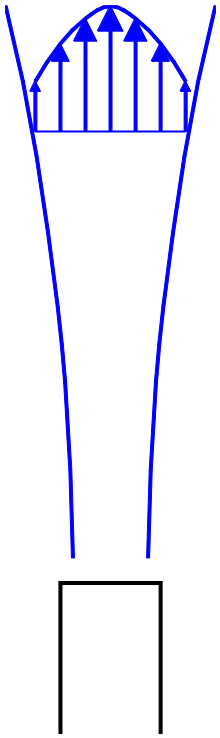
7. Turbulence

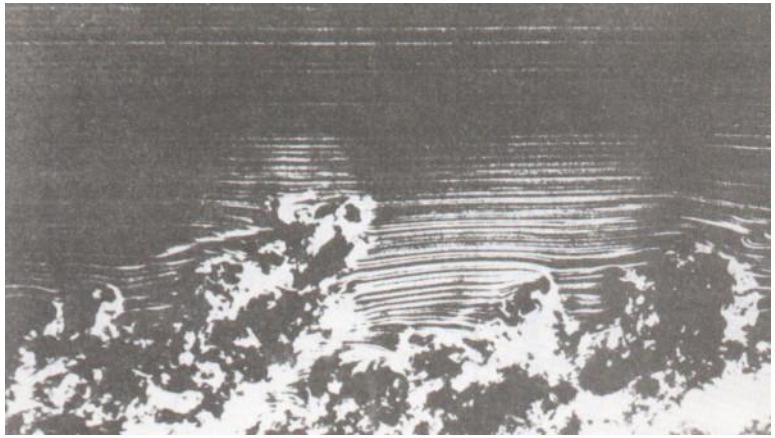
Turbulent Jet

Instantaneous

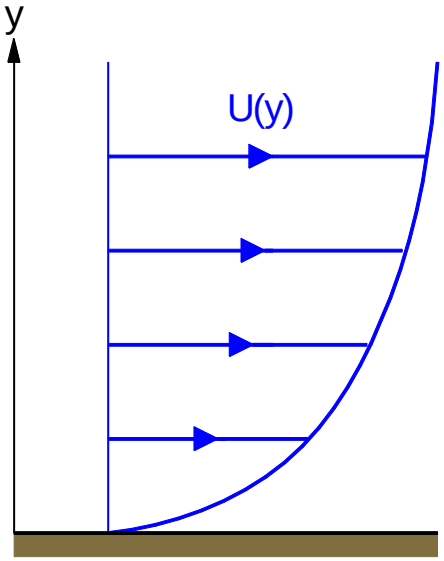


Mean

Turbulent Boundary Layer



Instantaneous



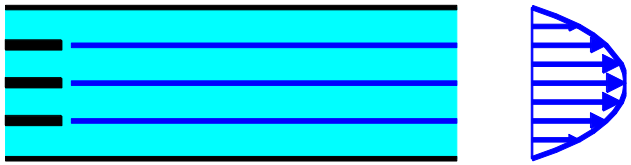
Mean

What is Turbulence?

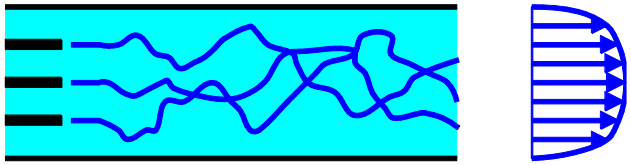
- A "random", 3-d, time-dependent eddying motion with many scales, superposed on an often drastically simpler **mean** flow
- A solution of the Navier-Stokes equations
- The natural state at **high Reynolds numbers**
- An efficient transporter and **mixer**
- A major source of **energy loss**
- A significant influence on **drag**:
  - **increases frictional drag** in non-separated flow
  - delays, or sometimes prevents, boundary-layer separation on curved surfaces, so **reducing pressure drag**.
- "*The last great unsolved problem of classical physics*"

Momentum Transfer in Laminar and Turbulent Flow

- **Laminar:**
  - smooth
  - **no mixing** of fluid
  - momentum transfer by **viscous forces**



- **Turbulent:**
  - chaotic
  - **mixing** of fluid
  - momentum transfer mainly by the **net effect of intermingling**



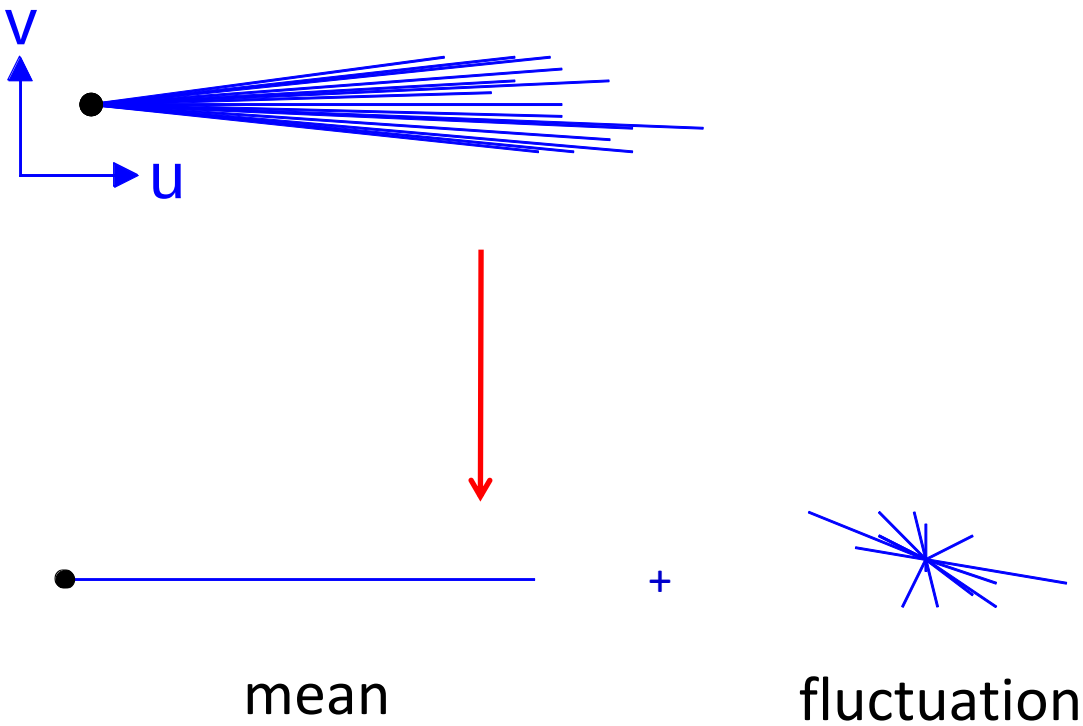
Regime determined by **Reynolds number**:

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$

"Low" $\mathrm{Re} \Rightarrow$ laminar; "high" $\mathrm{Re} \Rightarrow$ turbulent

"High" or "low" depends on the flow and the choice of U and L

Reynolds' Decomposition



$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$

$$p = \bar{p} + p'$$

Alternative Notations

- **Overbar** for mean, **prime** for fluctuation:

$$\overline{u} + u' \qquad\qquad -\rho\overline{u'v'}$$

- **Upper case** for mean, **lower case** for fluctuation:

$$U + u \qquad\qquad -\rho\overline{uv}$$

Reynolds Averaging of a Product

$$u = \overline{u} + u' \qquad \overline{u'} = 0$$

$$v = \overline{v} + v' \qquad \overline{v'} = 0$$

$$uv = (\overline{u} + u')(\overline{v} + v') \qquad = \overline{u}\,\overline{v} + \overline{u}v' + u'\overline{v} + u'v'$$

$$\overline{uv} = \overline{u}\,\overline{v} + \cancel{\overline{u}\overline{v'}} + \cancel{\overline{u'}\overline{v}} + \overline{u'v'}$$

$$\overline{uv} = \overline{u}\,\overline{v} + \overline{u'v'}$$

$\overline{u'v'}$ is the **covariance**

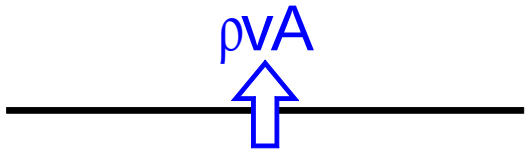
$$\overline{u^2} = \overline{u}^2 + \overline{u'^2}$$

$\overline{u'^2}$ is the **variance**

Effect of Turbulence on the Mean Flow

(i) Mass

Mass flux: $\rho v A$

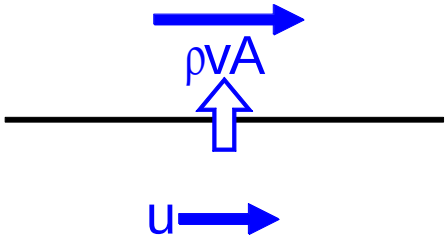


Average mass flux: $\rho \bar{v} A$

The **mean** velocity satisfies the same continuity equation as the **instantaneous** velocity

Effect of Turbulence on the Mean Flow

(ii) Momentum



(x-)momentum flux: $(\rho v A)u \quad = \rho(uv)A$

Average momentum flux : $\overline{(\rho v A)u} \quad = \rho(\overline{u}\,\overline{v} + \overline{u'v'})A$

extra term

Net rate of transport of momentum per unit area $\rho\overline{u'v'}$ from LOWER to UPPER ...

... is equivalent to $-\rho\overline{u'v'}$ from UPPER to LOWER ...

... has the same **dynamic** effect as a **stress** $-\rho\overline{u'v'}$

The **mean** velocity satisfies the same momentum equation as the **instantaneous** velocity, except for additional **apparent** stresses; e.g.

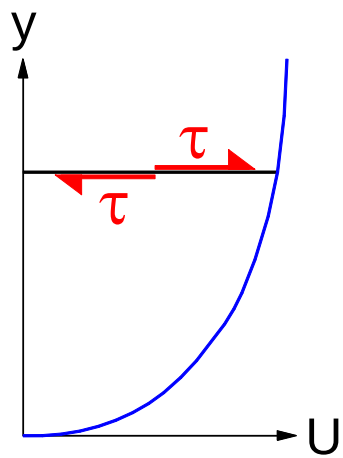
$$-\rho\overline{u'v'}, \qquad -\rho\overline{u'u'}$$

These are called the **Reynolds stresses**

Total Stress

In simple shear:

$$\tau = \underbrace{\mu \frac{\partial \bar{u}}{\partial y}}_{\text{viscous stress}} \quad \underbrace{-\rho \overline{u'v'}}_{\text{turbulent stress}}$$

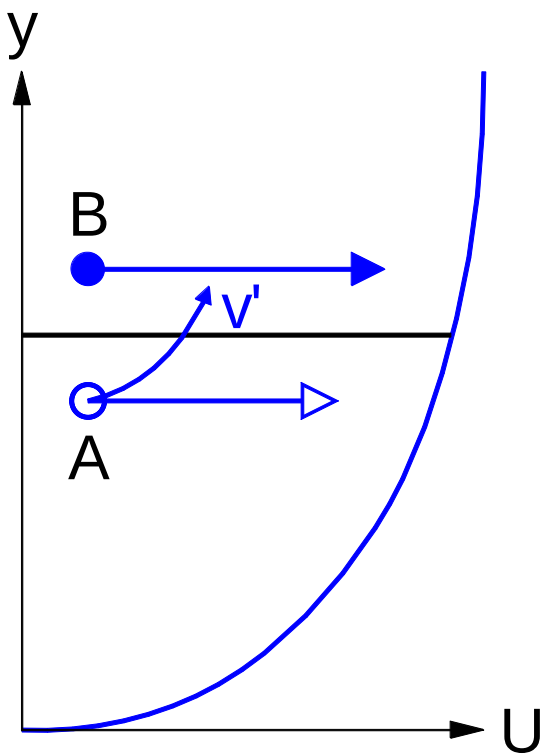


τ may be regarded as EITHER:

- the **force** per unit area exerted by upper on lower fluid OR
- the **rate of transport of momentum** (per unit area) from upper to lower fluid

The **dynamic** effect (average rate of transfer of momentum) is the same

Turbulent Transport

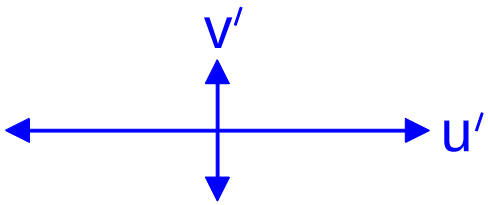


$\overline{u'v'}$ is negative

$\tau \equiv -\rho\overline{u'v'}$ is positive

Velocity Fluctuations

Normal stresses: $\overline{u'^2},\ \overline{v'^2},\ \overline{w'^2}$



Shear stresses: $\overline{v'w'},\ \overline{w'u'},\ \overline{u'v'}$

Turbulent kinetic energy:

$$k = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

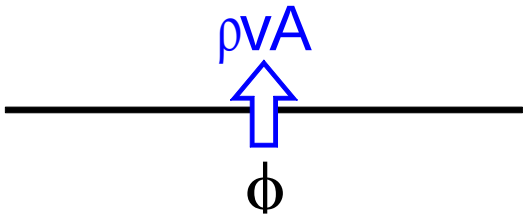
Turbulence intensity:

$$\frac{\text{root} - \text{mean} - \text{square fluctuation}}{\text{mean velocity}} = \frac{u'_{\text{rms}}}{U} = \frac{\sqrt{\frac{2}{3}k}}{U}$$

Effect of Turbulence on the Mean Flow

(iii) General Scalar

Scalar flux: $$(\rho v A)\phi = \rho(v\phi)A$$



Average scalar flux: $$\overline{(\rho v A)\phi} = \rho(\overline{v}\,\overline{\phi} + \overline{v'\phi'})A$$

extra term

The **mean** concentration satisfies the same transport equation as the **instantaneous** concentration, except for additional **turbulent fluxes**

$$\rho\overline{v'\phi'}$$

The Closure Problem

- To close the **mean** flow equations one must specify the **Reynolds stresses**

$$-\rho\overline{u'^2},\ -\rho\overline{u'v'},\ \ldots$$

  and other turbulent fluxes

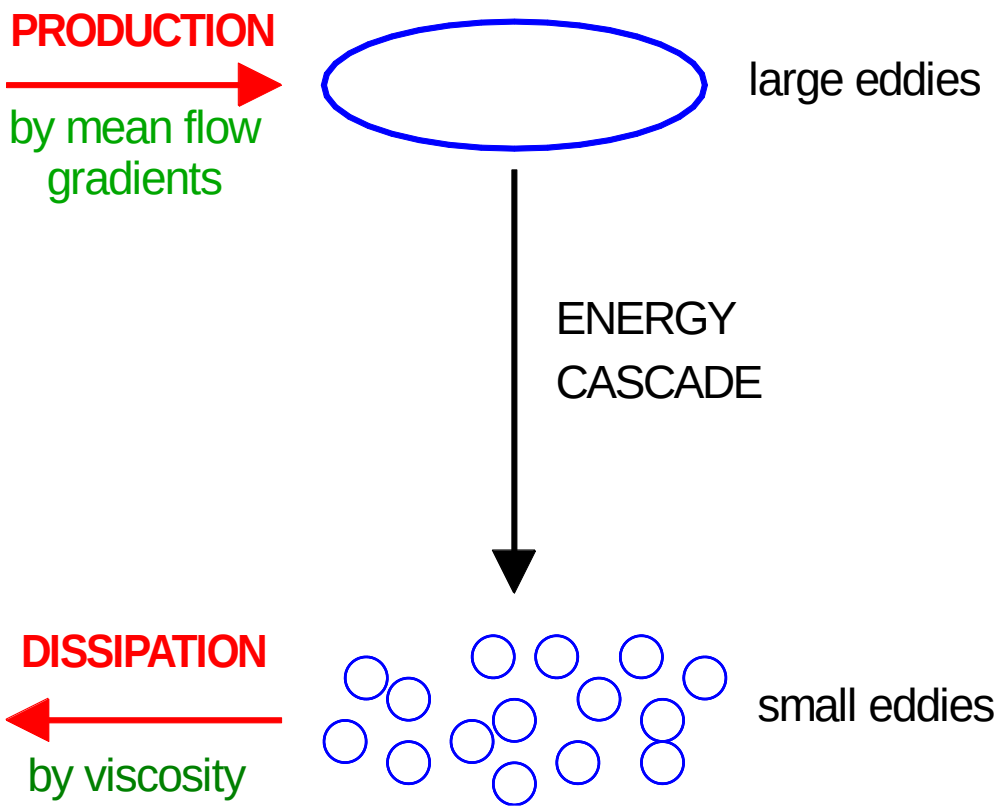
- A means of doing so is called a **turbulence model** or **turbulence closure**

- The simplest model assumes that the turbulent stress can be modelled in a similar manner to the viscous stress. In simple shear:

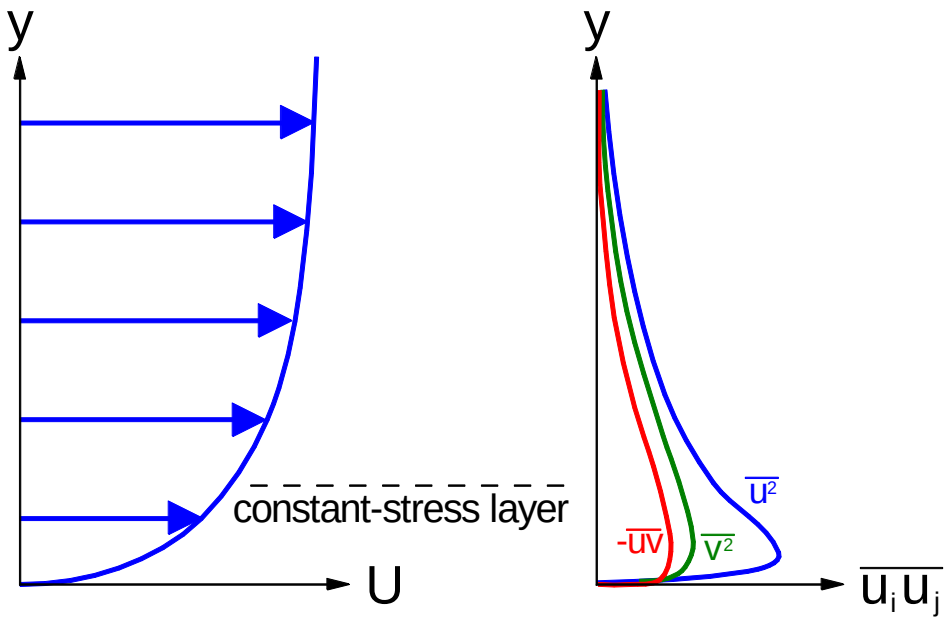
$$-\rho\overline{u'v'} = \mu_t \frac{\partial \overline{u}}{\partial y}$$

  This is called an **eddy-viscosity model**

Turbulence Energy: Production and Dissipation



Anisotropy



$$\overline{u'^2} \neq \overline{v'^2} \neq \overline{w'^2}$$

Turbulence Transport

- Turbulence is transported by the flow
- Each Reynolds stress satisfies its own scalar-transport equation
- The "source" term is a balance between:
  - **production** by **mean-velocity gradients**
  - **dissipation** by **viscosity**
  - **redistribution** by **pressure fluctuations**
- Turbulence **anisotropy** is a result of:
  - anisotropic **production** by particular **velocity gradients**
  - anisotropic **production** or damping by particular **body forces** (e.g. buoyancy)
  - selective **damping** of **wall-normal** fluctuations

Production of Turbulence Energy

Production of $\overline{u^2}$: $$P_{11} = -2(\overline{uu}\frac{\partial U}{\partial x} + \overline{uv}\frac{\partial U}{\partial y} + \overline{uw}\frac{\partial U}{\partial z})$$

Production of $\overline{uv}$: $$P_{12} = -(\overline{uu}\frac{\partial V}{\partial x} + \overline{uv}\frac{\partial V}{\partial y} + \overline{uw}\frac{\partial V}{\partial z}) - (\overline{vu}\frac{\partial U}{\partial x} + \overline{vv}\frac{\partial U}{\partial y} + \overline{vw}\frac{\partial U}{\partial z})$$

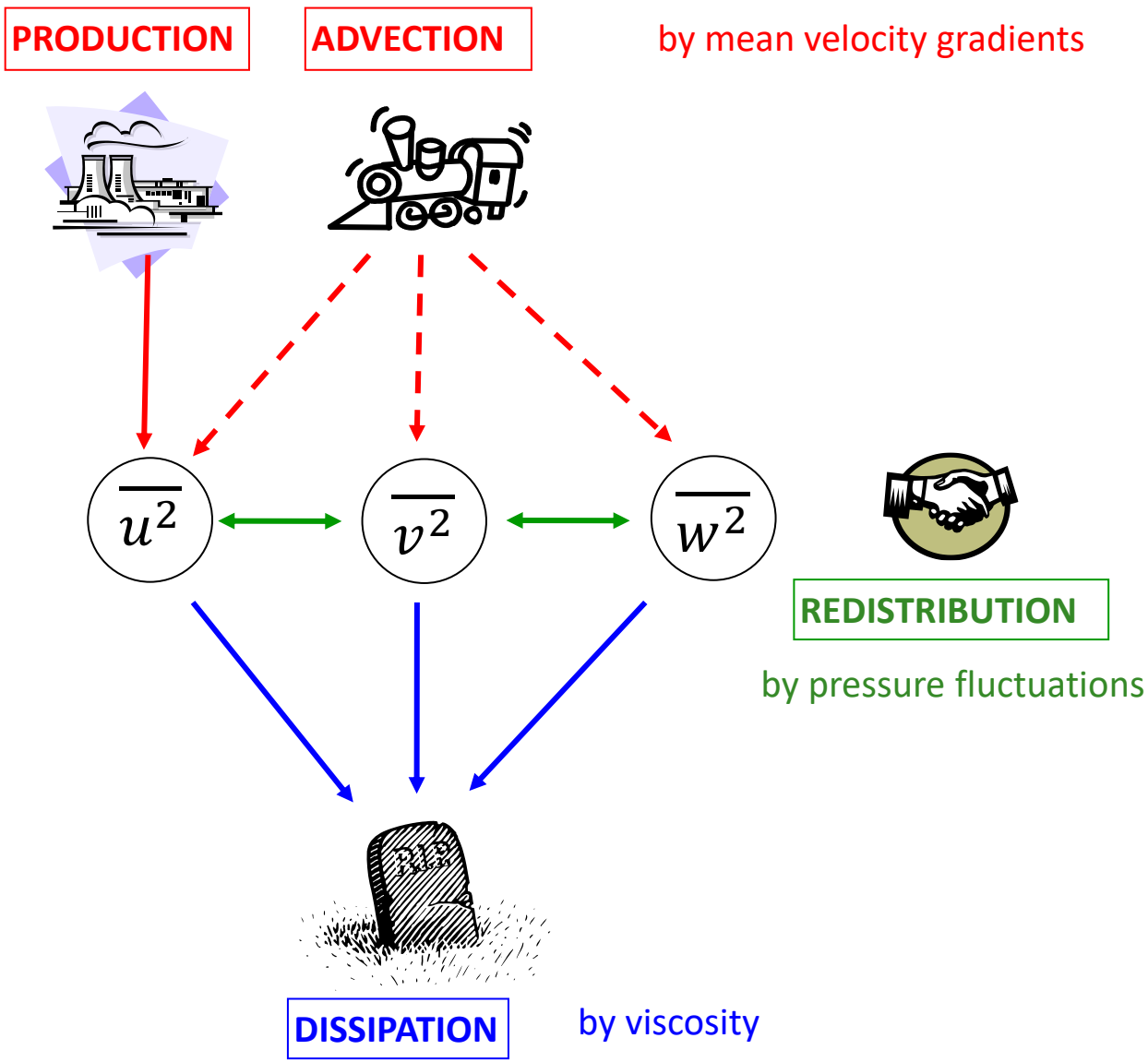
Production of $\overline{u_i u_j}$: $$P_{ij} = -(\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k})$$

Production of k: $$P^{(k)} = \frac{1}{2}(P_{11} + P_{22} + P_{33})$$

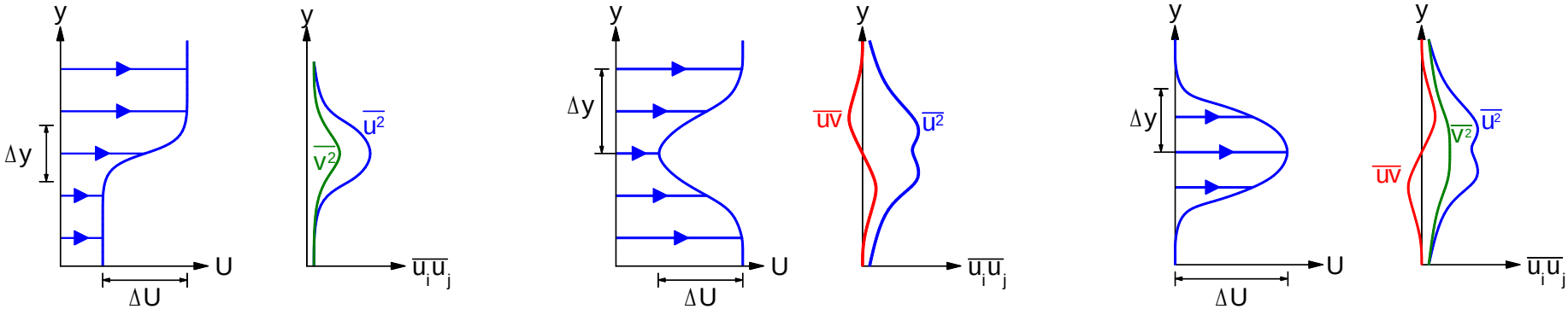
In **simple shear**: $$P_{11} = -2\overline{uv}\frac{\partial U}{\partial y}, \quad P_{22} = P_{33} = 0$$

$$P^{(k)} = -\overline{uv}\frac{\partial U}{\partial y}$$

Energy is preferentially supplied to the streamwise normal stress, $\overline{uu}$


PRODUCTION
ADVECTION
by mean velocity gradients
$\overline{u^2}$
$\overline{v^2}$
$\overline{w^2}$
REDISTRIBUTION
by pressure fluctuations
RIP
DISSIPATION
by viscosity

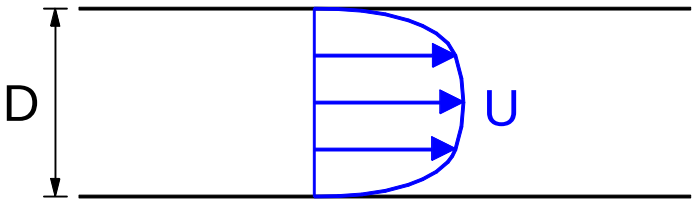
Free Shear Flows



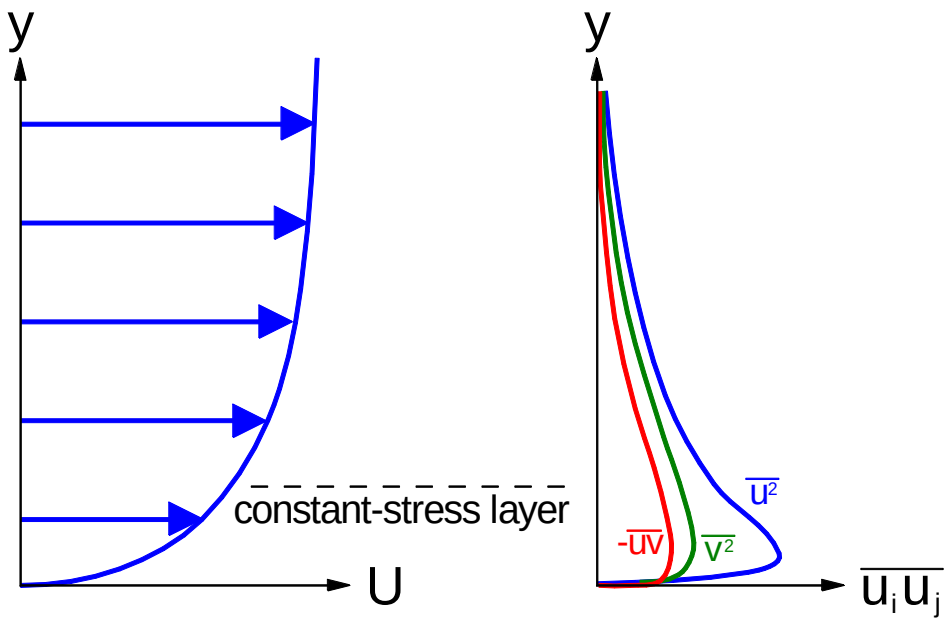
Mixing layer **Wake** **Jet**

- Maximum turbulence where velocity gradient $|\partial U/\partial y|$ is largest
- $\overline{uv}$ has the opposite sign to $\partial U/\partial y$ and vanishes where this is zero
- Turbulence is anisotropic: $\overline{u^2} > \overline{v^2}$

Wall-Bounded Flows

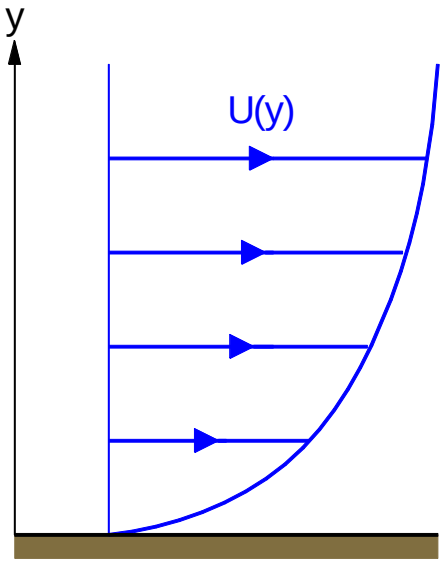


Pipe flow



Flat-plate boundary layer

Parameters in Wall-Bounded Flows



Wall shear stress: τ_w

Friction velocity: $\tau_w = \rho u_\tau^2$

$$u_\tau = \sqrt{\tau_w/\rho}$$

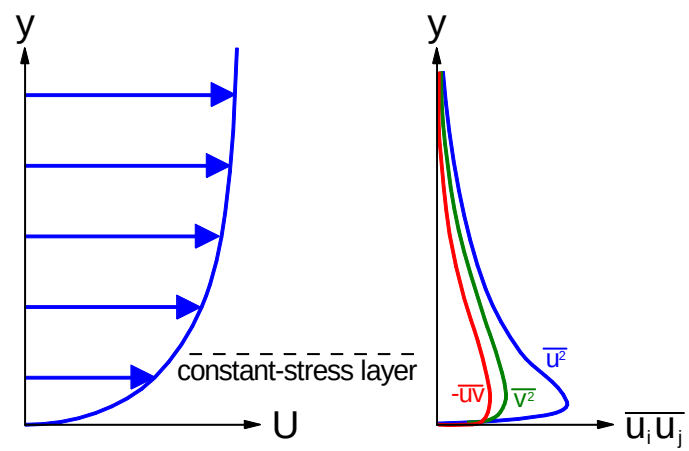
Non-dimensional height: $y^+ = \dfrac{y u_\tau}{\nu}$

Non-dimensional velocity: $U^+ = \dfrac{U}{u_\tau}$

wall units

Key Regions of a Turbulent Boundary Layer

$$\tau_w \approx \tau = \mu \frac{\partial U}{\partial y} - \rho \overline{uv}$$



Viscous sublayer ($y^+ < 5$):

– viscous stress dominant

$$\tau_w = \mu \frac{\partial U}{\partial y} \qquad U = \frac{\tau_w}{\mu} y$$

Log layer ($30 < y^+ < 500$):

– direct effect of viscosity negligible

$$\frac{\partial U}{\partial y} \propto \frac{u_\tau}{y} \qquad \Rightarrow \quad \frac{\partial U}{\partial y} = \frac{u_\tau}{\kappa y} \qquad \Rightarrow \quad U = \frac{u_\tau}{\kappa} \ln y + \text{constant}$$

For smooth walls:

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln \frac{y u_\tau}{\nu} + B \qquad \text{or} \qquad \frac{U}{u_\tau} = \frac{1}{\kappa} \ln E \frac{y u_\tau}{\nu}$$

In **wall units**:

$$U^+ = \frac{U}{u_\tau}, \qquad y^+ = \frac{y u_\tau}{\nu}$$

$$\boxed{\begin{array}{ll} \text{(viscous sublayer)} & U^+ = y^+ \\ \text{(log layer)} & U^+ = \frac{1}{\kappa} \ln y^+ + B \end{array}}$$

Summary (1)

- Turbulence is a 3-d, time-dependent eddying motion with many scales, causing continuous mixing
- Any instantaneous flow variable may be decomposed as **mean** + **fluctuation**
- The process of averaging turbulent variables is called **Reynolds averaging** and leads to the **Reynolds-averaged Navier-Stokes (RANS) equations**
- The **product** of turbulent fluctuations makes a net contribution to the transport of momentum and other quantities via **Reynolds stresses** and **turbulent fluxes**

Summary (2)

- A **turbulence model** is a means of specifying turbulent fluxes in order to close the mean-flow equations

- Turbulent energy is:
  - **generated** (anisotropically, at large scale) by mean-velocity gradients
  - **redistributed** amongst components by pressure fluctuations
  - **dissipated** (at small scale) by viscosity

- Turbulence modelling is guided by theory and experiments for fundamental **free** and **wall-bounded** flows: in particular, by the **log law**