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# Minimum-variance cone-excitation ratios and the limits of relational color constancy

SÉRGIO M.C. NASCIMENTO,[1] VASCO M.N. DE ALMEIDA,[2] PAULO T. FIADEIRO,[2]
AND DAVID H. FOSTER[3]

[1]Department of Physics, Gualtar Campus, University of Minho, Braga, Portugal
[2]Remote Sensing Unit—Department of Physics, University of Beira Interior, Covilhã, Portugal
[3]Visual and Computational Neuroscience Group, University of Manchester Institute of Science and Technology, Manchester, UK

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## Abstract

Relational color constancy refers to the constancy of the perceived relations between the colors of surfaces of a scene under changes in the spectral composition of the illuminant. Spatial ratios of cone excitations provide a natural physical basis for this constancy, as, on average, they are almost invariant under illuminant changes for large collections of natural surfaces and illuminants. The aim of the present work was to determine, computationally, for specific surfaces and illuminants, the constancy limits obtained by the application of a minimum-variance principle to cone-excitation ratios and to investigate its validity in predicting observers' surface-color judgments. Cone excitations and their changes due to variations in the color of the illuminant were estimated for colored surfaces in simulated two-dimensional scenes of colored papers and real three-dimensional scenes of solid colored objects. For various test surfaces, scenes, and illuminants, the estimated levels of relational color constancy mediated by cone-excitation ratios varied significantly with the test surface and only with certain desaturated surfaces corresponded to ideal matches. Observers' experimental matches were compared with predictions expressed in CIE 1976 $(u',v')$ space and were found to be generally consistent with minimum-variance predictions.

**Keywords:** Color constancy, Relational color constancy, Cone-excitation ratios, Color vision

## Introduction

The ability of the eye to perceive surface color independent of the illuminant ("color constancy") has been measured in a variety of ways. Typically, two colored patterns or scenes representing the same set of surfaces but under two different lights are presented to the observer, either side-by-side or sequentially. In a surface-color-matching task, the observer adjusts the color of a test patch embedded in one of the scenes so that it appears to be made of the same material as the corresponding patch in the other scene (Arend & Reeves, 1986; Cornelissen & Brenner, 1995; Bäuml, 1999; Foster et al., 2001*a*). In a closely related task, the observer could be asked instead if the two scenes are related by an illuminant change or if any of the objects have changed in their surface materials (Craven & Foster, 1992; Foster et al., 2001*b*; de Almeida et al., 2004).

Both of these tasks involve the comparison of the colors of surfaces within and across scenes, and it has been suggested (Foster & Nascimento, 1994) that performance in each task depends not so much on the constancy of their perceived colors under changes in illuminant but on the constancy of the perceived color relations between them ("relational color constancy"). These perceived color relations might be encoded by the ratios of cone excitations—within the same cone class—arising from light reflected from the different surfaces in the scene; for with natural surfaces these spatial ratios are almost invariant under wide range of illuminant changes (Foster & Nascimento, 1994; Nascimento et al., 2002). Neurophysiological mechanisms that might mediate cone-excitation-ratio effects have been considered in Hurlbert and Wolf (2003).

What limits are imposed, therefore, on the level of color constancy if it is determined by the invariance or otherwise of spatial ratios of cone excitations and to what extent might they influence performance in surface-color matching or discriminating illuminant from material changes? To address these questions, predictions of performance were obtained with simulated two-dimensional scenes comprising simple patterns of Munsell surfaces under different illuminants and with real three-dimensional scenes of solid colored objects under different illuminants, used in experiments described elsewhere (de Almeida et al., 2004). In both situations, observers were assumed to make surface-color matches by minimizing the variance in spatial cone-excitation ratios. It is

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Address correspondence and reprint requests to: Sérgio M. C. Nascimento, Department of Physics, Gualtar Campus, University of Minho, 4710-057 Braga, Portugal. E-mail: smcn@fisica.uminho.pt

emphasized that for observers the task was the traditional one—the assumption here concerns the mechanisms underlying observers' performance (Foster, 2003). The predicted degree of constancy was found to vary significantly with the color of the test surface and only with certain desaturated surfaces were matches close to ideal. These variations from ideal performance closely fitted the variations in observers' actual experimental matches, in both color direction and degree of constancy.

## Materials and methods

For the analysis of simulated two-dimensional scenes, surfaces were drawn from the complete set of 1269 samples in the *Munsell Book of Color* (Munsell Color Corporation, 1976) with each spectral-reflectance function sampled at 10-nm intervals. This Munsell set was chosen for its approximately uniform distribution of chromaticities and because individual Munsell spectra or their combinations may represent some naturally occurring spectra (Jaaskelainen et al., 1990). It does not contain extremely saturated surfaces, but these are rare in natural environments (see, e.g. Nascimento et al., 2002). The illuminants represented different phases of natural daylight and their spectral power distributions were generated from three basis functions taken from a principal components analysis of 622 samples (Judd et al., 1964). The levels of cone excitations for each illuminated surface were computed using the Vos-Walraven cone spectral-sensitivity data and conversions from cone excitations to chromaticities effected with a standard conversion formula (Vos, 1978).

The two-dimensional scenes were the simplest possible, consisting of a variable Munsell (test) surface embedded in a uniform surround with flat spectral reflectance. With this arrangement, it was possible to examine the effects of variations in test-surface reflectance unconfounded by other variations in the spectral composition of the scene. The neutral surround was intended to represent a spectral average (the "grey-world" assumption), but chromatic surrounds were also tested. The CIE 1976 $(u',v')$ chromaticity coordinates and corresponding levels of cone excitations of the test surface and surround were calculated under pairs of different illuminants with correlated color temperatures chosen from 25,000 K, 6700 K, and 4000 K. The chromaticity coordinates producing exactly invariant cone-excitation ratios with respect to the surround for all three cone classes simultaneously were estimated by calculating the corresponding excitations and converting back to chromaticities (see Appendix 1). The degree of color constancy was quantified in CIE 1976 $(u',v')$ chromaticity space by a commonly used constancy index (Arend & Reeves, 1986), with value 0 corresponding to no constancy and 1 to ideal constancy.

For the analysis of real three-dimensional scenes, data were taken from an asymmetric surface-color matching experiment detailed in de Almeida et al. (2004). Five colored objects were used as test objects and seven pairs of illuminants, which separately and simultaneously illuminated each half of the scene, were drawn from the range 25,000 K–4000 K. Only the 25,000 K and 6700 K pair was used here. Observers were instructed to adjust the color of a three-dimensional test object on one side of the scene under the 6700 K illuminant so that the paper covering it looked as if it was cut from the same sheet as the paper covering the corresponding object on the other side of the scene under the 25,000 K illuminant.

The $(u',v')$ chromaticity coordinates and corresponding levels of cone excitations of these scenes were calculated in the same way as for the simulated scenes, that is, based on minimizing the variance in cone-excitation ratios, except that the test surfaces, surround, and the illuminants were drawn from those actually used in the experiment. Because the objects used in these experiments were real three-dimensional objects, their chromatic specification was not uniform across the surfaces; for that reason, the colors of the objects used in the computations were averages determined with a telespectroradiometer (SpectraColorimeter, PR-650, Photo Research, Inc., Chatsworth, CA).

## Results

Fig. 1(a) shows a histogram representing the distribution of predicted constancy indices based on minimizing the variance in cone-excitation ratios in the simulated two-dimensional scenes.

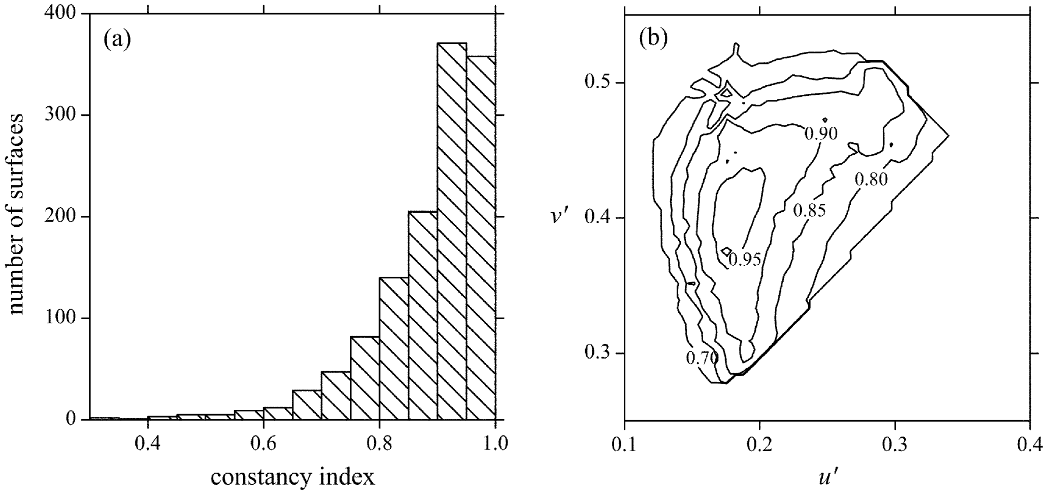

**Fig. 1.** Distribution of predicted constancy indices for 1269 Munsell test surfaces: (a) relative frequencies and (b) iso-index lines in CIE 1976 $(u',v')$ chromaticity space. Each test surface was assumed to be embedded in a neutral surround, with both illuminated by a daylight with correlated color temperature 25,000 K and then 6700 K.

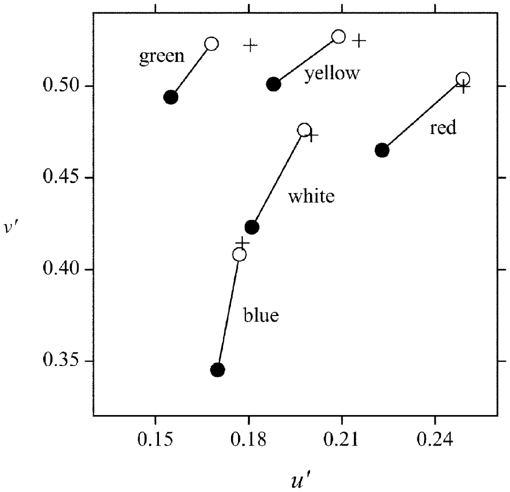

**Fig. 2.** Locations in CIE 1976 $(u',v')$ chromaticity space of five colored test surfaces used in real three-dimensional scenes under daylight illuminants with correlated color temperatures of 25,000 K (solid circles) and 6700 K (open circles), and predicted locations under the 6700 K illuminant (crosses) based on minimizing the variance in spatial cone-excitation ratios.

Each of the 1269 counts was obtained for a different Munsell surface surrounded by a neutral surface, with both illuminated by a daylight with correlated color temperature 25,000 K and then 6700 K. Values of the indices ranged from 0.998 for a pale pink surface to 0.35 for a yellow surface, with an average value across all Munsell surfaces of about 0.89. Replacing the neutral surround by a chromatic surround reduced the average index to the range 0.6–0.9, with the lower values obtained with more saturated colors. Both with neutral and with chromatic surrounds, more than half of the surfaces had predicted constancy levels of $<0.95$.

Fig. 1(b) shows for these scenes how the predicted constancy indices depend on the $(u',v')$ chromaticity coordinates of the test surface. The contours represent iso-index lines. Constancy indices are high with surfaces of low saturation and progressively decrease with surfaces of increasing saturation. The variation is anisotropic, with a slower decrease along a bluish-yellow axis than along a reddish-green axis. Larger shifts in correlated color temperature of 25,000 K to 4000 K produced similar anisotropies, but, on average, slightly higher constancy indices.

Fig. 2 shows, for the real three-dimensional scenes, the $(u',v')$ chromaticity coordinates of the five colored test surfaces under an illuminant with correlated color temperature 25,000 K and then 6700 K (solid and open circles, respectively). Also shown are the corresponding chromaticities of the surfaces under the 6700 K illuminant (crosses) based on minimizing the variance in cone-excitation ratios.

The color-matching performance of observers was compared with predictions in two complementary ways, first with respect to constancy indices and then with respect to the direction of bias in color space. Fig. 3(a) shows the predicted indices (solid bars) and observers' indices (hatched bars). The average prediction error was 0.06, and the maximum was 0.09, obtained with the yellow test surface. The particularly low value obtained by observers with the green test surface closely followed the predicted value.

Fig. 3(b) shows predicted and observers' direction of bias defined by the angle between a horizontal line and a line joining the chromaticity coordinates of the match to the coordinates of each of the five test surfaces under the 6700 K illuminant. The predicted bias follows the experimental data, although least well with the green test surface. Observers' matches were always biased more towards the direction of the color of the fixed comparison surface under the 25,000 K illuminant.

A similar analysis of constancy indices and bias direction was undertaken with data from a sequential illuminant-material-change discrimination experiment also using real three-dimensional objects (de Almeida et al., 2002). Solid colored objects were used as

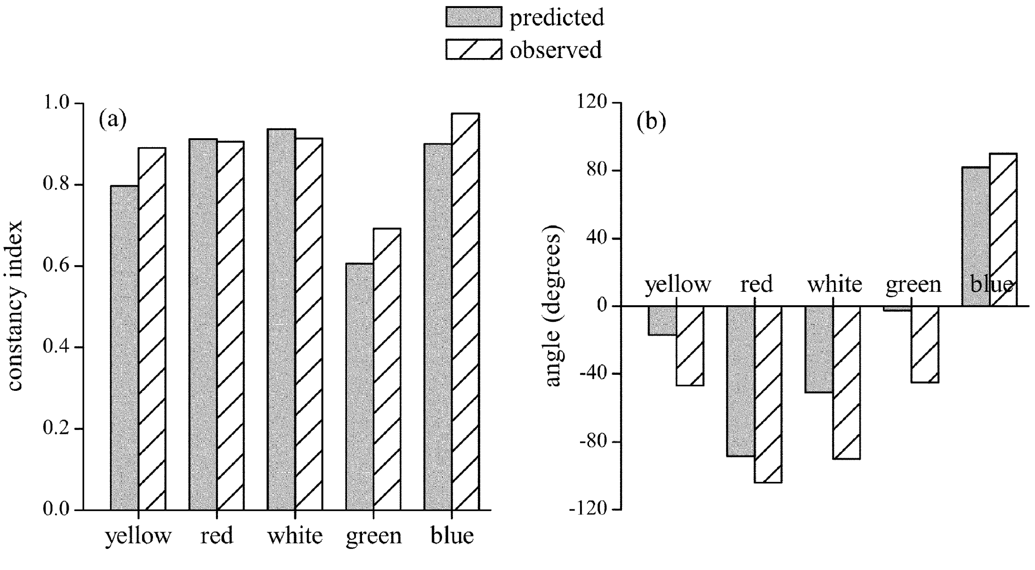

**Fig. 3.** Predicted surface-color-matching performance based on minimizing variance in spatial cone-excitation ratios (solid bars) and observers' performance (hatched bars) in real three-dimensional scenes under illuminants with correlated color temperatures of 25,000 K and 6700 K: (a) constancy indices and (b) angular direction of color bias. The color of each test surface is indicated.

test objects similar to those used here, and the illuminant pair had correlated color temperatures of 25,000 K and 6700 K. The relationship between predictions and observed performance was similar to that shown in Fig. 3 with simultaneous presentation of the comparison scenes.

## Discussion

The principle of minimizing the variance in cone-excitation ratios suggests significant limits on observers' performance in surface-color matching and in discriminating changes in illuminant and materials in scenes. With simulated two-dimensional scenes of Munsell surfaces and daylight illuminants, the predicted level of constancy varied with the color of the test surface and came close to ideal values only with desaturated surfaces. The fact that more than half of the surfaces produced constancy-index levels lower than 0.95 with a neutral surround and that the mean index fell to 0.6–0.9 with a chromatic surround suggests that the expected degree of constancy in practical situations could be lower than anticipated from everyday experience, particularly so with more saturated colors. Thus, the low values reported in some experimental studies may be due more to the selection of surfaces rather than to other experimental factors. A possible mechanism underlying the anisotropy in the distribution of constancy indices has been considered elsewhere (Foster et al., 2003).

Quantitative modeling of observers' experimental performance in surface-color matching (e.g. Brainard et al., 1997; Bäuml, 1999) usually involves the adjustment of various empirically determined parameters. In contrast, the application here of the minimum-variance principle to observers' matching performance required only the chromatic specification of the stimuli: there were no free parameters. Yet the variation in the degree of constancy and the directional biases in the experimental data were generally well reproduced. The residual bias in observers' performance was directed towards the color of the illuminant of the fixed comparison surface, suggesting some hue-and-saturation-based matching, which has also been reported elsewhere (Bäuml, 1999).

The success of this minimum-variance principle adds to previous data (Nascimento & Foster, 1997) pointing to a critical role for spatial cone-excitation ratios in judging surface-color properties under changes in illuminant.

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## Appendix 1

The CIE 1976 $(u',v')$ chromaticity coordinates of the test surface producing exactly invariant cone-excitation ratios against the surround in all three cone classes were calculated as follows. Suppose that $q_s^1$ and $q_s^2$, respectively, are the excitations in one cone class produced by light from the surround surface under the first and second daylight illuminants of 25,000 K and 6700 K, and that $q_t^1$ and $q_t^2$ are the corresponding excitations produced by light from the test surface. In Cartesian coordinates, the point defined by the pair $(q_s^1, q_s^2)$ defines a straight line passing through the origin. If the pair $(q_t^1, q_t^2)$ falls on that line, then the transformation exactly preserves cone-excitation ratios, for then $q_t^1/q_s^1 = q_t^2/q_s^2$. Although very stable, these ratios are not exactly invariant, but it is possible to use this relationship to define the ideal level $q_i^2$ of excitation for the test surface under 6700 K, that is, $q_i^2 = q_s^2\, q_t^1/q_s^1$, exactly preserving cone-excitation ratios. This value was transformed to $(u',v')$ chromaticity coordinates with aid of the (Vos, 1978) conversion formula.