Pergamon

PII: S0042-6989(97)00137-5

Vision Res., Vol. 37, No. 21, pp. 2975–2979, 1997

Printed in Great Britain
0042-6989/97 $17.00 + 0.00

Binary Masks Yielding Gaussian Light Distributions in Maxwellian View

SÉRGIO M. C. NASCIMENTO,* DAVID H. FOSTER,† PAUL MCKEE‡

Received 13 June 1996; in revised form 11 March 1997

Gaussian light distributions are important stimuli for vision research, but are difficult to produce for Maxwellian viewing conditions. In this study, two *binary* Maxwellian-view masks which produce smooth light distributions on the retina were considered: one was an out-of-focus circular aperture; the other was an in-focus pattern of opaque squares with a prescribed size distribution. The diffraction images of the two masks on the retina were computed numerically and shown to be well described by gaussian functions. As an experimental test, a mask with the desired pattern of opaque squares was manufactured by evaporation of chromium over a quartz substrate; its quality, as assessed by micro-transmittance measurements, was found to be adequate for most visual applications. © 1977 Elsevier Science Ltd

Gaussian stimulus Maxwellian viewing Defocused aperture Colour mechanisms

INTRODUCTION

Gaussian spatial distributions of light are optimally localized in both space and spatial-frequency domains; that is, they have smooth spatial variation but constrained spatial extent. This property and the particular characteristics of their frequency spectrum, also gaussian, make them important stimuli for vision research, both as distributions in their own right and as envelopes of other, spatially more rapidly varying, stimuli. In colour-vision research, where monochromatic light of variable wavelength may need to be imaged on the retina, stimuli are commonly provided by a Maxwellian-view optical system. With such systems, gaussian light distributions have been used to stimulate preferentially chromatic visual mechanisms; thus, the foveal test spectral sensitivity function for such stimuli reveals at around 580 nm a pronounced minimum that is associated with a loss of opponent-colour sensitivity and low, non-opponent, luminance sensitivity (Thornton & Pugh, 1983; Snelgar *et al.*, 1987; Calkins *et al.*, 1992; Nascimento, 1995).§

One way of producing a gaussian stimulus with a Maxwellian-view system is to use a photographic mask that has a spatially continuously varying transmittance function (Thornton & Pugh, 1983; Calkins *et al.*, 1992). The contrast function of the photographic process is, however, difficult to control, and consequently its effects may be difficult to predict. Smooth light distributions can also be produced by a defocused circular aperture, as was suggested by Snelgar *et al.* (1987) and used in other studies (Verdon & Haegerstrom-Portnoy, 1996); but the extent to which the distributions produced by this method can approximate gaussian functions has not yet been fully investigated. Alternatively, an in-focus binary mask made, for example, of a pattern of discrete opaque features in a transparent medium (somewhat like the halftone process of letterpress or lithographic printing) can be designed to produce smooth light distributions on the retina. Such masks avoid the need for defocusing and, because they are based on only two optical states (two transmittance values), they might give more predictable results than masks based on continuous variation of transmittance.

So that both methods could be assessed quantitatively, numerical calculations were made of the theoretical retinal diffraction images of a circular aperture for several out-of-focus conditions and of an in-focus pattern of opaque squares of variable size. Both of these

---

*Department of Physics, University of Minho, 4709 Braga Codex, Portugal.

†Department of Vision Sciences, Aston University, Birmingham B4 7ET, U.K.

‡Microlithography and Coatings Group, BT Laboratories, Martlesham Heath, Ipswich IP5 7RE, U.K.

§Sharp-edged stimuli generally produce a much weaker minimum at around 580 nm or a small inflexion, reflecting the greater contribution of non-opponent mechanisms (Foster, 1981; Foster & Snelgar, 1983; Snelgar *et al.*, 1987). A stimulus which psychophysically has an effect similar to that of a gaussian light distribution can be produced from a sharp-edged stimulus by edge masking (Foster & Snelgar, 1983; Snelgar *et al.*, 1987; Nacer *et al.*, 1989; Nacer *et al.*, 1995).

theoretical distributions were found to be well described by gaussian functions. Although creating an accurate circular aperture in a surface is easy, making a pattern of accurately positioned microscopic features on a suitable substrate requires the use of special techniques. To test the quality of a particular physical implementation, a mask consisting of a pattern of opaque squares was manufactured by thermal evaporation of chromium over a quartz substrate masked with its tone-reversed resist image (a multi-level-resist high-resolution lift-off method); the transmittance over a small area of the mask was assessed by direct measurement with a travelling microscope. The final quality of the mask obtained by this particular process was found to be adequate for most visual applications.

CIRCULAR APERTURE: THEORY

The mask was assumed to consist of a circular aperture of radius r in an opaque surface. Images were calculated for an ideal diffraction-limited eye, focused at infinity and with a pupil diameter of 2 mm. For such a pupil, aberrations are small and the quality of the eye's optics is, to a good approximation, limited only by diffraction effects (Campbell & Green, 1965; Campbell & Gubisch, 1966; Artal & Navarro, 1994). The aperture was assumed to be imaged with monochromatic light of 555 nm under Maxwellian viewing conditions and localized at the first principal focal plane of the lens closest to the artificial pupil (in-focus condition) or displaced from that point by a variable amount (out-of-focus condition). Light in the Maxwellian-view system is partially coherent owing to the non-zero extent of the source, but it can be assumed to be incoherent if the image of the source at the pupil is larger than 2.5 mm; in this condition mutual coherence between different points of the light source is negligible (Westheimer, 1966). This condition was assumed to be satisfied (as it usually is in practice), and linear-systems incoherent diffraction theory applied.

Modulation transfer functions (MTFs) for the diffraction-limited in-focus and out-of-focus eye, derived by Hopkins (1955), were evaluated by numerical integration with Romberg's method; the product of the Fourier transform of the aperture by the eye's MTF was then calculated, in the appropriate coordinate system, and a fast inverse Fourier transform algorithm applied to determine the light distribution on the retina. The formula for the Fourier transform of a circular aperture of radius r is standard. Thus, if w_x, w_y are the spatial-frequency coordinates in the x- and y-directions, respectively, and J_1 represents the Bessel function (of the first kind) of order one, then, apart from a scaling factor, the formula is

$$\frac{rJ_1\left(r\sqrt{w_x^2 + w_y^2}\right)}{\sqrt{w_x^2 + w_y^2}}.$$

Figure 1 shows the profiles of the computed images of a circular aperture subtending 1.0 deg when focused, illuminated by monochromatic light of wavelength 555 nm. The continuous lines show the normalized intensities for four levels of defocus (0, 8, 12, 16 D) and the dotted line shows the gaussian function best fitting the data, in the sense of least squares, for 12 D defocus. The fitted line had a half-height full-width of 1.14 deg and accounted for 99.70% of the variance.

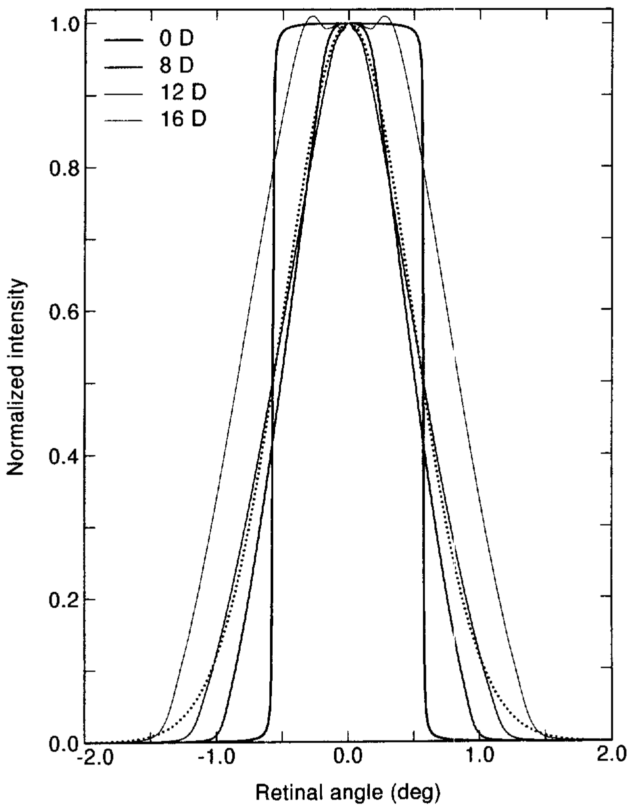

FIGURE 1. Profiles of the computed diffraction images of a circular aperture, subtending 1.0 deg when focused, for four levels of defocus. The dotted line shows the gaussian function that best fits the data for 12 D; it had a half-height full-width of 1.14 deg and accounted for 99.70% of the variance in the data. The mask was assumed to be imaged on the retina, under Maxwellian viewing conditions, with monochromatic light of wavelength 555 nm, and by a diffraction-limited eye with a 2-mm pupil; it was assumed to be positioned in the first principal focal plane of the lens closest to the artificial pupil (in-focus condition) or displaced by the appropriate amount (out-of-focus condition).

It is clear that a gaussian light distribution can be well approximated by appropriately defocusing a circular aperture. The method is simple to implement but has some disadvantages. First, for each required gaussian distribution, some calculation may be necessary to find the proper combination of aperture size and amount of defocus. Second, the amount of defocus may be difficult to control experimentally, especially in conditions where combinations of in-focus and out-of-focus stimuli are used.

PATTERN OF OPAQUE SQUARE PATCHES: THEORY

The halftone process used in printing to produce shades of gray can be implemented with a variety of possible patterns or screens; for example, with equally spaced dark patches of variable size or with patches of the same size but with variable spacing. In any case, the individual patches should not be visually resolvable; their size and position should therefore be specified with higher resolution than 60 cycles/deg. For a typical Maxwellian-view optical system, a linear resolution of the order of 1000 lines/mm is required, which is close to the limit of conventional photography. An electron-beam lithographic technique with much higher resolution, about 5000 lines/mm, was used instead.

The mask was assumed to consist of a rectangular array of variously sized opaque squares on a transparent substrate. Other non-rectangular geometries were rejected. For example, an array of opaque circles of variable spacing or of variable size on a transparent substrate, or the opposite, that is, an array of transparent circles on an opaque substrate, could each have been used, but both methods would have required some overlap of the circles (in the tail of the gaussian function with the transparent substrate and in the center of the gaussian function with the opaque substrate), resulting in less predictable results with the lithographic technique used.

The assumptions and method of calculating the diffraction image of this mask were the same as for the circular aperture in the in-focus condition. If the squares, spatially indexed by integers m, n, are arranged on a grid with spacing d, then their mid-points have Cartesian coordinates $(x, y) = (dm, dn)$. The arrangement is illustrated in Fig. 2, which shows an idealized version of the central region of the mask. Assume that the local transmittance of the mask is required to vary with position (x, y) according to some radially symmetric function f. Then the area a_{mn} of the square patch (m, n) should satisfy

$$1 - \frac{a_{mn}}{d^2} = f(d\sqrt{m^2 + n^2}). \qquad (1)$$

Each side of this equation represents the fraction of energy transmitted through a square region of area d^2 centred at (dm, dn).

The Fourier transform of the mask was determined analytically as follows. The pattern of squares can be considered as a superposition of elements having transmittance zero everywhere except in a square "frame", centred at (dm, dn), with inner and outer sides $\sqrt{a_{mn}}$ and d, respectively. As before, let w_x, w_y be the spatial-frequency coordinates in the x- and y-directions, respectively. Then the real part of the Fourier transform of each frame is, apart from a scaling factor,

$$\frac{\cos(w_x md + w_y nd)}{w_x w_y}\left(\sin\left(\frac{w_x d}{2}\right)\sin\left(\frac{w_y d}{2}\right) - \sin\left(\frac{w_x\sqrt{a_{mn}}}{2}\right)\sin\left(\frac{w_y\sqrt{a_{mn}}}{2}\right)\right).$$

The transform of the whole pattern is obtained by summing over all m, n, or—given its symmetry—over just one quadrant.

The choice of pattern grid spacing d and the function f depends on the desired retinal distribution of light, on the resolution of the technique used to physically implement the pattern, and on the Maxwellian-view optics. The computations reported here were carried out with $d = 7.8$ μm and f a gaussian function defined by

$$f(d\sqrt{m^2 + n^2}) = b\,\exp(-c(m^2 + n^2)d^2), \qquad (2)$$

where the constants b, c were chosen so that the smallest area of each opaque patch was approximately 4 μm^2 and the half-height full-width of the gaussian function subtended 1.0 deg visual angle. The linear dimensions of the square patches were allowed to vary in steps of 0.2 μm. These parameters for the mask were compatible with the production technique, which is described in the next section.

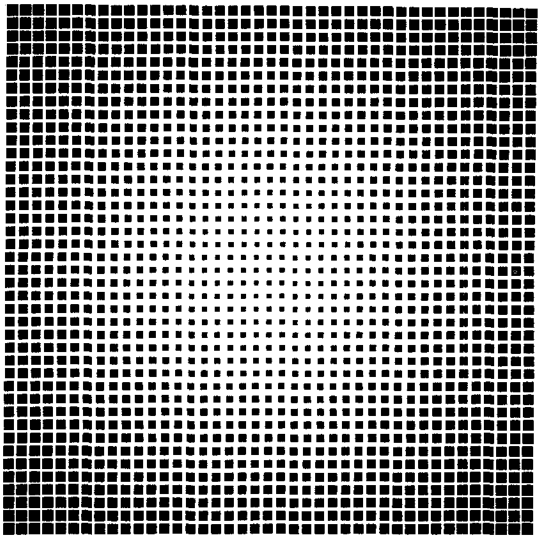

FIGURE 2. Idealized version of the central region of a binary mask. The actual pattern of variable-sized opaque squares was made by a thin layer of chromium on a quartz substrate and was produced by a multi-level-resist high-resolution lift-off method. The area of the squares of the resulting pattern was in the range 2.4^2–7.8^2 μm^2.

Figure 3 shows the profile of the computed image of this ideal mask when illuminated by monochromatic light of wavelength 555 nm (heavy continuous line). The best-fitting gaussian function had half-height full-width of 1.09 deg (dotted line) and accounted for 99.96% of the variance. (The profiles shown by the light continuous and dotted lines are discussed later.) The residual irregularities in the function, noticeable at intensities of 10% and less, are due to the discreteness of the imposed variation in square size. If the production technique allows higher resolution, the resulting profile will be smoother.

PATTERN OF OPAQUE SQUARE PATCHES: EXPERIMENTAL TEST

A mask with the prescribed pattern of opaque squares was produced as follows. A tone-reversed pattern was generated according to equation (1) and equation (2) and used to produce a high-resolution negative tone resist; the resist image was developed and a layer of chromium was thermally evaporated over the image; the resist was then removed, leaving the desired pattern by lift-off.

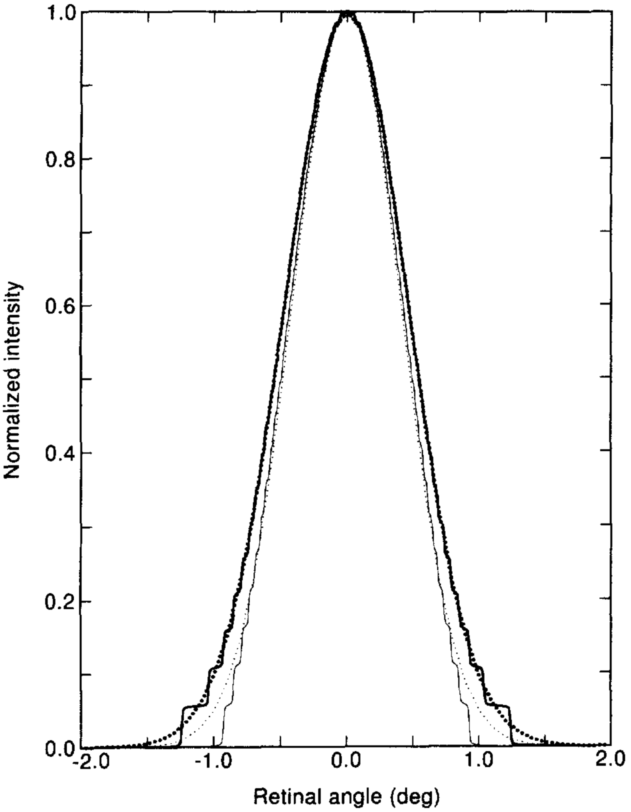

FIGURE 3. Profiles of the computed diffraction images of the ideal binary mask (heavy continuous line) and of the manufactured binary mask shown in Fig. 2 (light continuous line). Dotted lines show the best-fitting gaussian functions with half-height full-widths of 1.09 and 0.96 deg, respectively; the proportion of variance accounted for by the fits was 99.96% and 99.76%, respectively. Other details as for Fig. 1.

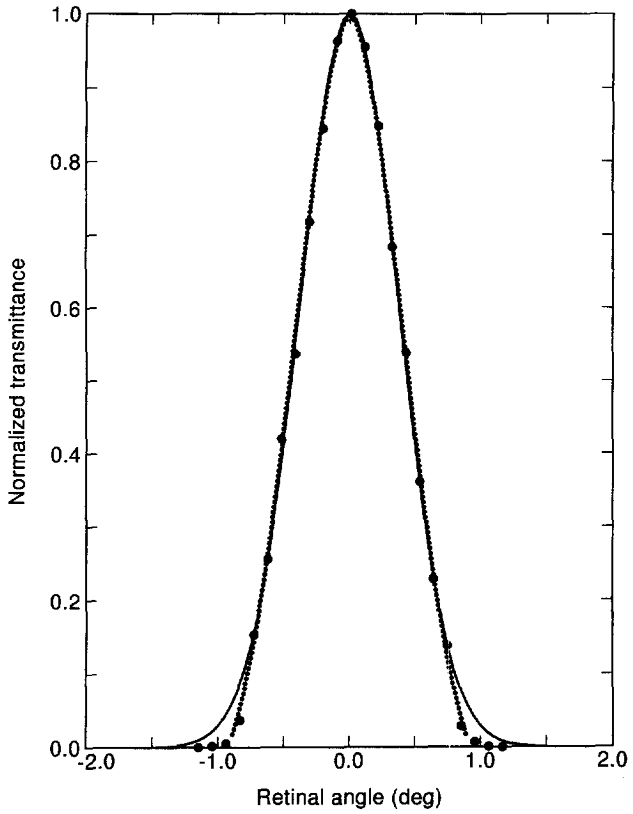

FIGURE 4. Measured transmittance profile of the binary mask shown in Fig. 2. Symbols represent transmittances and the continuous line the best-fitting gaussian function (without the lowest four points in each tail) with half-height full-width 0.89 deg; it accounted for 99.64% of the variance of all data points. The dotted line shows the modified function predicted by an over-exposure of the mask during the manufacturing process.

To assess the quality of the final result, the mask was imaged with a Leitz Dialux EB20 light microscope (× 40 objective) and the luminance over a circular region with radius of about 20 μm measured at points across the mask; these measurements were made in the photographic plane of the camera (final magnification of × 128) with a photometer (LMT GmbH, Berlin, type L1003). Figure 4 shows the measured transmittance profile (symbols). The gaussian function best fitting all the data points except the lowest four values on each side of zero (continuous line) had a half-height full-width of 0.89 deg and accounted for 99.64% of the variance in the data set.

The mask was slightly over-exposed; as a result the profile was narrower and had a faster fall-off than predicted. A fixed increase in the side of each patch by 0.4 μm accounts better for the data, as can be seen by the dotted line in Fig. 4. The diffraction image of a mask with this amount of distortion was computed and is shown in Fig. 3 (light continuous line). The best fitting gaussian function (light dotted line) had a half-height full-width of 0.96 deg and accounted for 99.76% of the variance.

CONCLUSION

Both a defocused circular aperture and an in-focus pattern of opaque squares of variable size can produce light distributions on the retina that can be well described by gaussian functions. Both methods require some calculation to find the proper combination of spatial parameters for the masks. The circular-aperture method is obviously easy to implement, but the amount of defocus may be difficult to control in a Maxwellian-view system where both in-focus and out-of-focus stimuli are used. The multi-level-resist high-resolution lift-off method of producing the required pattern of opaque squares is less easy to implement, and some fine adjustment of the exposure conditions may be needed to produce a gaussian light distribution to within 10% of a specified half-height full-width. The shape of the distribution is, however, appropriate for most visual applications, and the method has the further advantage of being completely flexible; that is, the spatial parameters of the pattern can in principle be adjusted to produce arbitrary continuously varying light distributions.

REFERENCES

Artal, P. & Navarro, R. (1994). Monochromatic modulation transfer function of the human eye for different pupil diameters: an analytic expression. *Journal of the Optical Society of America A, 11*, 246–249.

Calkins, D. J., Thornton, J. E. & Pugh, E. N. Jr. (1992).

Monochromatism determined at a long-wavelength/middle-wavelength cone-antagonistic locus. *Vision Research, 32*, 2349–2367.

Campbell, F. W. & Green, D. G. (1965). Optical and retinal factors affecting visual resolution. *Journal of Physiology, 181*, 576–593.

Campbell, F. W. & Gubisch, R. W. (1966). Optical quality of the human eye. *Journal of Physiology, 186*, 558–578.

Foster, D. H. (1981). Changes in field spectral sensitivities of red-, green- and blue-sensitive colour mechanisms obtained on small background fields. *Vision Research, 21*, 1433–1455.

Foster, D. H. & Snelgar, R. S. (1983). Test and field spectral sensitivities of colour mechanisms obtained on small white backgrounds: action of unitary opponent-colour processes? *Vision Research, 23*, 787–797.

Hopkins, H. H. (1955). The frequency response of a defocused optical system. *Proceedings of the Royal Society of London A, 231*, 91–103.

Nacer, A., Murray, I. J. & Carden, D. (1989) Interactions between luminance mechanisms and colour opponency. In Kulikowski, J. J., Dickinson, C. M. & Murray, I. J. (Eds), *Seeing contour and colour* (pp. 357–360). Oxford: Pergamon Press.

Nacer, A., Murray, I. J. & Kulikowski, J. J. (1995). Balancing sensitivity of human chromatic opponent mechanisms by adaptation. *Journal of Physiology, 485*, 21P.

Nascimento, S. M. C. (1995) Surface colour perception under illuminant transformations. PhD Thesis, Keele University, U.K.

Snelgar, R. S., Foster, D. H. & Scase, M. O. (1987). Isolation of opponent-colour mechanisms at increment threshold. *Vision Research, 27*, 1017–1027.

Thornton, J. E. & Pugh, E. N. Jr (1983). Red/green color opponency at detection threshold. *Science, 219*, 191–193.

Verdon, W. & Haegerstrom-Portnoy, G. (1996). Mechanisms underlying the detection of increments in parafoveal retina. *Vision Research, 36*, 373–390.

Westheimer, G. (1966). The Maxwellian view. *Vision Research, 6*, 669–682.

Acknowledgements—We are grateful to D. Travis and C. Dix of BT Research Laboratories, Martlesham Heath, Ipswich, U.K., for collaborating in the manufacture of the binary mask; to C. M. Hackney and D. N. Furness of the Department of Communication and Neuroscience, Keele University, for providing the light-microscope facilities and to R. Knapper and S. Murray of the same department for technical assistance; and to L. M. Doherty, K. J. Linnell, and S. A. Randle for critical reading of the manuscript. S. M. C. N. was supported by the Junta Nacional de Investigação Científica e Tecnológica (grant no. BD/1328/91-RM), and by the Universidade do Minho, Braga, Portugal. Additional support was provided by the Wellcome Trust (grant No. 034807).