Q1.

(a)

Mass

Conservative form (given in the question):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

Expanding the second and third terms using the product rule:

$$\frac{\partial \rho}{\partial t} + \left(\frac{\partial \rho}{\partial x}u + \rho\frac{\partial u}{\partial x}\right) + \left(\frac{\partial \rho}{\partial y}v + \rho\frac{\partial v}{\partial y}\right) = 0$$

Collecting terms into those containing derivatives of ρ and those containing multiples of ρ:

$$\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right) + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

Hence, using the definition of the material derivative:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

Momentum

Conservative form (given in the question):

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho v u) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u$$

(Partially) expanding the derivatives of products $\rho u$, $\rho u \times u$ and $\rho v \times u$ on the LHS:

$$\text{LHS} = \left(\frac{\partial \rho}{\partial t}u + \rho\frac{\partial u}{\partial t}\right) + \left(\frac{\partial(\rho u)}{\partial x}u + \rho u\frac{\partial u}{\partial x}\right) + \left(\frac{\partial(\rho v)}{\partial y}u + \rho v\frac{\partial u}{\partial y}\right)$$

Collecting terms (as either multiples of $u$ or multiples of ρ):

$$\text{LHS} = \underbrace{\left(\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y}\right)}_{=0\ by\ mass\ conservation} u + \rho \underbrace{\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)}_{=\mathrm{D}u/\mathrm{D}t}$$

Thus, the LHS of the momentum equation is just $\rho \mathrm{D}u/\mathrm{D}t$. The equation is then

$$\rho\frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{\partial p}{\partial x} + \mu\nabla^2 u$$

(b) *Incompressible*: flow-induced changes to pressure (or temperature) do not cause significant changes in density; i.e. density does not change along a streamline:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = 0$$

(Note that this does not necessarily imply that ρ is uniform and constant, although that is often the case.)

From part (a) the 2-d continuity equation then reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(c)

Mass: $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Momentum: $$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho v u) + \frac{\partial}{\partial z}(\rho w u) = -\frac{\partial p}{\partial x} + \mu \nabla^2 u$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

(d)

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho u w) + \frac{\partial}{\partial y}(\rho v w) + \frac{\partial}{\partial z}(\rho w w) = -\frac{\partial p}{\partial z} - \rho g + \mu \nabla^2 w$$

Note the extra term on the RHS, corresponding to the gravitational force per unit volume. The LHS can also be written in the more compact non-conservative form as $\rho \frac{\mathrm{D}w}{\mathrm{D}t}$.

(e) Since $\partial z/\partial x = \partial z/\partial y = 0$, and $\partial z/\partial z = 1$, the pressure and gravitational force densities in the momentum equation can be combined as the gradient of a single scalar field:

$$-\frac{\partial p}{\partial x} = -\frac{\partial}{\partial x}(p + \rho g z)$$

$$-\frac{\partial p}{\partial y} = -\frac{\partial}{\partial y}(p + \rho g z)$$

$$-\frac{\partial p}{\partial z} - \rho g = -\frac{\partial}{\partial z}(p + \rho g z)$$

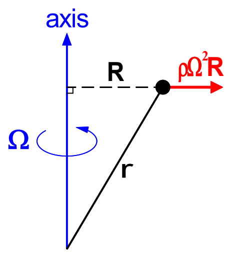
or, more concisely,

$$-\nabla p - \rho g \mathbf{e}_z = -\nabla(p + \rho g z), \qquad \text{since} \qquad \nabla z = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) z = (0,0,1) = \mathbf{e}_z$$

Hence, pressure and gravitational forces can be combined as the gradient of a single modified pressure $p + \rho g z$ (called the *piezometric pressure*).

(f)
Centrifugal Force
The centrifugal force is

$$-\rho\mathbf{\Omega}\wedge(\mathbf{\Omega}\wedge\mathbf{r}) = \rho\Omega^2\mathbf{R}$$
$$= \nabla(\tfrac{1}{2}\rho\Omega^2 R^2)$$




where **R** is the part of the position vector **r** perpendicular to the axis of rotation.

Hence, in the momentum equations the pressure and centrifugal forces can again be combined as the gradient of a single scalar field:

$$-\nabla p + \rho\Omega^2\mathbf{R} = -\nabla[p - \frac{1}{2}\rho\Omega^2 R^2]$$

corresponding to a modified pressure $p - \frac{1}{2}\rho\Omega^2 R^2$.

In rigid-body rotation (i.e. at rest in the rotating frame), there must be no net apparent force; as its gradient is zero, $p - \frac{1}{2}\rho\Omega^2 R^2$ must be a constant, meaning that pressure $p$ itself will increase with radius $R$. (Physically, it must do so to provide the *centripetal* acceleration necessary to keep moving in a circle).

CFD codes usually solve internally for the modified pressure $p - \frac{1}{2}\rho\Omega^2 R^2$, but will need to recover the actual pressure $p$ at boundaries in order to calculate forces on structures etc.

Coriolis Force

$$-2\rho\mathbf{\Omega}\wedge\mathbf{u} = -2\rho\begin{pmatrix}0\\0\\\Omega\end{pmatrix}\wedge\begin{pmatrix}u\\v\\w\end{pmatrix}$$
$$= -2\rho\begin{pmatrix}-\Omega v\\\Omega u\\0\end{pmatrix}$$
$$= \begin{pmatrix}2\rho\Omega v\\-2\rho\Omega u\\0\end{pmatrix}$$

The $x$ and $y$ components are, therefore, $2\rho\Omega v$ and $-2\rho\Omega u$

This is perpendicular to the velocity and causes a turn to the right when looking down the axis of rotation. It is why the geostrophic winds on a weather map blow clockwise around a high-pressure region in the northern hemisphere and anticlockwise in the southern hemisphere; the Coriolis force (perpendicular to velocity) is then in balance with the pressure force (from high pressure to low pressure). Since the rotation about the high-pressure centre is in the opposite sense to the earth's rotation this is known as an *anticyclone*.

(g)

Mass: $$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{u}) = 0$$

Momentum: $$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \bullet (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

(h)

Mass: $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0$$

Momentum: $$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

Q2.

Since the flow is fully developed, there is no acceleration: $\frac{\mathrm{D}u}{\mathrm{D}t} = 0$

Since $u$ is only a function of $y$, the Laplacian $\nabla^2 u$ reduces to $\frac{\partial^2 u}{\partial y^2}$

Hence, the $x$-momentum equation reduces to

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

Since $u$ and its derivatives are at most functions of $y$, then $\partial p/\partial x$ is independent of $x$ or $z$. Since $v = 0$ , the corresponding $y$-momentum equation reduces to

$$0 = -\frac{\partial p}{\partial y}$$

so that $p$, and hence $\partial p/\partial x$, are not functions of $y$ either. Hence, the pressure gradient $\partial p/\partial x$ is a constant: call it $-G$; (negative since pressure must decrease in the direction of the flow).

Thus, the mean velocity profile satisfies

$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} = -\frac{G}{\mu} \quad (= \textit{constant})$$

(Partial derivatives may be replaced by ordinary derivatives since $u$ is only a function of $y$ here). This can be integrated twice to give

$$u = -\frac{1}{2}\frac{G}{\mu} y^2 + Ay + B$$

where $A$ and $B$ are constants of integration.

In both cases (a) and (b), $u = 0$ on $y = 0$, so that $B = 0$, and the profile may be written

$$u = y(A - \frac{1}{2}\frac{G}{\mu} y) \qquad (*)$$

The value of $A$ depends on the boundary condition at the top of the channel, $y = h$.

(a) Plane Poiseuille flow: $u = 0$ at $y = h$ implies that

$$A = \frac{1}{2}\frac{G}{\mu} h$$

and the solution (*) becomes

$$u = \frac{G}{2\mu} y(h - y) \qquad \text{(a parabola)}$$

(b) Couette flow: with zero pressure gradient, $G = 0$, (*) becomes

$$u = Ay \qquad \text{(linear)}$$

and the boundary condition $u = U_w$ on $y = h$ gives $A = U_w/h$, or

$$u = \frac{U_w y}{h} \qquad \text{(a straight line)}$$

A pressure gradient is not required to drive the flow in this instance as the fluid is "dragged along" by the upper wall.

(c) Poiseuille flow (axisymmetric)

The pressure gradient is $-G$ as above, but the more complex expression for the Laplacian gives an equation for the streamwise velocity as

$$0 = G + \mu \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u}{\mathrm{d}r}\right)$$

i.e. $$\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u}{\mathrm{d}r}\right) = -\frac{G}{\mu}r$$

Integrating once:

$$r\frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{G}{2\mu}r^2 + A$$

$A = 0$ since all other terms are zero when $r = 0$. Hence, dividing by $r$,

$$\frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{G}{2\mu}r$$

Integrating again,

$$u = -\frac{G}{4\mu}r^2 + B$$

If $u = 0$ on the pipe wall $r = R$, then $B = GR^2/4\mu$ and the solution is

$$u = \frac{G}{4\mu}(R^2 - r^2)$$

This was the solution derived by other means in Hydraulics 1 and 2 and, like plane Poiseuille flow, is a parabolic profile.

And the really difficult bit about this question? How to pronounce "Poiseuille"! Try googling the internet arguments about this one. Commonest suggestions are:

"pwahs-WEE" (rhymes with "wee");

"pwah-ZOY" (rhymes with "employ");

Some of my Mechanical Engineering colleagues also like a version that ends in "ILL"!

My preference is the second (based on how I would pronounce "feuille" in French and linking "s" with the second syllable rather than the first), but answers on a postcard, please!

Q3.

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho\phi \ \mathrm{d}V + \oint_{\partial V} (\rho\mathbf{u}\phi - \Gamma\nabla\phi) \bullet \mathrm{d}\mathbf{A} = \int_V s \ \mathrm{d}V$$

For a fixed control volume, take d/d$t$ under the integral sign, apply the divergence theorem to turn the surface integral into a volume integral and take the source to the LHS;

$$\int_V \left\{\frac{\partial}{\partial t}(\rho\phi) + \nabla \bullet (\rho\mathbf{u}\phi \ - \Gamma\nabla\phi) - s\right\} \mathrm{d}V = 0$$

Since this is true for all $V$ the integrand must vanish identically. Hence, we have the conservative differential form

$$\frac{\partial}{\partial t}(\rho\phi) + \nabla \bullet (\rho\mathbf{u}\phi \ - \Gamma\nabla\phi) = s$$

For the non-conservative form expand the first two terms:

$$\left(\frac{\partial\rho}{\partial t}\phi + \rho\frac{\partial\phi}{\partial t}\right) + (\nabla \bullet (\rho\mathbf{u})\ \phi \ + \rho\mathbf{u} \bullet \nabla\phi) - \Gamma\nabla\phi = s$$

$$\Rightarrow \qquad \underbrace{\left\{\frac{\partial\rho}{\partial t} + \nabla \bullet (\rho\mathbf{u})\right\}}_{0\ by\ mass\ conservation} \ \phi + \rho\underbrace{\left\{\frac{\partial\phi}{\partial t} + \mathbf{u} \bullet \nabla\phi\right\}}_{\mathrm{D}\phi/\mathrm{D}t} - \Gamma\nabla\phi = s$$

Hence, the non-conservative form (combined time and advective derivatives) is

$$\rho\frac{\mathrm{D}\phi}{\mathrm{D}t} - \nabla \bullet (\Gamma\nabla\phi) = s$$

Q4.
(a) (ii) = pressure coefficient, $c_P$ (pressure loading)

(b) (ii) = Reynolds number (viscous flows).

(c) (i) = gradient Richardson number (stratified or density-varying flows).

The numerator is the *buoyancy frequency* (aka *Brunt-Väisälä frequency*), the frequency of small-amplitude gravity waves in density-stratified flow, such as often occur in the atmosphere or oceans. The denominator is a frequency based on the mean velocity gradient (aka *shear frequency*).

(d) (iii) = Rossby number (flow in a rotating reference frame).

Some authors refer to this as (Rossby number)$^{-1}$. C'est la vie!

Q5.
(a)

$$-2\rho\boldsymbol{\Omega}\wedge\mathbf{u} = -2\rho\begin{pmatrix}0\\0\\\Omega\end{pmatrix}\wedge\begin{pmatrix}u\\v\\w\end{pmatrix}$$
$$= -2\rho\begin{pmatrix}-\Omega v\\\Omega u\\0\end{pmatrix}$$
$$= \begin{pmatrix}2\rho\Omega v\\-2\rho\Omega u\\0\end{pmatrix}$$

The $x$ and $y$ components of the Coriolis force are $2\rho\Omega v$ and $-2\rho\Omega u$, respectively.

(b)

$x$-component: $$\rho\frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{\partial p}{\partial x} + \mu\nabla^2 u + 2\rho\Omega v$$

$y$-component: $$\rho\frac{\mathrm{D}v}{\mathrm{D}t} = -\frac{\partial p}{\partial y} + \mu\nabla^2 v - 2\rho\Omega u$$

(c) Write variables in terms of their non-dimensional counterparts (denoted by an asterisk, *) and scales $\rho_0$ (density), $U_0$ (velocity) and $L_0$ (length), together with a reference pressure $p_{ref}$:

$$\rho = \rho_0\rho^*$$
$$\mathbf{u} = U_0\mathbf{u}^*$$
$$p = p_{ref} + \rho_0 U_0^2 p^*$$
$$\mathbf{x} = L_0\mathbf{x}^*$$
$$t = \left(\frac{L_0}{U_0}\right)t^*$$

Substituting in Equation (*),

$$\frac{\rho_0 U_0^2}{L_0}\rho^*\frac{\mathrm{D}\mathbf{u}^*}{\mathrm{D}t^*} = -\frac{\rho_0 U_0^2}{L_0}\nabla^* p^* + \frac{\mu U_0}{L_0^2}\nabla^{*2}\mathbf{u}^* - 2\rho_0\Omega U_0\rho^*\mathbf{e}_\Omega\wedge\mathbf{u}^*$$

where $\mathbf{e}_\Omega$ is a unit vector along the rotation axis.

Multiplying through by $L_0/(\rho_0 U_0^2)$:

$$\rho^*\frac{\mathrm{D}\mathbf{u}^*}{\mathrm{D}t^*} = -\nabla^* p^* + \frac{\mu}{\rho_0 U_0 L_0}\nabla^{*2}\mathbf{u}^* - 2\frac{\Omega L_0}{U_0}\rho^*\mathbf{e}_\Omega\wedge\mathbf{u}^*$$

Hence, in non-dimensional variables (dropping the asterisks), the momentum equation is

$$\rho\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} - \frac{2}{\mathrm{Ro}}\rho\mathbf{e}_\Omega\wedge\mathbf{u}$$

where:

Reynolds number: $\text{Re} \equiv \frac{\rho_0 U_0 L_0}{\mu}$

Rossby number: $\text{Ro} \equiv \frac{U_0}{\Omega L_0}$

(d) *Conservative* means that flux terms can be integrated directly – i.e. all derivatives of the dependent variable are "on the outside" of expressions.

For example, in one dimension a conservative differential equation has the form

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = s(x)$$

and can be integrated directly to give

$$\Big[f(x)\Big]_{x_1}^{x_2} = \int_{x_1}^{x_2} s \; \mathrm{d}x$$

or

$$f(x_2) - f(x_2) = \int_{x_1}^{x_2} s \; \mathrm{d}x$$

corresponding to "flux$_{out}$ – flux$_{in}$ = source".

*Non-conservative* means that flux terms are written in a form that cannot be integrated directly; i.e. are not total derivatives.

(e) The material derivative operator is defined by

$$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

The *x*-momentum equation is

$$\rho\frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{\partial p}{\partial x} + \mu\nabla^2 u + 2\rho\Omega v$$

Expanding the LHS and adding *u* times the continuity equation, the LHS is equivalent to

$$\underbrace{\left(\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z}\right)}_{\rho \mathrm{D}u/\mathrm{D}t} + u\underbrace{\left(\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right)}_{=0 \text{ by continuity}}$$

Combining corresponding terms in each bracket, by the product rule:

$$\text{LHS} = \frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z}$$

The *x*-momentum equation can therefore by written in a longer, but conservative, form:

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial p}{\partial x}+\mu\nabla^2 u+2\rho\Omega v$$

with all the flux terms on the LHS directly integrable (derivatives "on the outside").

(f)

| | | |
|---|---|---|
| concentration | ← | $u$ (velocity) |
| diffusivity | ← | μ (viscosity) |
| source | ← | non-viscous forces |

(g) They can not be treated as independent because each velocity component appears in all the other equations (via the mass fluxes).

(h)
(i) In high-speed flow of a compressible gas:

– the continuity equation gives the density, ρ;
– a transport equation for energy (or enthalpy) gives the temperature, $T$;
– pressure is then derived from an equation of state (e.g. the ideal gas law, $p = \rho RT$).

(ii) In incompressible flow, pressure is derived from the condition that solutions of the momentum equation be mass-consistent. The momentum equation gives a link between velocity and pressure which, when substituted into the continuity equation, gives an equation for pressure.

Q6.

(a) Since

$$\rho\mathbf{g} = -\rho g\mathbf{e}_z \quad = -\rho g \nabla z$$

and

$$\rho\Omega^2\mathbf{R} = \frac{1}{2}\rho\Omega^2\nabla(R^2)$$

we have, for constant ρ and Ω,

$$-\nabla p + \rho\mathbf{g} + \rho\Omega^2\mathbf{R} = -\nabla(p + \rho g z - \frac{1}{2}\rho\Omega^2 R^2)$$

Thus, pressure, gravitational and centrifugal forces (per unit volume) can be combined as a single gradient:

$$-\nabla p^*$$

for a modified pressure

$$p^* = p + \rho g z - \frac{1}{2}\rho\Omega^2 R^2$$

A CFD code would solve for the modified pressure $p^*$, separating the individual parts only for any post-processing necessary to determine the pressure forces on boundaries.

(b) In steady rotation the fluid is at rest in the rotating reference frame, and hence the net force in this frame, $-\nabla p^*$, is zero. The modified pressure $p^*$ is then constant. Since

$$p = p^* - \rho g z + \frac{1}{2}\rho\Omega^2 R^2$$

the actual pressure $p$:

- increases with depth; (this is due to hydrostatic forces);
- increases with radius; (this is because, in a non-rotating reference frame, the pressure gradient must supply a *centripetal* force to maintain motion in a circle).

Hence, the minimum and maximum pressure $p$ are at top-centre and bottom-outside, respectively, and the difference in pressure between them is

$$\rho g \times height + \frac{1}{2}\rho\Omega^2 \times radius^2$$

Here,

$$\Omega = \frac{600 \times 2\pi}{60} = 62.83 \text{ rad s}^{-1}$$

and hence the difference in pressure is

$$1100 \times 9.81 \times 0.15 + \frac{1}{2} \times 1100 \times 62.83^2 \times 0.05^2 \quad = 7047 \text{ Pa}$$

**Answer:** difference in pressure = 7.05 kPa.

Q7.

(a) The outward volume flux through each face, $Q_f$, is the sum of

*velocity component × projected area*

or, if the cell is traversed anticlockwise:

$$Q_f = u\Delta y - v\Delta x$$

| Face | $u$ | $v$ | $\Delta x$ | $\Delta y$ | $Q_f$ |
|---|---|---|---|---|---|
| e | 4 | 10 | 2 | 2 | **–12** |
| n | 1 | 8 | –7 | 0 | **56** |
| w | 2 | 2 | 1 | –2 | **–6** |
| s | 1 | 4 | 4 | 0 | **–16** |

The required volume fluxes are given in the final column of the table.

(b) The net outward volume flux is

$$\sum Q_f = 22$$

This is non-zero; i.e. there is net outflow of volume. Hence, the flow is not incompressible.

Continuity for the whole cell gives:

$$\frac{\mathrm{d}}{\mathrm{d}t}(mass) + \sum_{faces}(outward\ mass\ flux) = 0$$

$$\Rightarrow \qquad \frac{\mathrm{d}}{\mathrm{d}t}(\rho V) + \sum_{faces} \rho_f Q_f = 0$$

The "volume" of the cell (a trapezium in plan) is constant, with value

$$V = \frac{1}{2}(4+7) \times 2 \quad = 11$$

whilst the density everywhere at this instant is 1.0. Hence, at this instant:

$$11\frac{\mathrm{d}\rho}{\mathrm{d}t} + 22 = 0$$

$$\Rightarrow \qquad \frac{\mathrm{d}\rho}{\mathrm{d}t} = -2$$

**Answer:** time derivative of density (at this instant) = –2 units.