right)^{2/3} S^{1/2} \times A \]

\[ Q = KS^{1/2} \]

\[ K = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} \]

For compound channels (e.g. river plus flood plain) simply add the conveyances:

\[ K_{\text{eff}} = K_1 + K_2 + K_3 \]
# COMMON SHAPES OF CHANNEL

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area, $A$</th>
<th>Wetted Perimeter, $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangle</strong></td>
<td>$bh$</td>
<td>$b + 2h$</td>
</tr>
<tr>
<td><strong>Trapezoid</strong></td>
<td>$bh + \frac{h^2}{\tan \alpha}$</td>
<td>$b + \frac{2h}{\sin \alpha}$</td>
</tr>
<tr>
<td><strong>Circle</strong></td>
<td>$R^2(\theta - \frac{1}{2} \sin 2\theta)$</td>
<td>$2R\theta$</td>
</tr>
</tbody>
</table>

**Definitions:**
- $b$: Base width (for rectangle and trapezoid)
- $h$: Height
- $\alpha$: Angle of the trapezoid
- $\theta$: Angle of the circle
- $R$: Radius of the circle

## Diagrams:
- **Rectangle:** A rectangle with width $b$ and height $h$.
- **Trapezoid:** A trapezoid with base $b$, height $h$, and angle $\alpha$.
- **Circle:** A circle with radius $R$ and height $h$.
A culvert used to divert run-off has a rectangular cross section with base width 0.4 m and side heights of 0.3 m. Manning’s coefficient may be taken as \( n = 0.012 \text{ m}^{-1/3} \text{ s} \).

(a) Find the minimum slope \( S \) necessary to carry a discharge \( Q = 0.3 \text{ m}^3 \text{ s}^{-1} \).

(b) If the slope from part (a) is doubled for the same discharge, calculate depth of flow.
A concrete pipe 750 mm in diameter is laid to a gradient of 1 in 200. The estimated value of Manning’s $n$ is $0.012 \text{ m}^{-1/3} \text{ s}$.

Calculate the discharge when:

(a) the pipe is full;

(b) the depth is 90% of maximum.

Explain why the answer in (b) exceeds that in (a).