Classroom Example 1

Velocities, with correction:

$$u_A = 10 \quad \text{(boundary condition)}$$
$$u_B = 8 + 2(p'_1 - p'_2)$$
$$u_C = 11 + 2(p'_2 - p'_3)$$

Apply mass conservation for each cell. Since densities and areas are the same on each face they may be omitted.

<u>Cell 1</u>:

$$u_B - u_A = 0$$
$$\Rightarrow \quad 8 + 2(p'_1 - p'_2) - 10 = 0$$
$$\Rightarrow \quad 2p'_1 - 2p'_2 = 2$$

<u>Cell 2</u>:

$$u_C - u_B = 0$$
$$\Rightarrow \quad 11 + 2(p'_2 - p'_3) - 8 - 2(p'_1 - p'_2) = 0$$
$$\Rightarrow \quad -2p'_1 + 4p'_2 - 2p'_3 = -3$$

This gives two equations for three unknowns (p'_1,p'_2 and p'_3). Since pressure is defined only up to a constant (i.e. only pressure *differences* matter) we can, WLOG, take any one of them to be what we like. Corresponding to a common outflow boundary condition with pressure fixed we take $p'_3 = 0$. (Alternatively, we can just leave them to "float" in value – the unknown pressure will simply cancel when we subtract pressures to compute velocity corrections). Then, simplifying,

$$p'_1 - p'_2 = 1$$
$$-p'_1 + 2p'_2 = -\frac{3}{2}$$

These have solution $p'_2 = -1/2$, $p'_1 = 1/2$.

Substituting in the velocity-correction formulae:

$$u_B = 8 + 2(p'_1 - p'_2) \quad = 10$$
$$u_C = 11 + 2(p'_2 - p'_3) \quad = 10$$

i.e. the velocity has the constant value 10 (as expected).

Classroom Example 2

(a) Velocities – with correction:

$$u_B = 11 + 2(p'_A - p'_B)$$
$$u_D = 14 + 2(p'_C - p'_D)$$
$$v_C = 8 + 3(p'_A - p'_C)$$
$$v_D = 5 + 3(p'_B - p'_D)$$

Apply mass conservation for each cell in turn. Since densities and areas are the same on each face they may be omitted.

Cell A:

$$u_B - 5 + v_C - 15 = 0$$
$$\Rightarrow \quad 11 + 2(p'_A - p'_B) - 5 + 8 + 3(p'_A - p'_C) - 15 = 0$$
$$\Rightarrow \quad 5p'_A - 2p'_B - 3p'_C = 1$$

Cell B:

$$5 - u_B + v_D - 10 = 0$$
$$\Rightarrow \quad 5 - 11 - 2(p'_A - p'_B) + 5 + 3(p'_B - p'_D) - 10 = 0$$
$$\Rightarrow \quad -2p'_A + 5p'_B - 3p'_D = 11$$

Cell C:

$$u_D - 10 + 5 - v_C = 0$$
$$\Rightarrow \quad 14 + 2(p'_C - p'_D) - 10 + 5 - 8 - 3(p'_A - p'_C) = 0$$
$$\Rightarrow \quad -3p'_A + 5p'_C - 2p'_D = -1$$

Cell D:

$$20 - u_D + 10 - v_D = 0$$
$$\Rightarrow \quad 20 - 14 - 2(p'_C - p'_D) + 10 - 5 - 3(p'_B - p'_D) = 0$$
$$\Rightarrow \quad -3p'_B - 2p'_C + 5p'_D = -11$$

Assembling these into a single matrix equation:

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ -2 & 5 & 0 & -3 \\ -3 & 0 & 5 & -2 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 11 \\ -1 \\ -11 \end{pmatrix}$$

Gaussian elimination as follows

$R2 \to 5R2 + 2R1$
$R3 \to 5R3 + 3R1$
$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & -6 & 16 & -10 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 57 \\ -2 \\ -11 \end{pmatrix}$$

$R2 \to R2/3$
$R3 \to R3/2$
$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & -3 & 8 & -5 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ -1 \\ -11 \end{pmatrix}$$

$R3 \to 7R3 + 3R2$
$R4 \to 7R4 + 3R2$
$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & 0 & 50 & -50 \\ 0 & 0 & -20 & 20 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ 50 \\ -20 \end{pmatrix}$$

$R3 \to R3/50$
$R4 \to R4/20$
$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ 1 \\ -1 \end{pmatrix}$$

$R4 \to R4 + R3$
$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 7 & -2 & -5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 1 \\ 19 \\ 1 \\ 0 \end{pmatrix}$$

p'_D can be chosen arbitrarily; we could set an arbitrary convenient value, but it is perfectly legitimate to leave it as undetermined.

Back-substituting:

$$p'_C - p'_D = 1$$
$$\Rightarrow \quad p'_C = 1 + p'_D$$

$$7p'_B - 2p'_C - 5p'_D = 19$$
$$\Rightarrow \quad p'_B = \frac{19 + 2p'_C + 5p'_D}{7} \quad = p'_D + 3$$

$$5p'_A - 2p'_B - 3p'_C = 1$$
$$\Rightarrow \quad p'_A = \frac{1 + 2p'_B + 3p'_C}{5} \quad = p'_D + 2$$

Finally, use these to correct the velocity:

$$u_B = 11 + 2(p'_A - p'_B) \quad = 9$$

$$u_D = 14 + 2(p'_C - p'_D) \quad = 16$$

$$v_C = 8 + 3(p'_A - p'_C) \quad = 11$$

$$v_D = 5 + 3(p'_B - p'_D) \quad = 14$$

with p'_D cancelling in each case.

(b) Although (non-unique) velocity corrections alone could have been computed to make the system mass-consistent, the only way to ensure that it also remains a solution of the momentum equation is to relate velocity and pressure corrections in the form implied by the momentum equation.

Q1.
(a)

$$(p_w - p_e)A$$

(b) Since $V = A\Delta x$, the net force per unit volume is

$$\frac{(p_w - p_e)A}{A\Delta x} = \frac{p_w - p_e}{\Delta x}$$

(c) The average pressure gradient is

$$\frac{p_e - p_w}{\Delta x}$$

which is minus the expression in part (b).

Hence, shrinking the control volume to a point, the pressure force in the x-direction, per unit volume, is $-\partial p/\partial x$.

Q2.
The net pressure force component in the x or y direction on a face is given by:

pressure × projected area *into* the cell.

Hence,

$$F_x = -\sum_{faces} p\Delta y$$

$$F_y = \sum_{faces} p\Delta x$$

where Δx and Δy are coordinate increments when the cell is traversed anticlockwise.

Starting from the lowest face (edge in 2d):

$$F_x = -\sum_{faces} p\Delta y \quad = 5\times 2 + 7\times(-4) + 9\times 2 \quad = 0$$

$$F_y = \sum_{faces} p\Delta x \quad = 5\times 4 + 7\times(-2) - 9\times 2 \quad = -12$$

Answer: net force = (0, –12).

In general,

$$\mathbf{F} = -\sum_{faces} p\mathbf{A}$$

where $\mathbf{A}$ is the outward face area vector (Section 9).

Q3.
The net pressure force component in the *x* or *y* direction on a face is given by:

pressure × *projected area* <u>*into*</u> *the cell*.

Hence,

$$F_x = -\sum_{faces} p\Delta y$$
$$F_y = \sum_{faces} p\Delta x$$

where Δx and Δy are coordinate increments when the cell is traversed anticlockwise.. Hence,

$$F_x = -\sum_{faces} p\Delta y \quad = -[2\times 3 + 4\times 1 + 10\times(-3) + 1\times(-1)] \quad = 21$$
$$F_y = \sum_{faces} p\Delta x \quad = 2\times 1 + 4\times(-4) + 10\times(-2) + 1\times 5 \quad = -29$$

Answer: F = (21,–29) units.

Q4.

(a) The discretised momentum equation for a single finite-volume cell can be written in the form (e.g. for the x-component of momentum):

$$a_P u_P - \sum_F a_F u_F = A(p_w - p_e) + other\ \ forces$$

Thus, there is a relationship between the velocity at any location and the centred pressure difference across it:

$$u_p = d_p(p_w - p_e) + \cdots$$

or, in terms of centred differences:

$$u = -d\Delta p + \cdots$$

If this relationship is translated to the velocities on cell faces then the mass-conservation equation yields (focusing on the x-directed faces):

$$\begin{aligned} 0 &= (\rho u A)_e - (\rho u A)_w + \cdots \\ &= (\rho d A)_e (p_P - p_E) - (\rho d A)_w (p_W - p_P) + \cdots \end{aligned}$$

hence producing a discrete equation between pressure values at nodes.

(b) If velocity and pressure are co-located then, because interpolation must be applied to determine pressures on cell faces, the momentum equation gives (in index notation):

$$\begin{aligned} u_i &= d_i \left[\frac{1}{2}(p_{i-1} + p_i) - \frac{1}{2}(p_i + p_{i+1})\right] + \cdots \\ &= \frac{1}{2} d_i [p_{i-1} - p_{i+1}] + \cdots \end{aligned}$$

Thus, the net pressure force depends only on the difference in pressure between alternate, not successive nodes.

If linear interpolation is used to get the velocities on cell faces in the continuity equation then

$$\begin{aligned} 0 &= (\rho u A)_{i+1/2} - (\rho u A)_{1-1/2} + \cdots \\ &= (\rho A)\left[\frac{1}{2}(u_i + u_{i+1}) - \frac{1}{2}(u_{i-1} + u_i)\right] + \cdots \\ &= \frac{1}{2}(\rho A)[u_{i+1} - u_{i-1}] + \cdots \\ &= \frac{1}{4}(\rho A)[d_{i+1}(p_i - p_{i+2}) - d_{i-1}(p_{i-2} - p_i)] + \cdots \end{aligned}$$

Hence, in a co-located arrangement with linear interpolation for advective fluxes both momentum and continuity equations lead to relations between pressures at alternate rather than successive nodes, leading to decoupling of the odd and even nodal values.

The interpolation procedure of Rhie and Chow separately interpolates pressure and non-pressure (*pseudo-velocity*) parts to the cell face. i.e. if the momentum equation predicts a velocity-pressure linkage of the form

$$u = \hat{u} - d\Delta p$$

then the value on a cell face is determined by

$$u_{face} = \hat{u}_{face} - d_{face}\Delta p$$

where Δ denotes a centred difference. The pseudo-velocity has first to be worked out at nodes by inverting the relationship:

$$\hat{u} = u + d\Delta p$$

with the centred pressure difference this time being worked out across the cell.

(c)

	•		•		•		•
u	5		4		3		2
p	0.6		0.7		1.1		1.6
p_{face}		0.65		0.9		1.35	
$\hat{u}_{node}$ $(= u + 3\Delta p)$			4.75		4.35		
$\hat{u}_{face}$				4.55			
$u_{face}(= \hat{u}_{face} - 3\Delta p)$				3.35			

Answer: the velocity on cell face *f* is 3.35.

(d) SIMPLE method:

(i) Solve the momentum equations with the current pressure p*.

(ii) Rewrite the mass-conservation equation as a pressure-correction equation using the velocity-correction formulae:

$$u \to u * -d\Delta p' \text{ (applied at scalar-cell faces)}$$

(iii) Solve the resulting pressure-correction equation and correct both velocity and pressure:

$$u \to u * -d\Delta p' \text{ (applied at velocity nodes)}$$

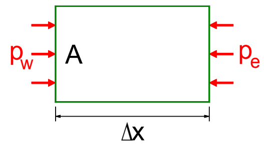
$$p \to p * +p'$$

Repeat (i)–(iii) until both momentum and continuity equations are satisfied simultaneously.

Q5.

(a) If the cell length in the x direction is Δx and the cross-sectional area is A then the net pressure force in the x direction is

$$(p_w - p_e)A$$



whilst the volume is

$$V = A\Delta x$$

The net force per unit volume is then

$$\frac{(p_w - p_e)A}{A\Delta x} = \frac{p_w - p_e}{\Delta x} \quad = -(\frac{p_e - p_w}{\Delta x})$$

In the limit as $\Delta x \to 0$ this tends to $-\partial p/\partial x$.

(b) The Rhie-Chow algorithm is used:

- to predict the advective velocities (the cell-face velocities in the mass flux);
- to prevent odd-even pressure decoupling that arises from centred differencing of pressure forces and linear interpolation of velocities.

(c)

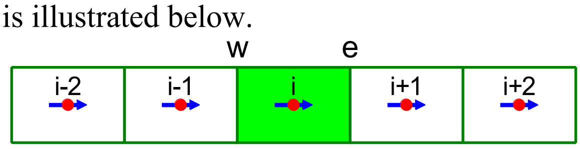
(i) By linear interpolation (just average the velocities either side) we have

$u_w = 2.5$; $u_e = 3.5$

(ii) By Rhie-Chow interpolation, the complete calculation sequence is:

- calculate pressures at cell *faces*, p_f, by interpolation;
- calculate pseudovelocities $\hat{u}$ at *nodes* by inversion: $\hat{u} = u + \frac{1}{2}\Delta p$;
- calculate pseudovelocities $\hat{u}$ at *faces* by interpolation;
- calculate final advective velocity at *faces* by $u = \hat{u} - \frac{1}{2}\Delta p$.

This is illustrated below.



	i-2		i-1	w	i	e	i+1		i+2
$p =$	2		6		5		7		4
$u =$	1		2		3		4		5
$p_f =$		4		5.5		6		5.5	
$\hat{u} =$			2.75		3.25		3.75		
$\hat{u}_f =$				3		3.5			
$u_f =$				3.5		2.5			

The final face velocities are:

$u_w = 3.5$; $u_e = 2.5$

Comparing the pairs of values for u_w and u_e it can be seen that the Rhie-Chow algorithm reverses the order of the two velocities. Inspection of the pressure field shows that it is trying to reduce the local peak to the left by increasing u_w and reduce the local peak to the right by decreasing u_e.

(d) Corrected velocities on the west and east faces are:

$$u_w = 3.5 - \frac{1}{2}(p'_i - p'_{i-1})$$

$$u_e = 2.5 - \frac{1}{2}(p'_{i+1} - p'_i)$$

Continuity requires that

$$u_e A - u_w A = 0$$

where A is a cell-face area. Substituting, and dividing by A, gives:

$$2.5 - \frac{1}{2}(p'_{i+1} - p'_i) - 3.5 + \frac{1}{2}(p'_i - p'_{i-1}) = 0$$

$$\Rightarrow \qquad -\frac{1}{2}p'_{i-1} + p'_i - \frac{1}{2}p'_{i+1} = 1$$

This is the required pressure-correction equation.

Q6.
Corrected velocities:

$$u_1 = 4$$
$$u_2 = 3 - 4(p'_2 - p'_1)$$
$$u_3 = 5 - 4(p'_3 - p'_2)$$
$$u_4 = 6 - 4(p'_4 - p'_3)$$

Mass conservation for cells centred on the internal pressure nodes yields pressure-correction equations as follows.

<u>Cell 1</u>

$$u_2 - u_1 = 0$$
$$\Rightarrow \quad 3 - 4(p'_2 - p'_1) - 4 = 0$$
$$\Rightarrow \quad 4p'_1 - 4p'_2 = 1$$

<u>Cell 2</u>

$$u_3 - u_2 = 0$$
$$\Rightarrow \quad 5 - 4(p'_3 - p'_2) - 3 + 4(p'_2 - p'_1) = 0$$
$$\Rightarrow \quad -4p'_1 + 8p'_2 - 4p'_3 = -2$$

<u>Cell 3</u>

$$u_4 - u_3 = 0$$
$$\Rightarrow \quad 6 - 4(p'_4 - p'_3) - 5 + 4(p'_3 - p'_2) = 0$$
$$\Rightarrow \quad -4p'_2 + 8p'_3 - 4p'_4 = -1$$

Solving à la tri-diagonal matrix algorithm …
From the first cell:

$$4p'_1 = 4p'_2 + 1$$

From the second cell:

$$-(4p'_2 + 1) + 8p'_2 - 4p'_3 = -2 \qquad \text{or} \qquad 4p'_2 = 4p'_3 - 1$$

From the third cell:

$$-(4p'_3 - 1) + 8p'_3 - 4p'_4 = -1 \qquad \text{or} \qquad 4p'_3 = 4p'_4 - 2$$

Hence,

$$p'_3 = p'_4 - \tfrac{1}{2}$$
$$p'_2 = p'_4 - \tfrac{3}{4}$$
$$p'_1 = p'_4 - \tfrac{1}{2}$$

(p'_4 can be chosen as anything convenient, since it is only pressure differences that matter).

Substituting these into the velocity-correction formulae gives

$$u_1 = u_2 = u_3 = u_4 = 4$$

as anticipated.

Q7.

(a) Pressure-correction methods are iterative methods for the simultaneous solution of mass and momentum equations, which work by making small corrections to the pressure field to "nudge" the velocity field toward mass conservation.

They consist of alternate iterations of:

- the momentum equation with the current pressure;
- a pressure-correction equation, derived by substituting the velocity-pressure relationship implied by the momentum equation into the continuity equation.

(b)

- SIMPLE is iterative, PISO is non-iterative;
- PISO is time-dependent; SIMPLE may be steady or time-dependent.

(c) Total flow rates:

inflow: $6A + 6A = 12A$

outflow: $12A + 4A = 16A$

where A is the face area of a cell.

For global mass conservation we require "total flow in = total flow out" and hence the outflow velocities must be multiplied by a scaling factor

$$\frac{12}{16} = 0.75$$

The outflow velocities then become

$u_E = 9, \quad u_F = 3$

(d)

Applying mass conservation to scalar cells A – D in turn we require (on dividing by the common cell face area A):

$$u_C - 6 + 0 - v_A = 0$$
$$u_D - 6 + v_A - 0 = 0$$
$$9 - u_C + 0 - v_C = 0$$
$$3 - u_D + v_C - 0 = 0$$

Substituting for the velocities in terms of the current velocities and pressure corrections:

$$2 + 3(p'_A - p'_C) - 6 + 0 - 1 - 2(p'_B - p'_A) = 0$$
$$1 + 3(p'_B - p'_D) - 6 + 1 + 2(p'_B - p'_A) - 0 = 0$$
$$9 - 2 - 3(p'_A - p'_C) + 0 - (-2) - 2(p'_D - p'_C) = 0$$
$$3 - 1 - 3(p'_B - p'_D) + (-2) + 2(p'_D - p'_C) - 0 = 0$$

Hence,

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ -2 & 5 & 0 & -3 \\ -3 & 0 & 5 & -2 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -9 \\ 0 \end{pmatrix}$$

Solve by Gaussian elimination:

$R2 \to 5R2 + 2R1$
$R3 \to 5R3 + 3R1$

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & -6 & 16 & -10 \\ 0 & -3 & -2 & 5 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ -30 \\ 0 \end{pmatrix}$$

$R3 \to 7R3 + 2R2$
$R4 \to 7R4 + R2$

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & 0 & 100 & -100 \\ 0 & 0 & -20 & 20 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ -150 \\ 30 \end{pmatrix}$$

$R4 \to 5R4 + R3$

$$\begin{pmatrix} 5 & -2 & -3 & 0 \\ 0 & 21 & -6 & -15 \\ 0 & 0 & 100 & -100 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p'_A \\ p'_B \\ p'_C \\ p'_D \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ -150 \\ 0 \end{pmatrix}$$

p'_D can be chosen arbitrarily; (pressure is fixed only up to a constant).

Back-substituting:

$$100p'_C - 100p'_D = -150 \quad \Rightarrow \quad p'_C = p'_D - 1.5$$
$$21p'_B - 6p'_C - 15p'_D = 30 \quad \Rightarrow \quad p'_B = p'_D + 1$$
$$5p'_A - 2p'_B - 3p'_C = 5 \quad \Rightarrow \quad p'_A = p'_D + 0.5$$

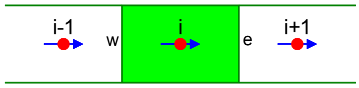
Finally, update velocity:

$$u_C = 2 + 3(p'_A - p'_C) \quad = 8$$
$$u_D = 1 + 3(p'_B - p'_D) \quad = 4$$
$$v_A = 1 + 2(p'_B - p'_A) \quad = 2$$
$$v_C = -2 + 2(p'_D - p'_C) \quad = 1$$

Q8.
(a)
(i) Pressure-correction methods are iterative methods for the simultaneous solution of mass and momentum equations, which work by making small corrections to the pressure field to "nudge" the velocity field toward mass conservation.

They consist of alternate iterations of:
- the momentum equation with the current pressure;
- a pressure-correction equation, derived by substituting the velocity-pressure relationship implied by the momentum equation into the continuity equation.

(ii) If there is co-located storage of velocity and pressure with linear interpolation for cell-face values then both momentum and mass equations only relate pressures at alternate nodes. This leads to large oscillations in the pressure field because pressure values at nodes with odd numbers are unconnected with those at even numbers – i.e. odd-even decoupling.

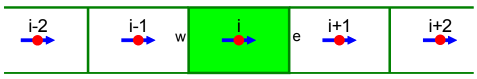


In the momentum equation the net pressure force involves

$$p_w - p_e = \frac{1}{2}(p_{i-1} + p_i) - \frac{1}{2}(p_i + p_{i+1})$$
$$= \frac{1}{2}(p_{i-1} - p_{i+1})$$

Hence, the discretised momentum equation has the form:

$$u_i = \frac{1}{2}d_i(p_{i-1} - p_{i+1}) + \cdots$$



In the continuity equation the net mass flux depends on

$$u_w - u_e = \frac{1}{2}(u_{i-1} + u_i) - \frac{1}{2}(u_i + u_{i+1})$$
$$= \frac{1}{2}(u_{i-1} - u_{i+1})$$
$$= \frac{1}{4}[d_{i-1}(p_{i-2} - p_i) - d_{i+1}(p_i - p_{i+2})] + \cdots$$

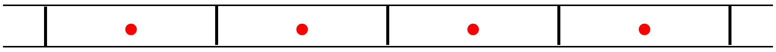
Thus, both mass and momentum equations only produce links between pressures at alternate nodes, leading to odd-even decoupling and large oscillations in the pressure field.

(b) In the Rhie-Chow algorithm we write

$$u = \hat{u} - d\Delta p$$

and interpolate pressure and non-pressure parts separately to the cell face. $\hat{u}$ is worked out at nodes by the inverse relation $= \hat{u} + d\Delta p$

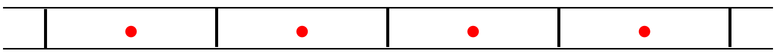
(i)



u	4		2		2		3	
p	0.3		0.5		0.4		0.3	
p_{face}		0.4		0.45		0.35		(by interpolation)
$\hat{u}$			2.2		1.6			$\hat{u} = u + 4(p_e - p_w)$
$\hat{u}_{face}$				1.9				(by interpolation)
u_{face}				<u>2.3</u>				$u_{face} = \hat{u}_{face} - 4(p_E - p_W)$

Answer: $u_f = 2.3$; (more than would be obtained by linear interpolation because the scheme is trying to smooth out the local pressure maximum to the left of the face).

(ii)



u	4		2		2		3	
p	0.3		0.3		0.4		0.3	
p_{face}		0.3		0.35		0.35		(by interpolation)
$\hat{u}$			2.2		2			$\hat{u} = u + 4(p_e - p_w)$
$\hat{u}_{face}$				2.1				(by interpolation)
u_{face}				<u>1.7</u>				$u_{face} = \hat{u}_{face} - 4(p_E - p_W)$

Answer: $u_f = 1.7$; (less than would be obtained by linear interpolation because the scheme is trying to smooth out the local pressure maximum to the right of the face).

(c)
(i) The net pressure force component in x or y direction on a face is given by:
pressure × projected area *into* the cell

Hence, accounting for direction if the cell is traversed anticlockwise:

$$F_x = \sum_{faces} p(-\Delta y)$$

$$F_y = \sum_{faces} p\Delta x$$

Thus, starting from the lowest face:

$$F_x = 3 \times 0 + 5 \times (-5) + 2 \times 5 \quad = -15$$
$$F_y = 3 \times 4 + 5 \times (-1) + 2 \times (-3) \quad = 1$$

Answer: force = (–15, 1).

(ii) Since "force = mass × acceleration" we need the mass of the cell.

Since it has unit depth and is a triangle in plan, the cell has volume

$$\frac{1}{2} \times base \times height \quad = \frac{1}{2} \times 4 \times 5 \quad = 10$$

Since ρ = 1.0, the mass is 10 units also. Hence,

$$\mathbf{a} = \frac{1}{m}\mathbf{F} \quad = \frac{1}{10} \times (-15, 1) \quad = (-1.5, 0.1)$$

Answer: acceleration = (–1.5, 0.1).