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# Perceptual limits on low-dimensional models of Munsell reflectance spectra

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**Abstract.** Some experimental and theoretical approaches to surface-colour perception depend on approximating surface reflectance spectra by low-dimensional models. In the psychophysical experiment reported here, observers had to discriminate between patterns of Munsell surfaces and their spectral approximations under either the same or different illuminants. The approximations were produced by principal component analysis, by independent component analysis, and by artificial neural networks trained with a supervised-learning rule. In all experimental conditions, observers required, on average, at least 5 basis functions for discrimination performance to be at chance, thus placing a lower limit on the dimensionality of models of Munsell reflectance spectra.

## 1 Introduction

When the illumination on a scene changes, the spectrum of the light reaching the eye also changes. Yet observers are generally able to make judgments about the colour of surfaces largely independent of those changes. Explanations of this colour-constancy ability have sometimes been based on models of surface reflectance spectra requiring just a few parameters (Maloney 1986, 1999; D'Zmura and Iverson 1993), so that any particular reflectance spectrum can be simply approximated by a suitably weighted combination of elementary spectra or 'basis functions'. If the number of basis functions is sufficiently small, then, despite the constraints imposed on normal vision by having just three cone classes, it is possible, in theory at least, to extract from the spectrum reaching the eye a perceptual correlate of surface spectral reflectance. This assumption depends, of course, on the population of surfaces being modelled, and the adequacy of the low-dimensional model representing them.

The *Munsell Book of Color* (Munsell Color Corporation 1976) has provided a popular resource for experiments on surface-colour perception. This set comprises a large number of colour samples spaced in approximately equal perceptual steps of hue, value, and chroma, and forms a comprehensive coverage of colour space. Some representations of its spectra have been shown to include natural spectra, such as those from flowers, flower clusters, leaves, and berries (Jaaskelainen et al 1990). Previous studies of the Munsell set have found that the number of basis functions needed to approximate its spectra ranges from 3 to 8, depending on the criterion of fit and whether part or all of the set was used (Cohen 1964; Maloney 1986; Parkkinen et al 1989; Jaaskelainen et al 1990; Usui et al 1992; Vrhel et al 1994; Lenz et al 1996; Owens et al 2000; Westland et al 2000; Romney and Indow 2003). In all these studies, however, the adequacy of the approximation was based on theoretical criteria, rather than on psychophysical measurement. An exception is a previous experiment by Nascimento et al (2001) on approximating images of natural scenes by principal component analysis.

The purpose of the present study was to construct spectral approximations of the Munsell set according to three common statistical models, and then test psychophysically how many basis functions were required with each model for the approximations and the original Munsell spectra to be indistinguishable, under either the same or

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different illuminants. Stimuli were generated in the form of geometrical Mondrian-like patterns so as to isolate the properties of the spectral reflectances from their spatial distribution and semantic associations, as might be found in natural scenes. The models were based on principal component analysis (PCA), independent component analysis (ICA), and certain artificial neural networks (ANNs) trained with a supervised-learning rule. Notice that these models are neutral with respect to the spectral characteristics of the visual mechanisms assumed to underlie surface-colour perception.

Preliminary reports of some of this work have appeared in abstract form (Oxtoby et al 2002, 2003).

## 2 Methods

### 2.1 Computational models

Spectral-reflectance data for the Munsell set were taken from a study by Parkkinen et al (1989). The 1269 spectra, with black excluded, were sampled between 400 and 700 nm at 10-nm intervals, and were mean-centred.

In the PCA model, the data were represented by statistically uncorrelated basis functions, obtained by singular value decomposition. Any particular Munsell spectrum was approximated to the  $n$ th order by adding to the spectral mean of the Munsell set the sum of the products of the first  $n$  basis functions and their respective weights.

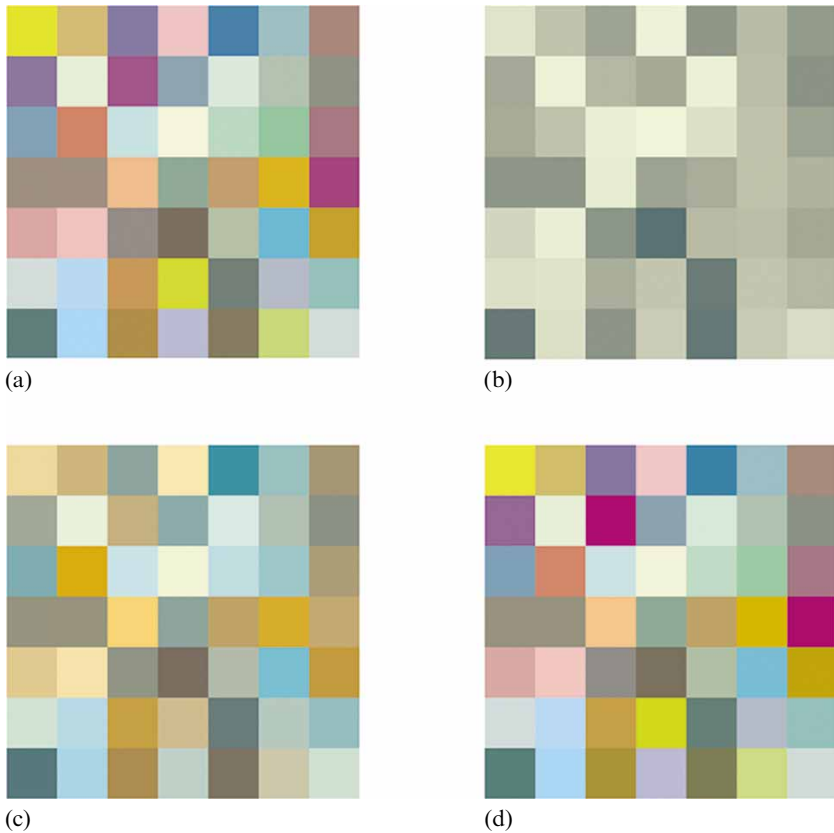
In the ICA model, the data were represented by statistically independent (as opposed to merely uncorrelated) basis functions, obtained with the FastICA algorithm (Hyvärinen and Oja 1997), which maximises non-Gaussianity. These basis functions, unlike those for PCA, depend on the number of model parameters. Spectral approximations were produced as for PCA.

In the ANN model, the data were represented nonlinearly. By contrast with the 5-layer ANN constructed by Usui et al (1992) for a similar purpose, the ANN used here had just 3 layers, trained for 500 epochs with the Levenberg–Marquardt back-propagation algorithm in conjunction with Bayesian regularisation (MacKay 1992; Foresee and Hagan 1997). The first layer consisted of 31 nodes with linear transfer functions, each node receiving a single reflectance value as input. The inputs to the second layer were nonlinearly transformed by a sigmoidal (hyperbolic tangent) function. The number  $n$  of nodes in the second layer varied from 1 to 8. The final layer had 31 nodes, the inputs to which were again nonlinearly transformed with a hyperbolic tangent function. The weights connecting a given node in the second layer to the nodes of the third layer defined the basis functions. The mean of the Munsell set was added to the outputs of the final layer to produce the spectral approximations. As with ICA, the basis functions depend on the number of model parameters.

In all three models, the parameter  $n$  varied from 1 to 8. The models were constructed within the Matlab programming environment (Matlab Ver. 6.5, The MathWorks, Inc., Natick, MA, USA). The FastICA algorithm was provided by the Laboratory of Computer and Information Science, Helsinki University of Technology, <http://www.cis.hut.fi/projects/ica/fastica/>.

### 2.2 Psychophysical experiments

Stimuli were computer simulations of Mondrian-like patterns. The patterns were presented simultaneously, as adjacent pairs, each comprising  $7 \times 7$  coloured surfaces with each surface subtending 1 deg visual angle at a viewing distance of 100 cm. Surfaces were selected from the Munsell set at random but constrained so that there were no duplications within a pattern. Patterns were presented for 5 s and viewed binocularly in a dark surround. The ambient illumination in the room did not exceed  $0.3 \text{ cd m}^{-2}$ . Some example patterns are shown in figure 1: (a) is an original pattern and (b)–(d) are low-order approximations, similar for all three models; the approximation (b) with  $n = 1$



**Figure 1.** Examples of Mondrian patterns produced from (a) original Munsell spectra and (b)–(d) from PCA approximations with 1–3 basis functions, respectively.

was grey-scale only, (c) with  $n = 2$  included blue–yellows, and (d) with  $n = 3$  included red–greens (cf Wandell 1995).

A computer-controlled RGB colour-graphics system with nominal 15-bit intensity resolution on each gun (VSG 2/3F, Cambridge Research Systems Ltd, Rochester, Kent, UK) was used to generate the stimuli on the screen of a 21-inch RGB monitor (GDM-F520, Sony Corp., Tokyo, Japan). Screen resolution was  $1024 \times 768$  pixels and refresh rate approximately 100 Hz. A telespectroradiometer (SpectraColorimeter, PR-650; Photo Research Inc., Chatsworth, CA), which had previously been calibrated by the UK National Physical Laboratory, was used to calibrate the display system. Errors in the displayed CIE  $(x, y, Y)$  coordinates of a white test patch were  $< 0.005$  in  $(x, y)$  and  $< 5\%$  in  $Y$ . The distinguishability of original and approximated spectra was not limited by the chromatic resolution of the display system, even with the largest value of  $n$  (Parkkinen et al 1989).

Observers completed experimental blocks of 80 trials. In half of these trials, the two patterns in the pair were constructed from identical reflectance spectra drawn from the Munsell set. In the remaining trials, one of the patterns in the pair was an approximation produced by one of the models. The order of the approximation (ie the number  $n$  of parameters in the model) varied randomly from trial to trial. The approximated pattern appeared on the left or right side of the screen with equal probability.

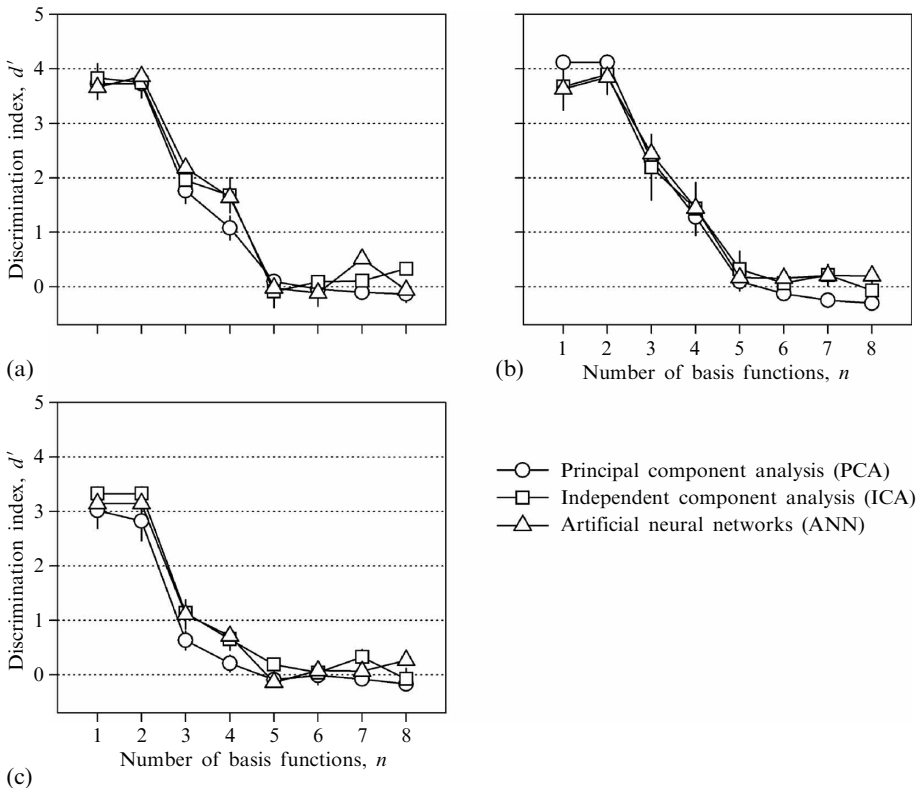
Three illuminant conditions were used. In two of the illuminant conditions, both patterns were illuminated by identical daylight conditions with a correlated colour temperature of either 6700 K or 25000 K and luminance of  $50 \text{ cd m}^{-2}$ . In the third illuminant condition,

the left pattern was illuminated by a daylight with a correlated colour temperature of 25 000 K and the right pattern was illuminated by a daylight with correlated colour temperature of 6700K.

Observers had to judge in each trial whether the two patterns were constructed from identical colour samples or whether one, some, or all of the samples were different. Observers made their responses, “same” or “different”, using a joystick input control to the computer. Responses could not be made until after the patterns had disappeared.

Performance was quantified with the discrimination index  $d'$  from signal-detection theory (Macmillan and Creelman 1991). This index has several advantages, including reducing the effects of observer bias. The value of  $d'$  is zero when performance is at chance level and it increases monotonically as discrimination performance improves.

Five observers participated in trials with the PCA approximations and completed 8 experimental blocks with each illuminant condition. Four observers participated in trials in which both the ICA and ANN approximations were interleaved. They completed 8 experimental blocks with each illuminant condition for both the ICA and ANN approximations. All observers had normal colour vision as assessed with the Ishihara test, Rayleigh and Moreland anomaloscopy, and the Farnsworth–Munsell 100-Hue test, by which each had an error score of less than 20. They all had normal or corrected-to-normal visual acuity.



**Figure 2.** Discriminability of Mondrian patterns as a function of the order  $n$  of their approximation. Discrimination index  $d'$  is averaged over 4–5 observers for each of the approximating models. (a) Both patterns under 6700 K daylight; (b) both patterns under 25000 K daylight; (c) one pattern under 6700 K daylight, the other under 25000 K daylight. Vertical bars indicate  $\pm 1$  SEM.

### 3 Results

Observers' discrimination performance is summarised in figure 2. Discrimination index  $d'$  averaged over observers is plotted against the order  $n$  of the approximation for each of the models indicated by the different symbols. In (a) both patterns were illuminated by the 6700 K daylight, in (b) both by the 25000 K daylight, and in (c) one by the 6700 K daylight and the other by the 25000 K daylight.

### 4 Discussion

For all three models and illuminant conditions, 5 spectral basis functions were sufficient for human observers to be unable to distinguish between members of the Munsell set and their spectral approximations on a colour monitor. There was little difference between the PCA, ICA, and ANN models in the variation in discrimination performance with the order of the approximation. Interestingly, although discrimination performance was somewhat reduced when comparisons were made across an illuminant difference, there was no reduction in the minimum number of basis functions required.

Despite the large gamut of the Munsell set, this estimate of a minimum of 5 spectral basis functions may underestimate the detectability of spectral approximations. Other experiments with Mondrian patterns made up of spectral reflectances drawn at random from natural scenes suggest that for rural scenes 6 basis functions are needed, and for urban scenes the number may increase to 10 (Oxtoby et al 2002; Oxtoby 2004). Similar experiments, but using natural scenes rather than Mondrian patterns (Nascimento et al 2001), indicate that at least 8 basis functions are necessary. For applications unconstrained by the gamut of colour monitors, still larger numbers of basis functions may need to be considered.

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