TOPIC 2: FLOW IN PIPES AND CHANNELS

OBJECTIVES

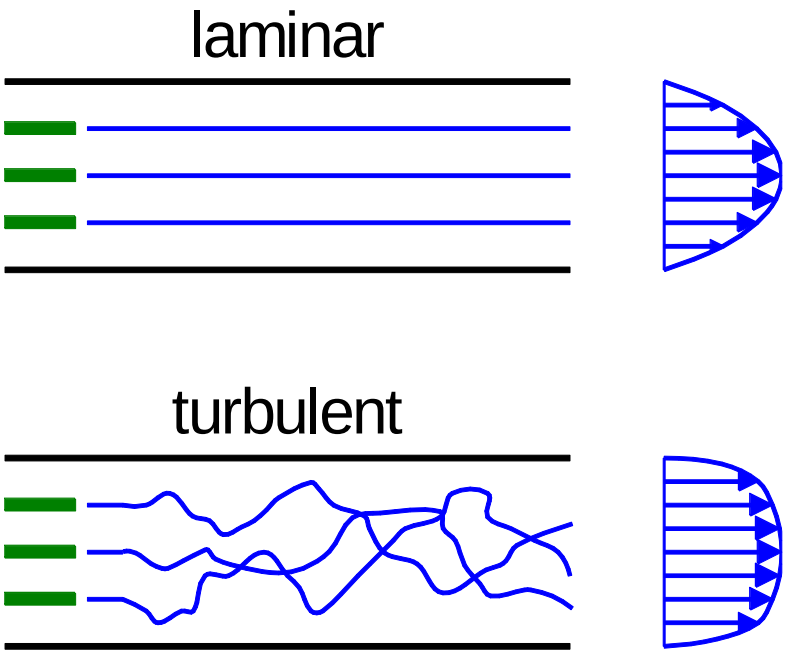
1. Calculate the **friction factor** for a pipe using the **Colebrook-White** equation.

2. Undertake **head loss**, **discharge** and **sizing** calculations for single pipelines.

3. Use **head-loss vs discharge** relationships to calculate flow in **pipe networks**.

4. Relate **normal depth** to discharge for **uniform flow in open channels**.

FLOW REGIMES



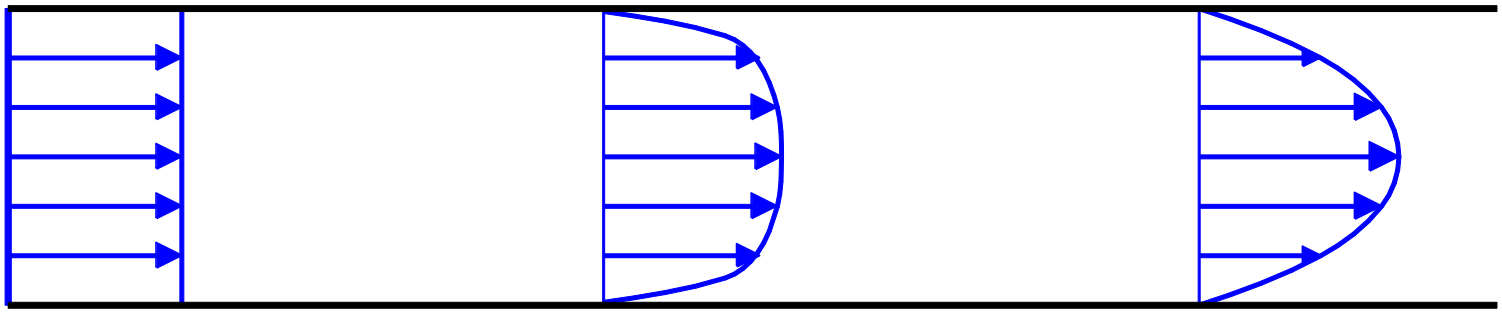
$$\mathrm{Re} \equiv \frac{VD}{\nu}$$

V = average (or bulk) velocity

D = diameter

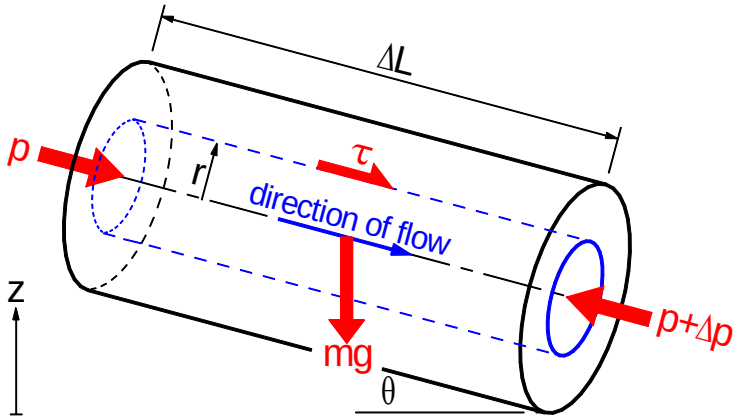
For a **pipe**, $\mathrm{Re}_{\mathrm{crit}} \approx 2300$

DEVELOPMENT LENGTH



$$\frac{L_{\text{dev}}}{D} = \begin{cases} 0.06\text{Re} & \text{(laminar)} \\ 4.4\text{Re}^{1/6} & \text{(turbulent)} \end{cases}$$

PIPE FLOW: BALANCE OF FORCES



- **pressure**
- **gravity**
- **friction**

$$\underbrace{p(\pi r^2) - (p+\Delta p)(\pi r^2)}_{\text{net pressure force}} + \underbrace{mg}_{\text{weight}} \sin\theta + \underbrace{\tau(2\pi r \Delta l)}_{\text{friction}} = 0$$

$$\left.\begin{aligned} m &= \rho \pi r^2 \Delta l \\ \sin\theta &= -\frac{\Delta z}{\Delta l} \end{aligned}\right\} \Rightarrow \quad -\Delta p(\pi r^2) - \rho \pi r^2 g\ \Delta z + \tau(2\pi r \Delta l) = 0$$

$$-\Delta(p + \rho g z)(\pi r^2) + \tau(2\pi r \Delta l) = 0$$

$$-\frac{\Delta(p+\rho g z)}{\Delta l} + 2\frac{\tau}{r} = 0$$

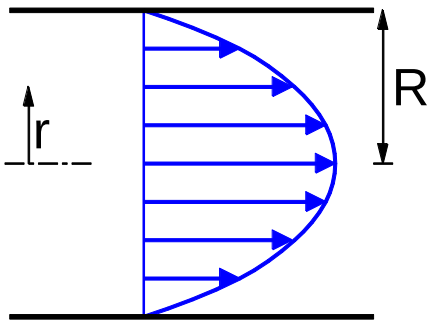
$$p^* = p + \rho g z \qquad \tau = \frac{1}{2} r \frac{\Delta p^*}{\Delta l}$$

(Streamwise) pressure **G**radient: $G = -\frac{\Delta p^*}{\Delta l} = -\frac{\mathrm{d}p^*}{\mathrm{d}l}$

$$\boxed{\tau = -\frac{1}{2} G r}$$

LAMINAR PIPE FLOW

Balance of forces: $\tau = -\frac{1}{2}Gr$ (stress $\propto$ pressure gradient)



Viscous stress: $\tau = \mu\frac{\mathrm{d}u}{\mathrm{d}r}$ (stress $\propto$ velocity gradient)

$$\frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{1}{2}\frac{G}{\mu}r$$

$$u = -\frac{Gr^2}{4\mu} + \text{constant}$$

$u = 0$ on $r = R$

$$u = \frac{G}{4\mu}(R^2 - r^2)$$

$$u = U_0\left(1 - \frac{r^2}{R^2}\right)$$

$$U_0 = \frac{GR^2}{4\mu}$$

EXAMPLE

$$u = \frac{G}{4\mu}(R^2 - r^2)$$

Pressure gradient: $$G = \frac{|\Delta p^*|}{L} = \frac{\rho g h_f}{L}$$

Find, from the velocity distribution given above:

(a) the centreline velocity, U_0;

(b) the average velocity, V;

(c) the volumetric flow rate, Q, in terms of head loss and pipe diameter;

(d) the friction factor, λ, defined by $h_f = \lambda \frac{L}{D}(\frac{V^2}{2g})$,

as a function of Reynolds number, Re.

QUESTIONS

Which forces are in balance in steady pipe flow?

pressure, gravity, friction

How can one combine the effects of pressure and weight?

via piezometric pressure $p^* = p + \rho g z$

How do we convert between **pressure** and **head**?

$$p = \rho g h \qquad\qquad h = \frac{p}{\rho g}$$

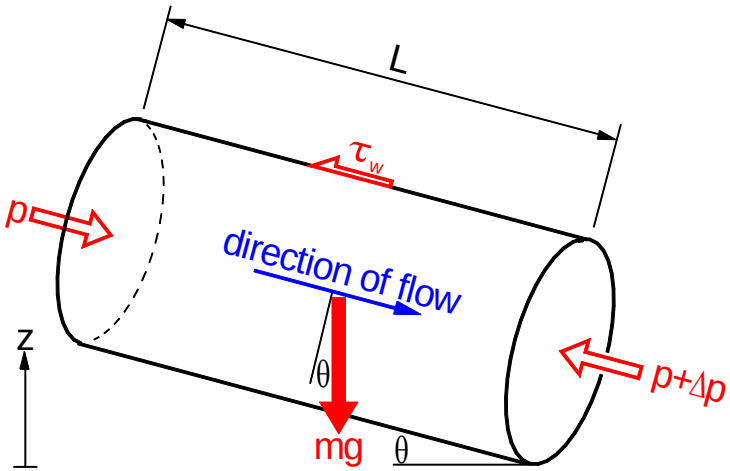
How do we define (a) **dynamic pressure**; (b) **dynamic head**?

$$\frac{1}{2}\rho V^2 \qquad\qquad \frac{V^2}{2g}$$

How do we define the **skin-friction coefficient**?

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho V^2}$$

PIPE FLOW: BALANCE OF FORCES



- **pressure**
- **gravity**
- **friction**

$$\underbrace{-\Delta p \times \frac{\pi D^2}{4}}_{\text{net pressure force}} + \underbrace{mg}_{\text{weight}} \sin\theta - \underbrace{\tau_w \times \pi D L}_{\text{wall friction}} = 0$$

$$\left.\begin{aligned} m &= \rho \frac{\pi D^2}{4} L \\ \sin\theta &= -\frac{\Delta z}{L} \end{aligned}\right\} \Rightarrow \quad -\Delta p \times \frac{\pi D^2}{4} - \rho g \Delta z \times \frac{\pi D^2}{4} = \tau_w \times \pi D L$$

$$-\Delta(p + \rho g z) \times \frac{\pi D^2}{4} = \tau_w \times \pi D L$$

$$p^* = p + \rho g z \qquad -\Delta p^* = 4 \frac{L}{D} \tau_w$$

Definition of skin-friction coefficient: $\tau_w = c_f (\frac{1}{2} \rho V^2)$

$$|\Delta p^*| = 4 c_f \frac{L}{D} (\frac{1}{2} \rho V^2)$$

DARCY-WEISBACH EQUATION

$$\lambda = 4c_f$$

Pressure loss due to friction = $\lambda \frac{L}{D} \times$ dynamic **pressure**

$$|\Delta p^*| = \lambda \frac{L}{D} (\frac{1}{2} \rho V^2)$$

Head loss due to friction = $\lambda \frac{L}{D} \times$ dynamic **head**

$$h_f = \lambda \frac{L}{D} (\frac{V^2}{2g})$$

h_f	=	frictional head loss
λ	=	friction factor
L	=	length of pipe
D	=	diameter
V	=	average velocity ($\propto Q$)

EXAMPLE SHEET

A 0.75 m diameter pipe carries 0.6 cumec.

At point A, elevation 40 m, a Bourdon gauge fitted to the pipe records 1.75 bar, while at point B, elevation 34 m and 1.5 km along the pipe, a similar gauge reads 2.1 bar.

Determine the flow direction and calculate the friction factor.

CALCULATING THE FRICTION FACTOR, λ

Defined by: $$h_f = \lambda \frac{L}{D}\left(\frac{V^2}{2g}\right)$$

Laminar flow: $$\lambda = \frac{64}{\mathrm{Re}}$$

Turbulent flow – two limits:

Smooth: $$\frac{1}{\sqrt{\lambda}} = 2.0 \log_{10} \frac{\mathrm{Re}\sqrt{\lambda}}{2.51}$$

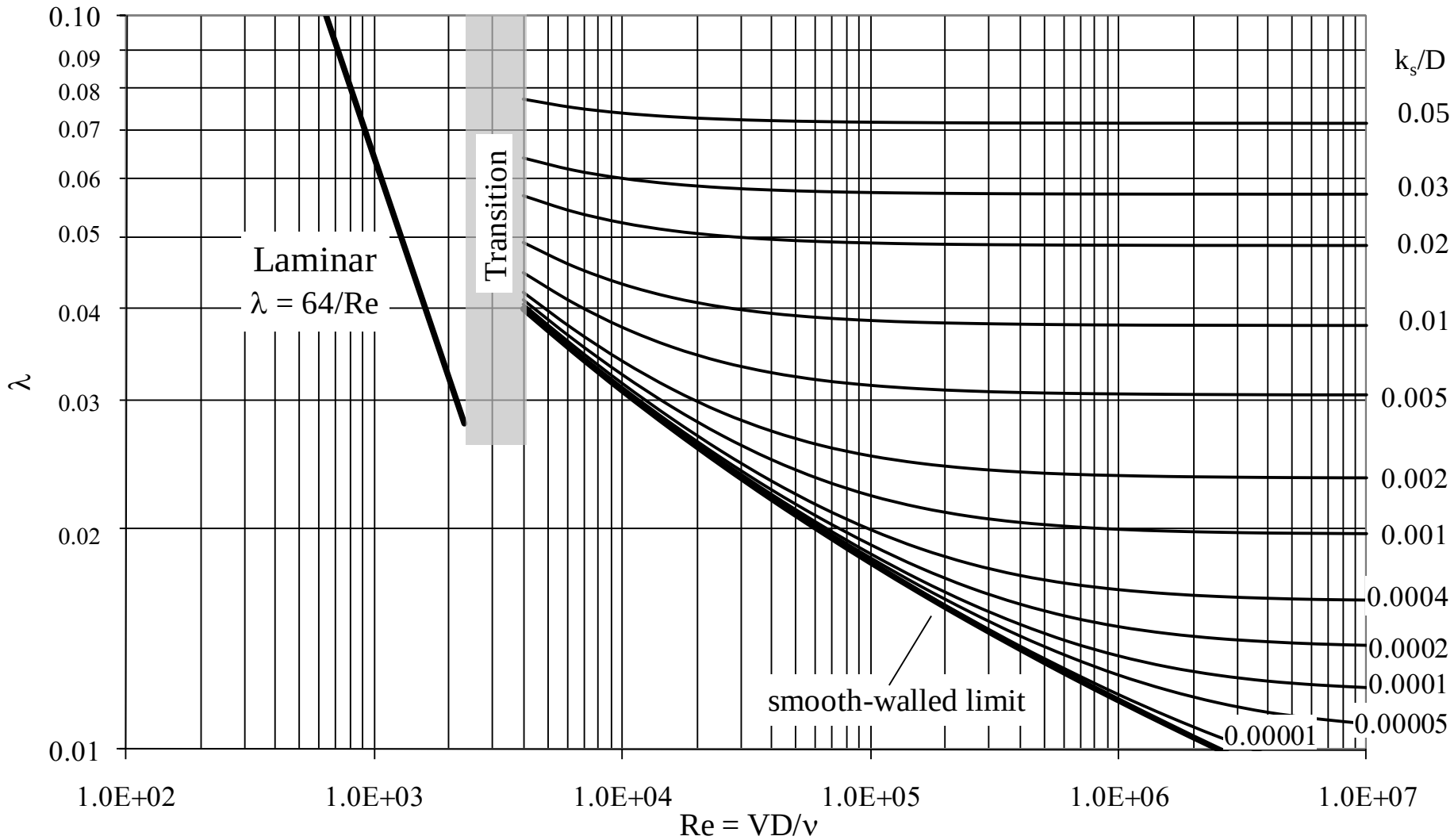
Rough: $$\frac{1}{\sqrt{\lambda}} = 2.0 \log_{10} \frac{3.7D}{k_s}$$

Colebrook-White Equation: $$\frac{1}{\sqrt{\lambda}} = -2.0 \log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

TYPICAL ROUGHNESS FOR COMMERCIAL PIPES

Material	k_s (mm)
Riveted steel	0.9 – 9.0
Concrete	0.3 – 3.0
Wood stave	0.18 – 0.9
Cast iron	0.26
Galvanised iron	0.15
Asphalted cast iron	0.12
Commercial steel or wrought iron	0.046
Drawn tubing	0.0015
Glass	0 (smooth)

MOODY CHART


0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
λ
Laminar
λ = 64/Re
Transition
smooth-walled limit
k_s/D
0.05
0.03
0.02
0.01
0.005
0.002
0.001
0.0004
0.0002
0.0001
0.00005
0.00001
1.0E+02
1.0E+03
1.0E+04
1.0E+05
1.0E+06
1.0E+07
Re = VD/ν

OTHER LOSSES

Loss coefficient, K

$$\text{head loss} = K \times (\text{dynamic head})$$

$$h = K\frac{V^2}{2g}$$

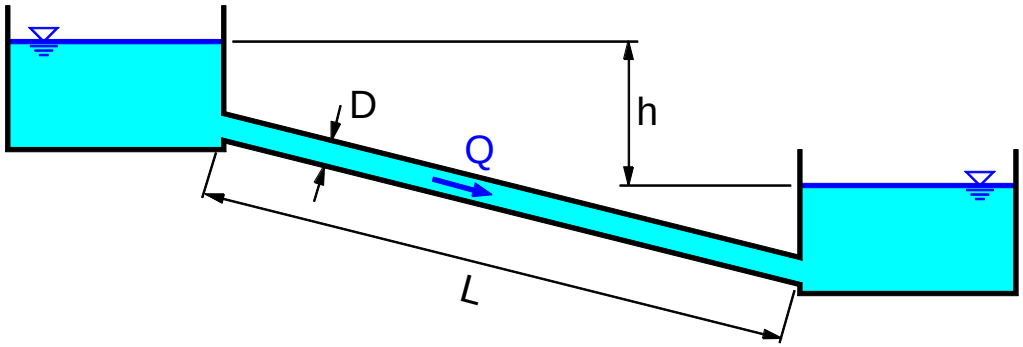
Commercial pipe fittings

Fitting	K
Globe valve	10
Gate valve – wide open	0.2
Gate valve – ½ open	5.6
90° elbow	0.9
Side outlet of T-junction	1.8

Entry/exit losses

Configuration	K
Bell-mouthed entry	0
Abrupt entry	0.5
Protruding entry	1.0
Bell-mouthed exit	0.2
Abrupt enlargement	0.5

PIPELINE CALCULATIONS



Main Design Parameters:

Head loss:	h
Quantity of flow:	Q
Diameter:	D

Other Parameters:

Length:	L
Roughness:	k_s
Kinematic viscosity:	ν
Minor loss coefficient:	K

Method: head difference = sum of head losses along the pipe

CALCULATION FORMULAE

1. Head Losses

$$h = (\lambda \frac{L}{D} + K)(\frac{V^2}{2g})$$

2. Loss coefficients

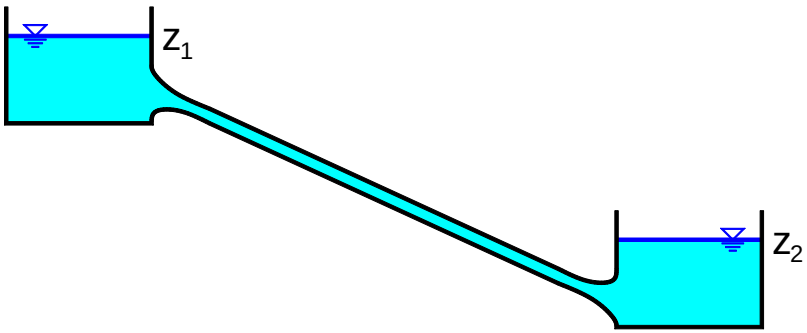
e.g. friction factor (Colebrook-White):

$$\frac{1}{\sqrt{\lambda}} = -2.0 \log_{10} \left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}} \right)$$

HEADS AT THE ENDS OF PIPES

Smooth exit to a downstream reservoir:

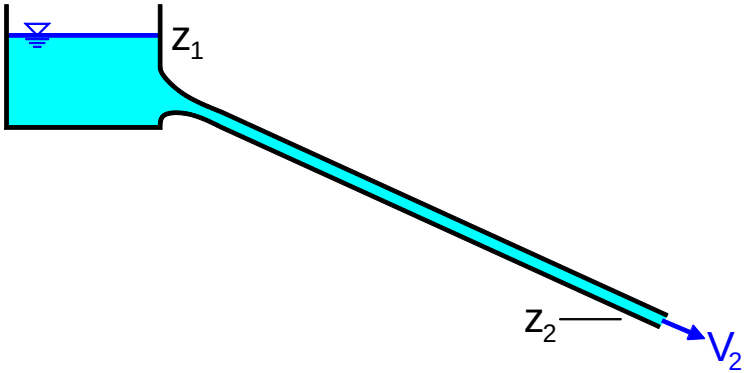
$$H_1 = z_1$$
$$H_2 = z_2$$



No residual dynamic head at exit.

Free jet to atmosphere (or abrupt exit to a tank):

$$H_1 = z_1$$
$$H_2 = z_2 + V_2^2/2g$$



Dynamic head must be included at exit.

TYPICAL PIPELINE CALCULATIONS

Type 1 – flow

Know: diameter, D, and head, h
Find: **discharge, Q**

Easy!

Type 2 – head

Know: diameter, D, and discharge, Q
Find: **head, h**

Solve Colebrook-White equation (iteratively)

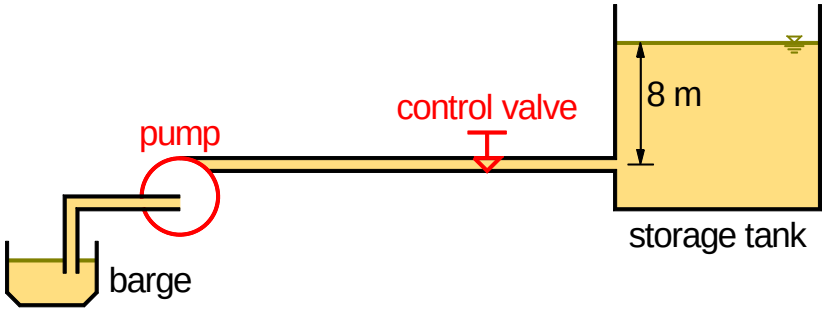
Type 3 – size

Know: discharge, Q, head, h
Find: diameter, D

Solve Colebrook-White and head-loss equations simultaneously and iteratively

EXAMPLE SHEET

Crude oil (specific gravity 0.86, kinematic viscosity $9.0 \times 10^{-5}\ \mathrm{m^2\ s^{-1}}$) is to be pumped from a barge to a large storage tank. The pipeline is horizontal and of diameter 250 mm, length 400 m and roughness 0.1 mm. It enters the tank 8 m below the level of oil in the tank. When the control valve is fully open the static pressure at pump delivery is $3 \times 10^5\ \mathrm{Pa}$ gauge. Ignore minor losses due to pipe fittings, entrance/exit losses etc.



Find:

(a) (using hydrostatics) the gauge pressure where the pipe enters the tank;

(b) (from the pressures at the two ends) the head loss along the pipeline;

(c) the volumetric flow rate in the pipeline.

If the pump delivery pressure remains the same but a valve reduces the flow by half, find:

(d) the head loss at the valve;

(e) the power loss at the valve.

EXAMPLE SHEET

(a) A pipeline is to be constructed to bring water from an upland storage reservoir to a town 30 km away, at an elevation 150 m below the water level of the reservoir. In summer the pipeline must be able to convey up to 5000 cubic metres per day. If the pipe is fabricated from material of roughness 0.3 mm, find the required diameter.

(b) During the winter, water requirements fall to only 3000 cubic metres per day and the excess head available can be used to drive a small turbine. If the turbine has an efficiency of 75%, find the maximum power output.

The Colebrook-White equation is

$$\frac{1}{\sqrt{\lambda}} = -2.0 \log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

where λ is the friction factor, k_s is the roughness, D is the pipe diameter, V is the average velocity and $\mathrm{Re} \equiv VD/\nu$ is the Reynolds number.

For water, take density $\rho = 1000\ \mathrm{kg\ m^{-3}}$ and kinematic viscosity $\nu = 1.0 \times 10^{-6}\ \mathrm{m^2\ s^{-1}}$.

EXAMPLE

A reservoir is to be used to supply water to a factory 5 km away. The water level in the reservoir is 60 m above the factory. The pipe lining has roughness 0.5 mm. Minor losses due to valves and pipe fittings can be accommodated by a loss coefficient $K = 80$. Calculate the minimum diameter of pipe required to convey a discharge of $0.3\ \mathrm{m}^3\ \mathrm{s}^{-1}$.

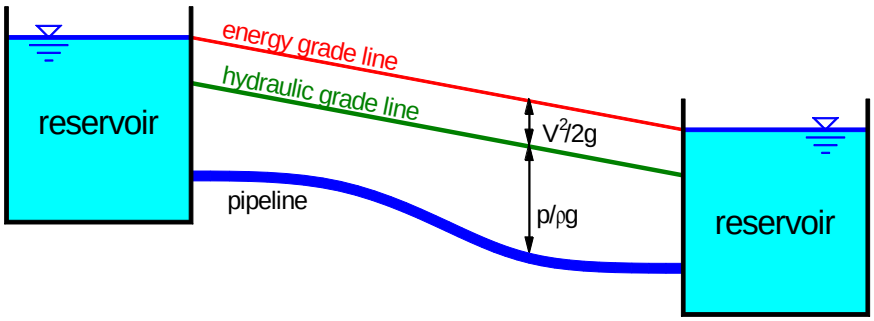
GRAPHICAL REPRESENTATION OF HEAD

Energy Grade Line (EGL) $$\frac{p}{\rho g} + z + \frac{V^2}{2g}$$ **total head**

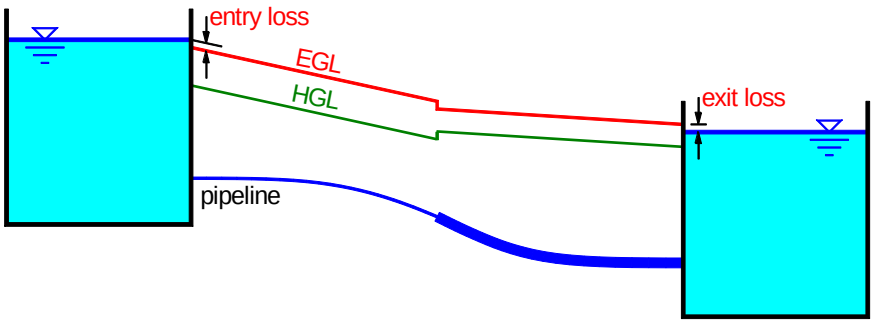
Hydraulic Grade Line (HGL) $$\frac{p}{\rho g} + z$$ **piezometric head**

GRAPHICAL REPRESENTATION OF HEAD

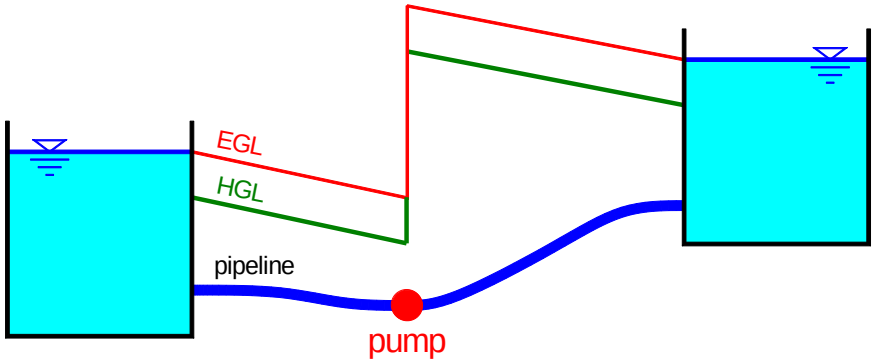
Pipe friction only



Pipe friction with minor losses (exaggerated), including change in pipe diameter.

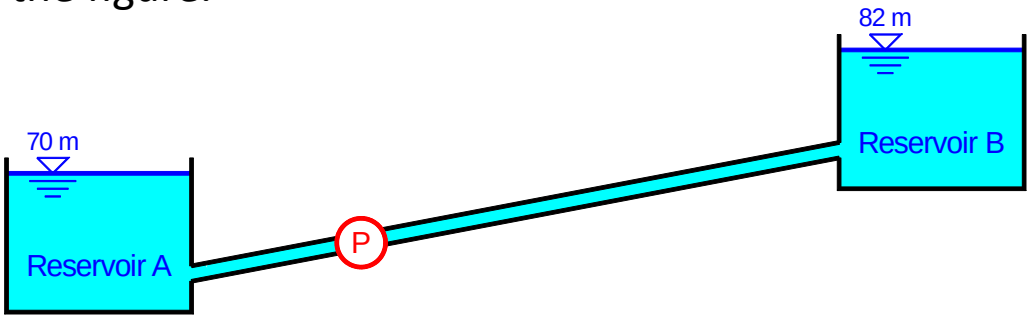


Pumped system



EXAMPLE

The two reservoirs illustrated are used for water storage and supply. The water levels in the reservoirs are constant and equal to 70 m AOD in the lower reservoir (Reservoir A) and 82 m AOD in the upper reservoir (Reservoir B). The reservoirs are connected by a 1.2 km long pipe with diameter $D = 200\ \mathrm{mm}$ and wall roughness $k_s = 0.2\ \mathrm{mm}$. A pump is installed in the pipe as illustrated in the figure.



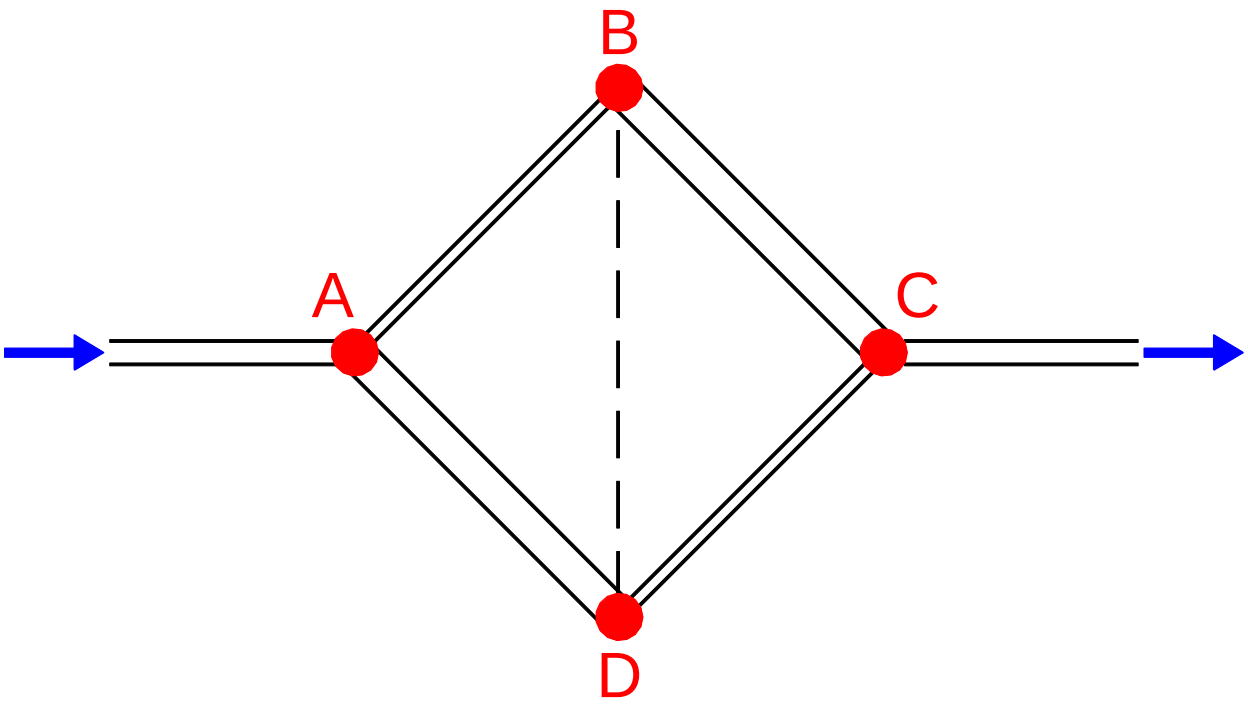
Neglecting minor losses,

(a) sketch the qualitative behaviour of the energy and hydraulic grade lines between Reservoir A and Reservoir B if the system operates under gravity alone (i.e., without the pump);

(b) sketch the qualitative behaviour of the energy and hydraulic grade lines between Reservoir A and Reservoir B when the pump is operating and the flow direction is from Reservoir A to Reservoir B;

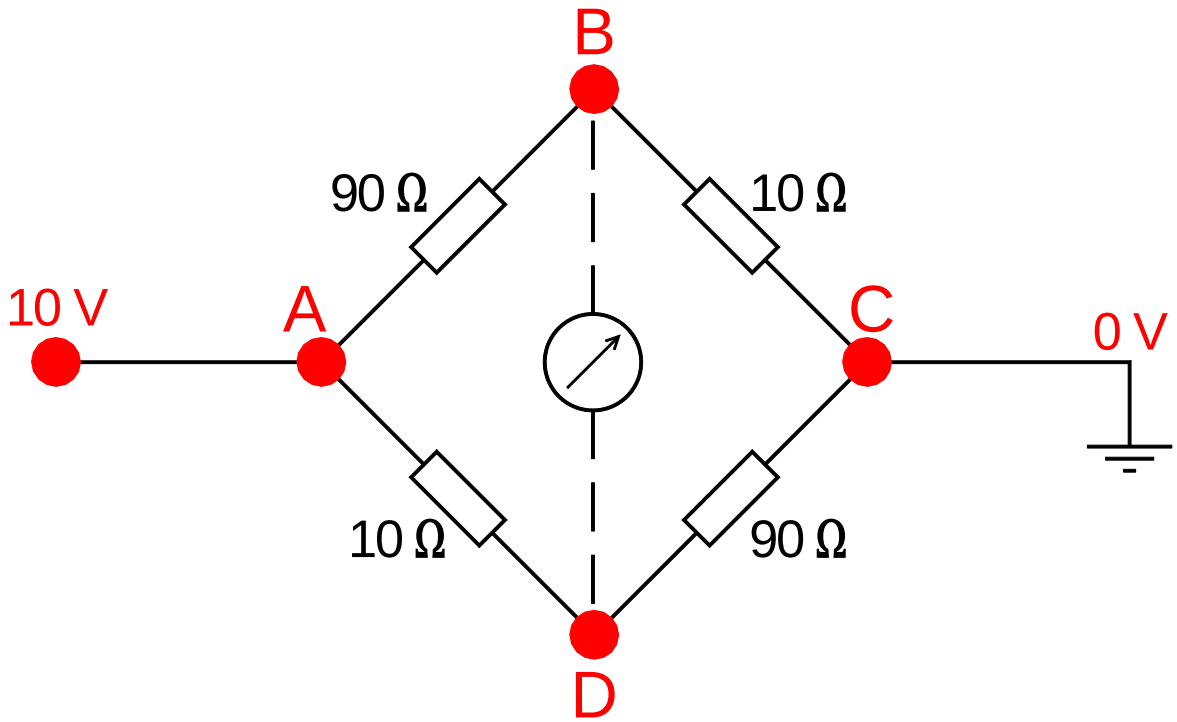
(c) find the pump head required to deliver a discharge of $0.025\ \mathrm{m^3\ s^{-1}}$ to reservoir B.

PIPE NETWORKS: EXAMPLE



Which way does the flow go in pipe BD?

PIPE NETWORKS: ELECTRICAL ANALOGUE



What are the voltages at B and D?

Which way does the current go in BD?

PIPE NETWORKS: BASIC RULES

1. **Continuity**: at any **junction**,

$$\sum Q_{\text{in}} = \sum Q_{\text{out}}$$

total flow in = total flow out

2. Each **point** has a unique **head**, H

3. Each **pipe** has a **head-loss vs discharge** (resistance) relation:

$$h = \alpha Q^2$$

ELECTRICAL ANALOGY

Continuity; unique head $\leftrightarrow$ Kirchoff's Laws

head, H $\leftrightarrow$ **potential** , V

discharge , Q $\leftrightarrow$ **current** , I

Resistance law:

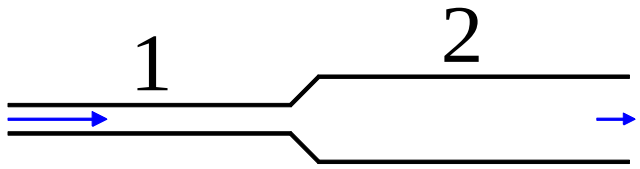
head loss , $\Delta H \propto Q^2$ $\leftrightarrow$ potential difference , $\Delta V \propto I$

What are the hydraulic analogues of:

- a **resistor**?
- a **capacitor**?
- an **inductor**?
- a **transistor**?

PIPES IN SERIES AND PARALLEL

Pipes in series



$$\Delta H = \alpha Q^2$$

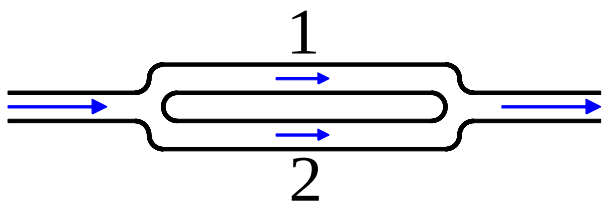
$Q = Q_1 = Q_2$ same flow

$\Delta H = \Delta H_1 + \Delta H_2$ add head changes

$$\alpha = \alpha_1 + \alpha_2$$

$$R = R_1 + R_2$$

Pipes in parallel



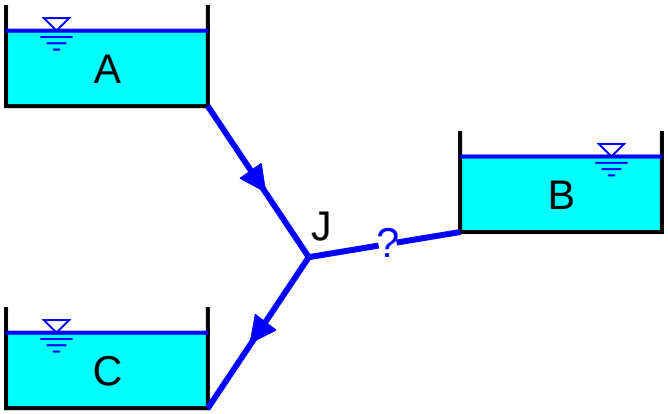
$\Delta H = \Delta H_1 = \Delta H_2$ same head change

$Q = Q_1 + Q_2$ add flows

$$\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{\alpha_1}} + \frac{1}{\sqrt{\alpha_2}}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

JUNCTION PROBLEMS: GENERAL METHOD



Method: Adjust H_J until **net** flow out of J is 0

(0) Establish the **head vs discharge** relations for all pipes: $\left|H_J - H_A\right| = \alpha Q_{JA}^2$ etc.

(1) Guess an initial value of **head** at the junction, H_J.

(2) Calculate **flow rates** in all pipes Q_{JA}, Q_{JB}, Q_{JC}

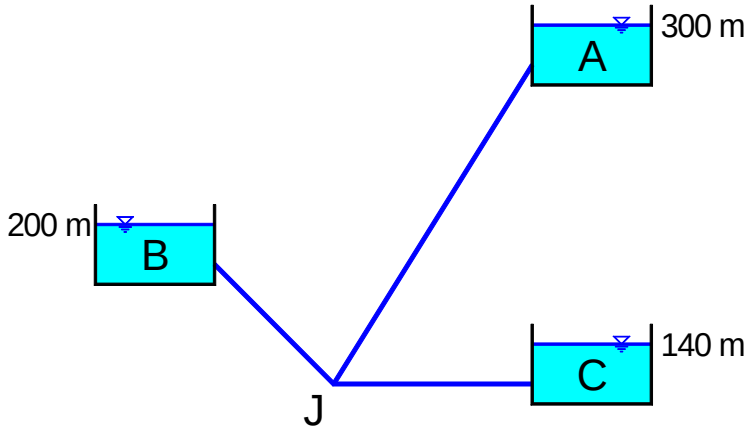
(3) Calculate **net flow** out of junction, $Q_{JA} + Q_{JB} + Q_{JC}$

(4) **Adjust** the head at the junction, H_J, until net flow out of junction = 0

EXAMPLE SHEET

In a water-storage scheme three reservoirs A, B and C are connected by a single junction J as shown. The water levels in A, B and C are 300 m, 200 m and 140 m respectively. The pipeline properties are given below. Friction factors may be assumed constant and minor losses may be neglected.

Pipeline	JA	JB	JC
Length L (m)	5000	3000	4000
Diameter D (m)	0.4	0.25	0.3
Friction factor λ	0.015	0.03	0.02



Calculate the total flow in each pipe and the direction of flow in pipe JB if:

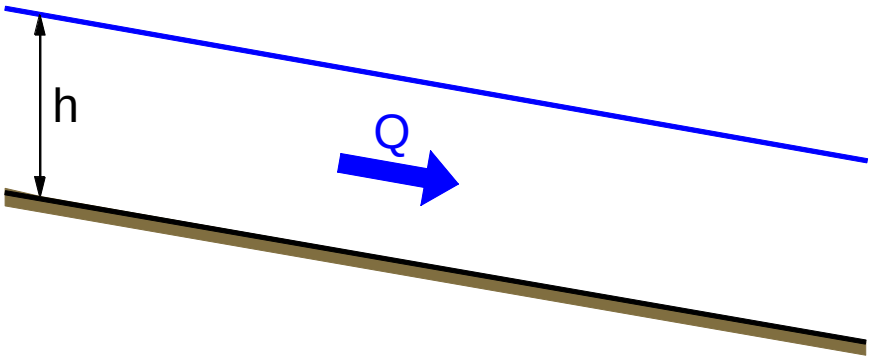
(a) there is a valve-regulated flow of 50 L s^{-1} to reservoir C but water flows freely under gravity in the other pipes;

(b) water flows freely under gravity in all pipes.

FLOW IN PIPES AND OPEN CHANNELS

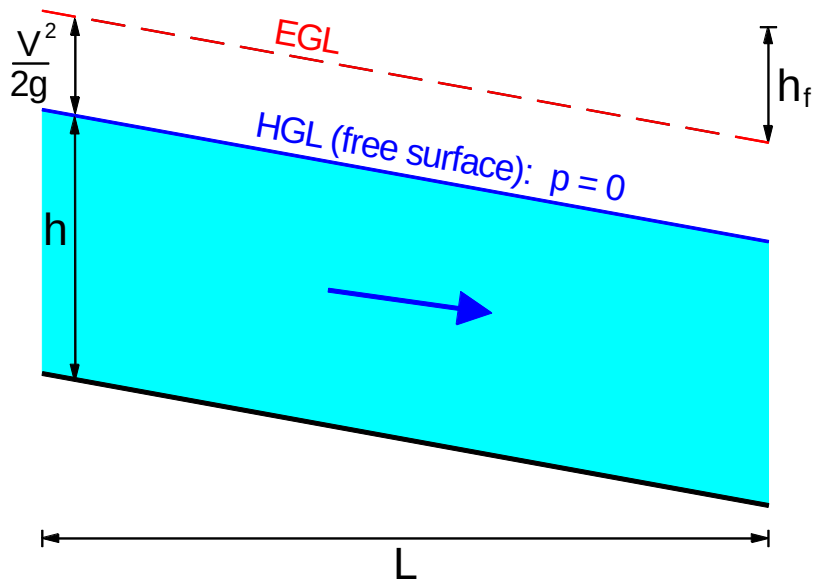
	PIPE FLOW	OPEN-CHANNEL FLOW
Fluid:	LIQUIDS or GASES	LIQUIDS (free surface)
Driven by:	PRESSURE, GRAVITY or BOTH	GRAVITY (down slope)
Size:	DIAMETER	HYDRAULIC RADIUS
Volume:	FILLS pipe	Depends on DEPTH
Equations:	DARCY-WEISBACH (head loss) COLEBROOK-WHITE (friction factor)	MANNING'S FORMULA

NORMAL FLOW



- **Normal flow** = steady, uniform flow (constant-depth flow under gravity)
- At best, an approximation for rivers / natural channels
- For any given Q there is a particular **normal depth**

NORMAL FLOW



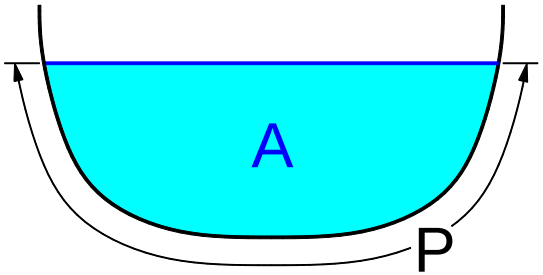
In normal flow:

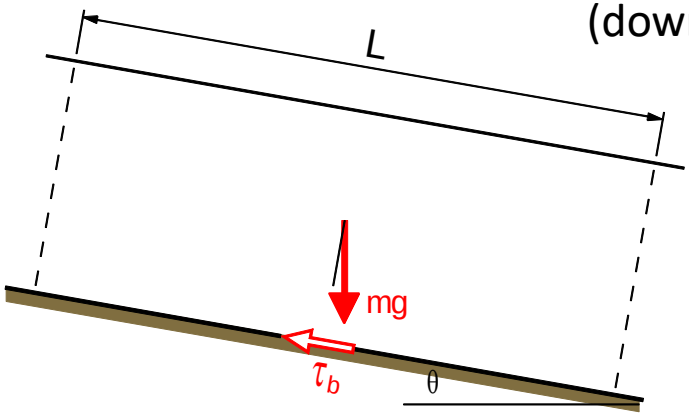
- Equal hydrostatic pressure forces at any cross section
- Downslope component of weight balances bed friction
- Channel bed, free surface (= HGL) and EGL are parallel; i.e. loss of fluid head equals drop in height
- Usual to assume small slopes

PART 1: BALANCE OF FORCES

A = area of fluid cross-section

P = wetted perimeter





(downslope) component of weight = friction on sides

$$mg\sin\theta = \tau_b \times \text{wetted surface area}$$

$$\rho ALg\sin\theta = \tau_b PL$$

$$\rho g\frac{A}{P}\sin\theta = \tau_b$$

Hydraulic radius (* depends on depth ***):**

$$R_h \equiv \frac{\text{cross} - \text{sectional area}}{\text{wetted perimeter}} = \frac{A}{P}$$

Normal-flow relationship:

$$\tau_b = \rho g R_h S$$

PART 2: EXPRESSION FOR FRICTION

$$\tau_b = \rho g R_h S$$

R_h is the **hydraulic radius**

$$c_f\left(\frac{1}{2}\rho V^2\right) = \rho g R_h S$$

definition of the **skin-friction coefficient**

$$V^2 = \frac{2g}{c_f} R_h S$$

Chézy's Formula:

$$V = C\sqrt{R_h S}$$

Robert Manning (compilation of experimental data):

$$C = R_h^{1/6} \times \text{function of roughness}$$

$$= \frac{R_h^{1/6}}{n}$$

Manning's Formula:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

MANNING'S ROUGHNESS COEFFICIENT

Channel type	Surface	n ($m^{-1/3}$ s)
Artificial lined channels	Glass	0.01
	Brass	0.011
	Steel, smooth	0.012
	painted	0.014
	riveted	0.015
	Cast iron	0.013
	Concrete, finished	0.012
	unfinished	0.014
	Planed wood	0.012
	Clay tile	0.014
	Brickwork	0.015
	Asphalt	0.016
	Corrugated metal	0.022
	Rubble masonry	0.025
Excavated earth channels	Clean	0.022
	Gravelly	0.025
	Weedy	0.03
	Stony, cobbles	0.035
Natural channels	Clean and straight	0.03
	Sluggish, deep pools	0.04
	Major rivers	0.035
Floodplains	Pasture, farmland	0.035
	Light brush	0.05
	Heavy brush	0.075
	Trees	0.15

CALCULATION FORMULAE (SUMMARY)

Manning's Formula: $$V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

V = average velocity

n = Manning's roughness parameter

S = slope (gradient)

R_h = **hydraulic radius** $= \frac{A}{P} = \frac{\text{cross} - \text{sectional area}}{\text{wetted perimeter}}$

Method

For a given channel:

1. Write area A and perimeter P as functions of a parameter (often depth, h)
2. Calculate hydraulic radius
3. Calculate average velocity
4. Calculate quantity of flow

Two Main Types of Problem

- Given h find Q
- Given Q find h

EXAMPLE SHEET

A V-shaped channel with sides sloping at $30°$ to the horizontal has a gradient of 1 in 100 and an estimated Manning's n of $0.012\ \mathrm{m}^{-1/3}\ \mathrm{s}$. Calculate:

(a) the discharge for a depth of 0.5 m;

(b) the depth when the discharge is $2\ \mathrm{m}^3\ \mathrm{s}^{-1}$.

CONVEYANCE

(a) Manning's formula: $V = \frac{1}{n} R_h^{2/3} S^{1/2}$ $\qquad R_h = \frac{A}{P}$

(b) Discharge: $Q = VA$

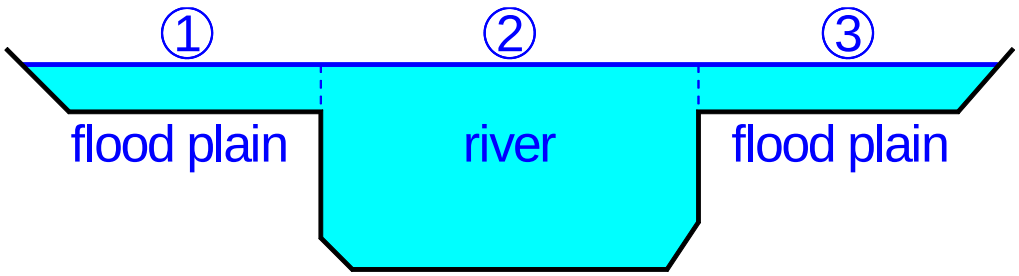
$$Q = \frac{1}{n}\left(\frac{A}{P}\right)^{2/3} S^{1/2} \times A$$

$$Q = KS^{1/2}$$

$$K = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}$$

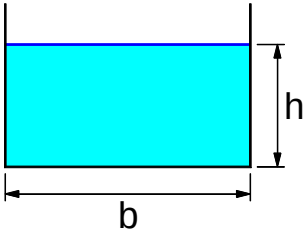
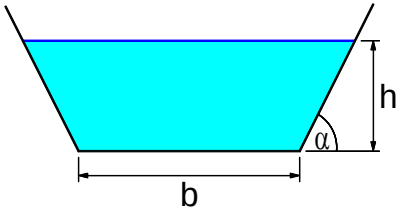
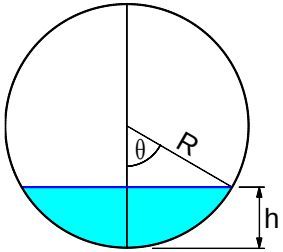
conveyance

For **compound channels** (e.g. river plus flood plain) simply add the conveyances:



$$K_{\text{eff}} = K_1 + K_2 + K_3$$

COMMON SHAPES OF CHANNEL

	rectangle	trapezoid	circle
			
area, A	bh	$bh + \frac{h^2}{\tan\alpha}$	$R^2(\theta - \frac{1}{2}\sin 2\theta)$
wetted perimeter, P	$b + 2h$	$b + \frac{2h}{\sin\alpha}$	$2R\theta$

EXAMPLE SHEET

A culvert used to divert run-off has a rectangular cross section with base width 0.4 m and side heights of 0.3 m. Manning's coefficient may be taken as $n = 0.012\ \mathrm{m}^{-1/3}\ \mathrm{s}$.

(a) Find the minimum slope S necessary to carry a discharge $Q = 0.3\ \mathrm{m}^3\ \mathrm{s}^{-1}$.

(b) If the slope from part (a) is doubled for the same discharge, calculate depth of flow.

EXAMPLE SHEET

A concrete pipe 750 mm in diameter is laid to a gradient of 1 in 200. The estimated value of Manning's n is $0.012\ \mathrm{m}^{-1/3}\ \mathrm{s}$.

Calculate the discharge when:

(a) the pipe is full;

(b) the depth is 90% of maximum.

Explain why the answer in (b) exceeds that in (a).