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Horizontal–Vertical Structure in the Visual Comparison of Rigidly Transformed Patterns

Jeremy I. Kahn and David H. Foster
Department of Communication and Neuroscience, University of Keele, Keele, England

Visual recognition of patterns reflected or rotated through 180° (point-inverted) depends critically on their positional symmetry and separation in the field. A possible explanatory scheme suggested a description of internal pattern representation structures and simple internal operations that naturally involved a horizontal–vertical reference system. Predictions of the scheme were tested here in three experiments. Subjects made *same–different* judgments on pairs of random-dot patterns briefly presented in various arrangements and related by reflection, point-inversion, or identity transformation, or paired at random. Experiment 1 tested reflected patterns and verified the importance of orientation of the reflection axis relative to display–configuration axis. Experiment 2 demonstrated an oblique effect of configuration on performance with reflected patterns, but not with identical or point-inverted patterns. Experiment 3 demonstrated a vertical shift effect of configuration on performance with point-inverted patterns, but not with identical or reflected patterns. We concluded that in *same–different* pattern comparisons, a horizontal–vertical reference system appears fundamental in determining the nature of and operations upon internal pattern representations.

Visual performance in recognizing patterns that have been spatially transformed, for example, rotated or reflected, depends on many factors, including the size and nature of the transformations, the position of the patterns in the visual field, and the population of stimuli from which the patterns are discriminated. For example, in the discrimination of patterns that are identical except for a planar rotation from patterns that are chosen at random, recognition accuracy depends on angle of rotation in a nonmonotonic fashion: It falls off with angle for rotations up to approximately 90° and then increases again for rotations up to 180°. This type of performance for the detection of *same* patterns has long been known (Dearborn, 1899; Mach, 1897/1959, chapter 6) and has been demonstrated with a variety of figures, including randomly contoured shapes (Dearborn, 1899; Rock, 1973), random-dot patterns (Foster, 1978; Kahn & Foster, 1981), and alphabetic characters (Aulhorn, 1948). It should be contrasted with the strictly monotonic performance obtained by Shepard and his colleagues in "mental rotation" experiments in which reaction time rather than accuracy is the dependent variable and in which discrimination typically involves the sense of the pattern, that is, whether it has been reflected or not (Cooper & Shepard, 1973; Shepard, 1975; Shepard & Cooper, 1982; Shepard & Metzler, 1971).

An explanation of the nonmonotonic angular dependence of *same*-detection performance has been proposed (Foster, 1978; Foster & Mason, 1979) in terms of a hybrid class of schemes for visual pattern recognition. In general, schemes for pattern recognition may be classified according to the extent to which an internal description formed by the visual system is "viewer-centered," in which spatial positions are specified relative to the observer, or "object-centered," in which spatial positions are defined with respect to the object (Foster, 1984; Marr, 1982). An object-centered description was proposed by Marr and Nishihara (1978) for simple three-dimensional figures. A principal axis was defined, and the positions of other axes were related to that axis by using a local coordinate system. An important property of object-centered descriptions is that they depend neither on the position nor on the orientation of the object with respect to the observer. Such descriptions may not always be free of viewer-centered labels (Foster, 1984), as was illustrated by Marr (1982, p. 42) in a description intended to be in an object-centered coordinate frame: The location of the tip of a certain cat's tail was said to be "above and to the left of its body." There is a natural viewer-centered interpretation of this description that is apparent when one considers the effect of viewing the cat from the opposite direction. Given the angular and positional dependence of visual recognition, a hybrid system of descriptions seems most appropriate: part viewer-centered and part object-centered (Foster, 1984). A similar point was made by Shepard (1981, p. 292), who argued that an internal representation achieves an "effective 'mesh' with the external object in the particular spatial relation that object currently bears to the subject."

For the explanation of the nonmonotonic angular dependence of *same*-detection performance, a "relational-structure" scheme has been suggested (Foster, 1978; Foster & Mason, 1979). This scheme was based on the assumption that patterns were represented in terms of local features, which for random-dot patterns might be dot clusters of a certain density and shape, and local spatial relations, such as "left of," "right of," "above,"

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Correspondence concerning this article should be addressed to David H. Foster, Department of Communication and Neuroscience, University of Keele, Keele, Staffordshire ST5 5BG, England.

and "below," defined within a horizontal–vertical reference system that described from the viewpoint of the observer how one local feature was related to another. For three-dimensional stimuli, other spatial order relations such as "behind" and "in front of" would be included. Two patterns were supposed to be recognized as the same if their internal representations coincided. A simple internal relabeling operation was also assumed to be available so that, if appropriate, representations could be transformed prior to comparison. This operation was a global reversal of the *sense* of spatial relations, so that "left of" could be replaced by "right of," "above" by "below," and so on. For 180°-rotated or point-inverted patterns (the latter description referring to the notion of the pattern's being inverted through a single point rather than an axis), this relabeling operation compensated precisely for the rotation. Using this scheme, Foster and Mason (1979) were able to predict numerically the detailed variation of recognition performance with rotation angle for a large family of random-dot patterns.

In subsequent experiments (Foster & Kahn, 1985; Kahn & Foster, 1981) it was shown that the observed upturn in recognition performance for 180° rotation was dependent on the symmetry of the display: When the symmetry of the pattern positions was disturbed, accuracy of recognition of point-inverted patterns diminished. The effects of positional symmetry and separation of the patterns were investigated for several pattern transformations. It was found that *same*-detection performance for identical patterns was strongly affected by the distance between the patterns and not by the symmetry of their positions with respect to the point of fixation: the greater the separation of the patterns, the worse the performance. In contrast, performance for point-inverted patterns and patterns related by a reflection in a vertical axis was strongly affected by the symmetry of the positions of the patterns with respect to the point of fixation and not by the distance between the patterns; when the patterns were positioned symmetrically about the point of fixation, performance was maximum (Foster & Kahn, 1985; Kahn & Foster, 1981).

Extended Internal Operations and Internal Representations

As a result of these experiments, an expanded scheme for internal pattern representations was developed (Foster & Kahn, 1985; Kahn & Foster, 1981) in which patterns were assumed to be represented in terms of local features, the local spatial relations between those local features, *and* the positions of the patterns with respect to the point of fixation. Representations were supposed to be transformable by two distinct kinds of internal operation. These operations were as follows: (a) a "continuous" operation by which any element in a representation could be modified, but only in a progressive, continuous fashion; (b) a "discrete" operation by which all elements of a given kind could be relabeled in a single step, providing that the relabeling was applied uniformly to all the elements of the given kind; thus all occurrences of the spatial relation "left of" would be replaced by "right of" and vice versa.

Both of these operations, it was assumed, could be effected in the internal comparison of two representations, but with efficiency depending on the "size" of the operation needed to bring the representations into coincidence. The following examples illustrate how these operations could be used to explain the dependence of *same*-detection performance on positional symmetry and separation of transformed patterns (Kahn & Foster, 1981). Pairs of symmetrically positioned patterns related by point-inversion would be detected as *same* by relabeling with the opposite term all those elements in the representation that specified spatial sense. By this simple global operation, the relation "left of" would become "right of," "above" would become "below," and "0.5° to the left of the fixation point" would become "0.5° to the right of the fixation point." The two representations would thus be brought into coincidence. If the two representations were not positioned symmetrically with respect to the point of fixation, the relabeling operation alone would not be sufficient to achieve coincidence of the representations. Further modification of the position component would be needed, and *same*-detection performance would be reduced.

The above scheme and its more restricted precursor both entailed the explicit assumption of a horizontal–vertical framework for the description of the spatial relations of stimulus patterns. This assumption and the assumption of an internal global sense-reversal operation imply certain constraints on visual performance that would not be expected from purely object-centered pattern descriptions, or indeed from viewer-centered descriptions that are rotationally isotropic, such as those using polar coordinate systems (Leibovic, Balslev, & Mathieson, 1971; Schwartz, 1980).

There is, however, another type of scheme for internal representations and operations that could generate similar constraints. The transformational approach advocated by Shepard, although differing in organization (Foster, 1980a, 1983; Shepard, 1981), could produce the equivalent of a sense-reversal operation by assuming a special status for the horizontal and vertical as rotation axes. How this might be done is described later. In a restricted, formal sense, the two types of schemes are mathematical duals of each other (Foster & Mason, 1979). In a relational-structure scheme, the critical structure is in the representation, specifically in the spatial-ordering information; in a transformational scheme, the critical structure is in the families of internal transformations that are applied to the representations. It is not the intention of this study to test which of these two types of schemes is more appropriate; rather, it is to determine how well the characteristic implications of a horizontal–vertical structure for representations and operations actually fit with observed performance.

Predictions of a Horizontal–Vertical Reference System

Consider the task of discriminating pairs of *same* patterns that are identical or related by a reflection or point-inversion, that is, planar rotation through 180°, from pairs of *different* patterns not related in this way and paired at random. The following group of predictions is specifically dependent on the assumption of a horizontal–vertical reference system.

Prediction 1: Reflection-Axis Effect

Suppose that *same* pattern pairs are related by reflection in an axis of variable orientation (Figure 1, inset, Sections b–e). If the patterns were positioned horizontally and symmetrically about the fixation point, as illustrated, the highest *same*-detec-

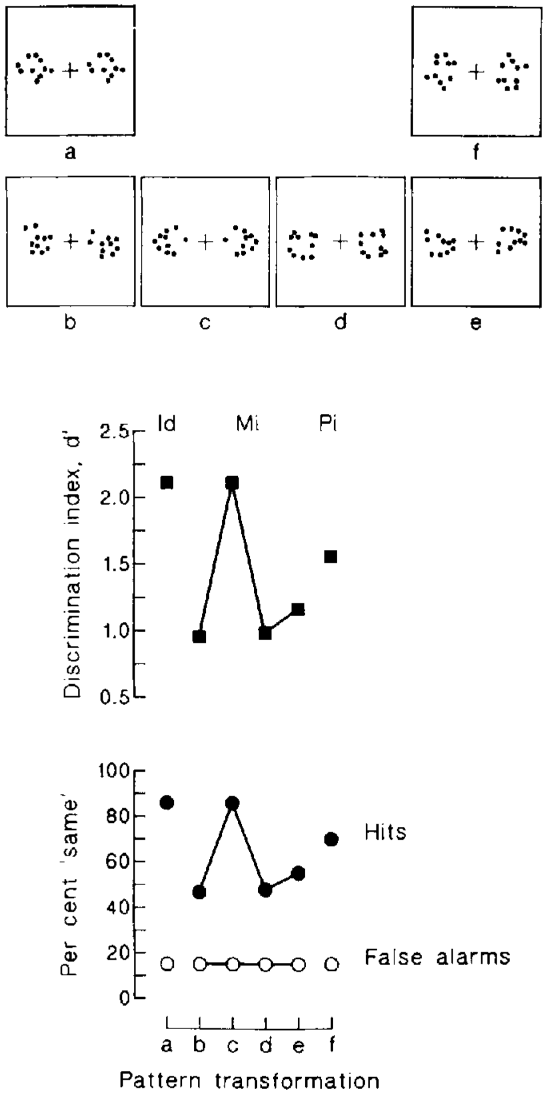

Figure 1. Inset panels: illustrations of *same* random-dot patterns and their transformations used in Experiment 1. (In Section a the patterns are related by identity [Id]; in each of Sections b–e one pattern is obtained from the other by reflection [Mi$_\theta$] in an axis oriented at an angle θ clockwise from the vertical, θ taking the values −45°, 0°, 45°, and 90° respectively; in Section f the patterns are related by point-inversion [Pi], that is, rotation through 180°. The cross shows the point of fixation and neither it nor the rectangular frame was visible during stimulus presentation.) Main figure: *same–different* discrimination performance is shown as a function of pattern transformation in Experiment 1. (The Transformations a–f correspond to those illustrated in the inset. *Different* patterns were obtained by pairing patterns at random. The *d'* values were calculated from the pooled *same* responses [over 4 subjects] to *same* patterns [hits] and to *different* patterns [false alarms]. Total number of *same* trials per transformation = 192; total number of *different* trials = 1,152.)

tion performance should occur when the reflection axis is perpendicular to an imaginary line joining the centers of the patterns (Figure 1, inset, Section c). This is so because the representations of the patterns can, in principle, be brought into coincidence without modification of the special global position relation; all that is needed is the simple operation of reversal of the sense of horizontal relations.

Prediction 2: Selective Oblique Effect

Prediction 2a. Suppose that the patterns were related by a reflection in an axis perpendicular to an imaginary line joining the centers of the patterns (Figure 2, inset, Sections e–h). If the patterns were positioned symmetrically about the fixation point, then *same*-detection performance should be lower when the imaginary line joining the centers of the patterns is oblique (Figure 2, inset, Sections e, g) than when it is horizontal or vertical (Figure 2, inset, Sections f, h). This is so because, when the line is oblique, the representations of the patterns cannot, in principle, be brought into coincidence by a simple reversal of the sense of horizontal relations, or vertical relations, or both. When the line is horizontal or vertical, these simple operations are sufficient.

Prediction 2b. Suppose that the patterns were identical (Figure 2, inset, Sections a–d). Then there should be no oblique effect as in 2a above. Independent of orientation of the imaginary line joining the patterns, the patterns differ by a constant separation, and their representations can, in principle, be brought into coincidence by continuous modification of the special global position relation alone.

Prediction 2c. Suppose that the patterns were related by point-inversion (Figure 2, inset, Sections i–l). Then there should also be no oblique effect as in 2a. Independent of orientation of the imaginary line joining the centers of the patterns, the representations can, in principle, be brought into coincidence just by reversing the sense of horizontal and vertical relations. This situation contrasts with that of 2a.

Prediction 3: Selective Midline Effect

Prediction 3a. Suppose that the patterns were related by reflection in a vertical axis (Figure 3, inset, Sections c, d). If the patterns were positioned symmetrically about the vertical midline, then *same*-detection performance should be independent (within some limits) of the vertical positions of the patterns. This is so because, independent of vertical position, the representations of the patterns can, in principle, be brought into coincidence by reversal of the sense of horizontal relations alone.

Prediction 3b. Suppose that the patterns were identical and positioned symmetrically about the vertical midline (Figure 3, inset, Sections a, b). Then, as in 2a, *same*-detection performance should also be independent (within limits) of vertical position.

Prediction 3c. Suppose that the patterns were related by point-inversion and positioned symmetrically about the vertical midline (Figure 3, inset, Sections e, f). Then *same*-detection performance should be lower for patterns above or below the horizontal midline than for patterns in line with the fixation point. This is so because only when the patterns are in line with the fixation point can the representations be brought, in principle, into coincidence by reversal of the sense of horizontal and vertical relations alone; when the patterns are above or below the fixation point, additional continuous modification of the special global position relation is required to bring the representations into coincidence.

Prediction 1 is not counterintuitive, but is a necessary prerequisite for Prediction 2. The most important predictions are 2a and 2c, and 3a and 3c, for the differences in predicted *same*-

detection performances with reflected and point-inverted patterns depend directly on the hypothesized internal horizontal-vertical framework. These predictions were tested in the following experiments. Random-dot patterns were used throughout so that stimuli would be unfamiliar to subjects and would have no particular meaning, name, or conventional handedness or orientation, which can be ascribed, for example, to letters and geometrical figures. Fresh random-dot patterns were generated in every trial for every subject. Because performance was measured for stimuli differing not only in transformation but also in position in the field and because discrimination was determined by responses to both *same* and randomly paired *different* patterns, the discrimination index *d′* from signal detection theory was used (Green & Swets, 1966). The index *d′* is zero when performance is at chance level and increases monotonically with improvement in performance. It has a number of advantages as a measure of discrimination performance (Swets, 1973), including its freedom from bias and its additivity (Durlach & Braida, 1969).

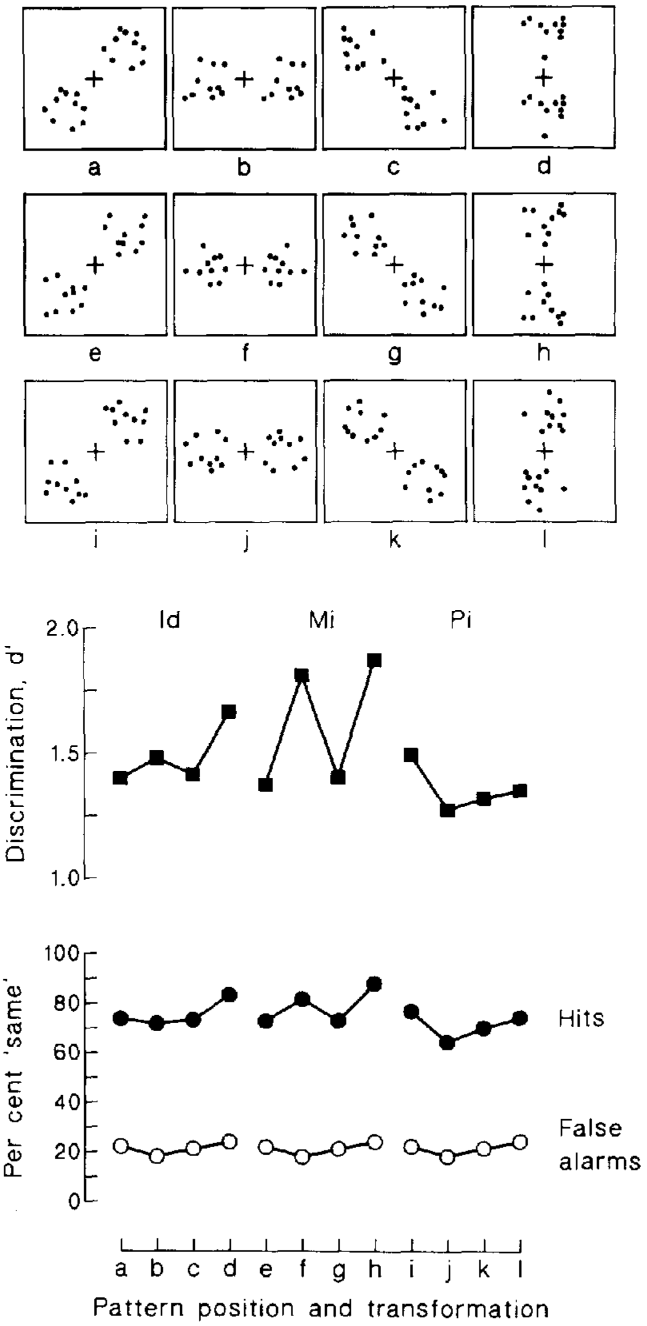

Figure 2. Inset panels: illustrations of *same* random-dot patterns and their positions and transformations used in Experiment 2. (In each of Sections a–d the patterns are related by identity [Id]; in each of Sections e–h one pattern was obtained from the other by reflection [Mi] in an axis perpendicular to an imaginary line joining the pattern positions; in each of Sections i–l one pattern was obtained from the other by point-inversion [Pi]. The cross shows the point of fixation and neither it nor the rectangular frame was visible during stimulus presentation.) Main figure: *same–different* discrimination performance as a function of pattern position (display orientation) and transformation in Experiment 2. (The display orientations and Transformations a–l correspond to those illustrated in the inset. *Different* patterns were obtained by pairing patterns at random. The *d′* values were calculated from the pooled *same* responses [over 5 subjects] to *same* patterns [hits] and to *different* patterns [false alarms]. Total number of *same* trials per position and transformation = 240; total number of *different* trials per position = 2,880.)

Experiment 1: Reflection-Axis Effect

In this experiment we used pairs of horizontally positioned patterns related by a reflection and determined the effect of varying the orientation of the axis of reflection. According to Prediction 1 above, *same*-detection performance should have been highest when this axis was vertical. For comparison, *same*-detection performance was also measured for identical patterns and patterns related by point-inversion.

Method

Subjects. Two male and 2 female subjects, from 23 to 26 years old, participated in the experiment. All were unpaid volunteers and were members of or visitors to the Department of Communication and Neuroscience. Each had normal or corrected-to-normal visual acuity. All except 1 subject (coauthor JIK) were unaware of the purpose of the experiment.

Apparatus. Stimuli were produced on the screen of an X-Y display oscilloscope (Hewlett-Packard, Type 1300A) with P4 sulfide phosphor (decay time 60 μs), controlled by a minicomputer (CAI Alpha LSI-2) with vector-graphics interface (Sigma Electronic Systems QVEC 2150). The screen was viewed binocularly at a distance of 1.7 m through a view tunnel and optical system which produced a uniform white background field subtending 7.4° × 6.2° at the eye and of luminance approximately 60 cd/m^2. The stimuli were white and appeared superimposed on the background field. The intensity of the stimuli was adjusted by each subject at the beginning of each experimental session to be 10 times luminance increment threshold (typically 50 μcd·s). This setting was achieved by introducing a 1.0-log-unit neutral density filter over the stimulus dots and by adjusting their intensity to increment threshold on the unattenuated background. Stimuli were thus adequately suprathreshold, but not so intense as to produce prolonged afterimages.

Fixation was aided by a computer-generated cross formed by two white lines, approximately 3° long, and a computer-generated white fixation spot superimposed at the center of the cross. The spot was displayed throughout each presentation, but the cross was extinguished at the start of each trial. The subject controlled the start of each trial and gave his or her responses on a hand-held push-button box connected to the computer.

Stimuli. Stimuli were random-dot patterns (as illustrated in reverse contrast in Figure 1, inset), each consisting of 10 dots distributed pseudo-randomly within an imaginary circle of diameter 0.5° visual an-

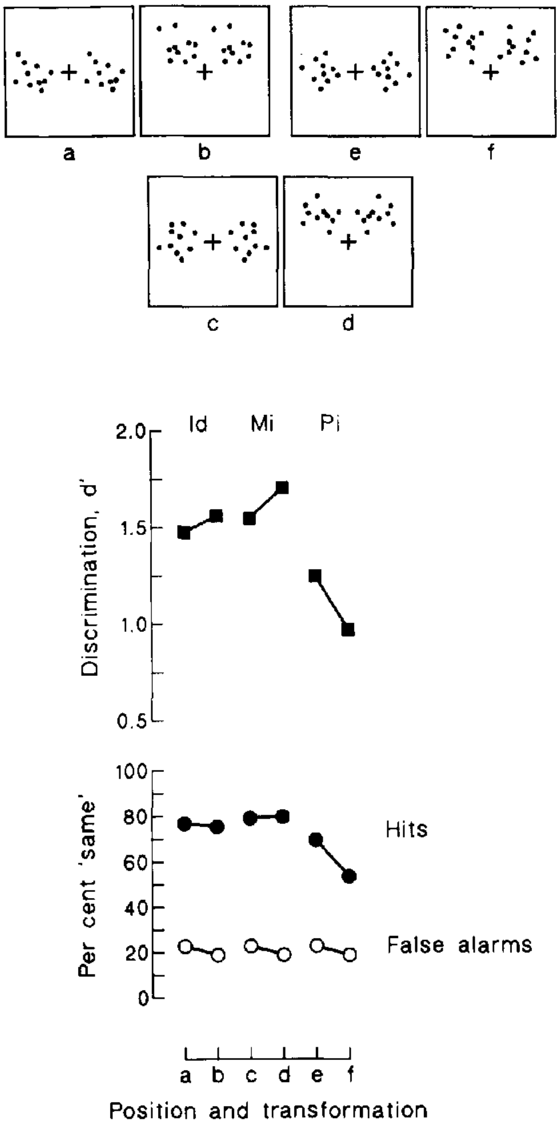

Figure 3. Inset panels: illustrations of *same* random-dot patterns and their positions and transformation used in Experiment 3. (In each of the Sections a, b the patterns are related by identity [Id]; in each of the Sections c, d one pattern was obtained from the other by reflection [Mi] in the vertical midline; in each of the Sections e, f one pattern was obtained from the other by point-inversion [Pi]. In Conditions b, d, and f, the vertical offset occurred upwards and downwards equally often. The cross shows the point of fixation and neither it nor the rectangular frame was visible during stimulus presentation.) Main figure: *same–different* discrimination performance as a function of pattern position (vertical offset) and transformation in Experiment 3. (The display positions and Transformations a–f correspond to those illustrated in the inset. *Different* patterns were obtained by pairing patterns at random. The *d'* values were calculated from the pooled *same* responses [over 9 subjects] to *same* patterns [hits] and to *different* patterns [false alarms]. Total number of *same* trials per position and transformation = 324; total number of *different* trials per position = 972.)

gle. Each dot subtended about 0.03°. Fresh random-dot patterns were generated for every trial.

Pattern positions. In each trial, two patterns appeared simultaneously, the one centered 0.5° to the left of the fixation point, the other 0.5° to the right of the fixation point.

Pattern transformations. There were six possible transformations (other than translations) relating the patterns in each *same* pair. These were the following: Id—the two patterns were identical (Figure 1, inset, Section a); Mi_θ—one pattern was obtained from the other by (mirror) reflection in an axis oriented at an angle θ clockwise from the vertical, θ taking the values −45°, 0°, 45°, and 90° (Figure 1, inset, Sections b–e, respectively); and Pi—one pattern was obtained from the other by point-inversion, that is, planar rotation through 180° about the center of the imaginary circle constraining the pattern (Figure 1, inset, Section f).

For *different* pairs, the two patterns were generated independently of each other.

Instructions. At the beginning of the experiment, subjects were informed of the nature of the stimuli and of the types of transformation. Subjects were instructed to indicate after the presentation of each pair of patterns whether they were the *same* or *different* according to the transformations listed above. It was emphasized that steady fixation was to be maintained throughout each presentation period and that responses should be made as quickly as possible while preserving accuracy. Subjects were given a preliminary run of 10–15 presentations to familiarize them with the types of stimuli and use of the response box. No feedback was given to subjects on their performances.

Presentation sequence. Following initiation of the trial by the subject, the fixation spot was extinguished, and, after a 1.0-s delay, the stimulus patterns were presented for 100 ms (a period too short for guided changes in fixation; see Bartz, 1962; Westheimer, 1954; White, Eason, & Bartlett, 1962). The subject's response was recorded by the computer. The time taken to make that response was also recorded as a control and to test for trade-off effects. After a 1.0-s delay, the fixation spot was redisplayed, indicating that the next trial could begin.

Experimental design. Trials were performed in sequences of 48. In each such run, each of the six *same* pattern transformations occurred four times, making 24 *same* pairs, and there were 24 *different* pairs. Each subject performed 12 runs in one experimental session. For the purpose of balancing the design, each run was split into four subruns of 12 trials. Within each subrun, the order of pattern transformations was chosen pseudo-randomly but balanced over runs to offset order and carry-over effects.

Results and Discussion

Figure 1 shows *same–different* pattern-discrimination performance as a function of pattern transformation: identity transformation, Id, in Section a; reflection, Mi_θ, about axis θ to the vertical in Sections b–e, as illustrated in the inset; and point-inversion, Pi, in Section f. The percentage of *same* responses to *same* patterns (hit rate) and to *different* patterns (false-alarm rate) is shown pooled over subjects. The discrimination index *d'* was calculated from these pooled scores. Differences between *d'* calculated thus and calculated as the weighted mean of individual subjects' *d'* values did not exceed 3% for any of the transformation conditions. A chi-squared test (Appendix) on individual subjects' data revealed no significant underlying differences between subjects in their variations over conditions, $\chi^2(14) = 16.3, p > .2$. Specific hypotheses concerning discrimination performance were tested by computing standardized contrasts. Two methods were used. The first used the variances of the *d'* estimated from the binomial scores and yielded a *z* statistic; the second used a less powerful but more robust method with subjects as the sampling unit and yielded a *t* statistic (Appendix). Results from both tests are presented.

Reaction times. A standard repeated-measures analysis of variance on the untransformed times showed that correct *same* responses were not significantly faster than incorrect *same* responses, 644 ms versus 767 ms, $F(1, 3) = 4.23$, $p > .1$, two-tailed test; there was no interaction between correctness of *same* responses and pattern transformation, $F(5, 15) = 1.93$, $p > .1$. Correct *same* reaction times (RTs) were not significantly different from correct *different* RTs, 644 ms versus 641 ms, $t(3) = 0.12$, $p > .5$, two-tailed test. There was no trade-off between performance (percent correct) and RT. With all data considered as a single group, RTs for correct *same* responses were significantly negatively correlated with percent correct *same* responses, gradient -3.10 ± 1.19 ms·percent^{-1}, $t(22) = 2.60$, $p < .05$, two-tailed test; differences between slopes or intercepts over transformations were not significant, $F(10, 12) = 0.56$, $p > .5$. RTs for incorrect *same* responses were not significantly correlated with percent incorrect *same* responses, gradient 1.86 ± 2.40 ms·percent^{-1}, $t(22) = 0.78$, $p > .2$, two-tailed test; differences between slopes or intercepts over transformations were not significant, $F(10, 12) = 0.82$, $p > .5$.

Best axis of reflection. *Same*-detection performance for patterns related by reflection, Mi_0, in a vertical axis (Figure 1, Section c) was higher than that for patterns related by any of the other reflections, Mi_{-45}, Mi_{45}, and Mi_{90} (Figure 1, Sections b, d, e, respectively)—$z = 8.7$, $p < .0001$; $t(3) = 7.1$, $p < .01$; two-tailed tests. This result confirms and extends previous results on the effects of axis reflection by Sekuler and Rosenblith (1964) and Foster and Mason (1979). There was no significant difference between performance for patterns related by Mi_{90} (Figure 1, Section e) and by Mi_{-45}, Mi_{45} (Figure 1, Sections b, d)—$\chi^2(2) = 3.17$, $p > .1$; linear contrast $t(3) = 0.47$, quadratic $t(3) = 1.2$, $p > .5$.

Other transformations. There was no significant difference in *same*-detection performance for patterns related by identity transformation, Id (Figure 1, Section a), and reflection, Mi_0, (Figure 1, Section c)—$z = 0.0$, $p > .5$; $t(3) = 0.26$, $p > .5$, two-tailed tests. There was a significant difference, however, between performance for patterns related by Id (Figure 1, Section a) and by point-inversion, Pi, (Figure 1, Section f)—$z = 3.80$, $p < .001$; $t(3) = 5.06$, $p < .05$, two-tailed tests.

As anticipated in Prediction 1 in the introduction, highest *same*-detection performance for patterns related by reflection occurred when the axis of reflection was vertical. The following experiment tested the effects of display orientation on *same*-detection performance.

Experiment 2: Selective Oblique Effect

The effect of display orientation on *same*-detection performance was measured for patterns related by reflection in an axis perpendicular to an imaginary line joining the patterns, both for identical patterns and for patterns related by point-inversion. By Prediction 2 in the introduction, an "oblique" effect should have occurred only for patterns related by reflection.

Method

Subjects. Five male subjects, from 23 to 27 years old, participated in the experiment. All were unpaid volunteers and were members of or visitors to the Department of Communication and Neuroscience. Each had normal or corrected-to-normal visual acuity. All except 1 subject (coauthor JIK) were unaware of the purpose of the experiment.

Apparatus. Apparatus and display were the same as in Experiment 1.

Stimuli. The stimuli in this experiment were random-dot patterns, but differed from those used in Experiment 1 in that they were "normalized." Thus, random-dot patterns were generated as in Experiment 1 and then scaled linearly, horizontally, and vertically, so that the horizontal separation of the horizontally extreme pair of dots was 0.5°, and the vertical separation of the vertically extreme pair of dots was 0.5°. This procedure was adopted after pilot experiments suggested the possibility that subjects might be able to use inappropriate cues for discriminating *same* patterns based on the equal horizontal or vertical extents of *same* patterns and the generally different horizontal and vertical extents of *different* patterns. After normalization, *same* patterns and *different* patterns all had the same horizontal and vertical extents (Figure 2, inset). Normalization is discussed more fully later.

Pattern positions. In each trial, two patterns appeared simultaneously, the one positioned 0.5° from the fixation point in one of four directions, the other positioned 0.5° from the fixation point in the opposite direction (Figure 2, inset). The four directions, −45°, 0°, 45°, and 90°, measured clockwise from the horizontal, defined four "display orientations," so that in display orientation of 0° the patterns were side by side, and in display orientation of 90° one pattern was above the other.

Pattern transformations. There were three possible transformations (other than translations) relating the patterns in each *same* pair. These were the following: Id—the two patterns were identical (Figure 2, inset, Sections a–d); Mi—one pattern was obtained from the other by reflection in an axis perpendicular to an imaginary line joining the pattern positions (Figure 2, inset, Sections e–h); and Pi—one pattern was obtained from the other by point-inversion, that is, planar rotation through 180° about the center of the imaginary circle constraining the pattern (Figure 2, inset, Sections i–l).

For *different* pairs, the two patterns were generated independently of each other. Fresh patterns were generated for every trial.

Instructions and presentation sequence. The instructions to the subject and the time course of each presentation sequence were as in the previous experiment.

Experimental design. There were 48 trials in each experimental run. In each run, every display orientation (−45°, 0°, 45°, 90°) occurred twice with each of the three *same* pattern transformations (Id, Mi, Pi) and six times with *different* pairs, so that a run consisted of 24 *same* pairs and 24 *different* pairs. Each subject performed 24 runs over a period of several days. For the purpose of balancing the design, each run was split into subruns within which the order of display orientations was chosen pseudo-randomly but balanced for order and carry-over effects over subruns. The sequence of pattern transformations occurring with a given display orientation was also chosen pseudo-randomly.

Results and Discussion

Figure 2 shows *same–different* pattern-discrimination performance. In each graph in the figure, discrimination index d' is plotted against display orientation, −45°, 0°, 45°, 90°, for each of the pattern transformations: identity transformation, Id, in Sections a–d; reflection, Mi, in Sections e–h; and point-inversion, Pi, in Sections i–l.

The d' data were calculated from the pooled *same* and *different* scores. Differences between d' calculated thus and calculated as the weighted mean of individual subjects' d' values did not exceed 7% in any of the conditions. A chi-squared test (Appendix) on individual subjects' data showed no significant underlying differences between subjects in their variations over conditions, $\chi^2(43) = 57.3$, $p > .05$.

Reaction times. Correct *same* responses were not significantly faster than incorrect *same* responses, 649 ms versus 747 ms, $F(1, 4) = 1.24$, $p > .2$, and there was no significant interaction between correctness of *same* responses and pattern transformation, $F(2, 8) = 1.10$, $p > .2$. There was a significant interaction between response correctness and pattern position, $F(3, 12) = 4.5$, $p < .05$, but not between response correctness, pattern position, and transformation, $F(6, 24) = 0.64$, $p > .5$. Correct *same* RTs were not significantly different from correct *different* RTs, 649 ms versus 659 ms, $t(4) = 0.16$, $p > .5$, two-tailed test. There was no trade-off between performance (percent correct) and RT. With all data considered as a single group, RTs for correct *same* responses were numerically negatively correlated with percent correct *same* responses, although not significantly, gradient -1.23 ± 0.81 ms·percent^{-1}, $t(58) = 1.52$, $p > .1$, two-tailed test; differences between slopes or intercepts over conditions were not significant, $F(22, 36) = 0.11$, $p > .5$. RTs for incorrect *same* responses were significantly negatively correlated with percent incorrect *same* responses, gradient -7.05 ± 1.81 ms·percent^{-1}, $t(58) = 3.89$, $p < .001$, two-tailed test; differences between slopes or intercepts over conditions were not significant, $F(22, 36) = 0.45$, $p > .5$.

Oblique effects. Oblique effects were tested for by contrasting performance for display orientations 0° and 90° with performance for display orientations −45° and 45°. There was no significant oblique effect for identity transformation, Id (Figure 2, Sections a–d), or for point-inversion, Pi (Figure 2, Sections i–l)—$z \leqslant 1.60$, $p > .1$; $t(4) \leqslant 1.70$, $p > .1$, for both, two-tailed tests; and, more generally, there were no significant deviations from constancy for either, $\chi^2(3) \leqslant 3.99$, $p > .2$. There was a highly significant oblique effect for reflection, Mi (Figure 2, Sections e–h)—$z = 4.23$, $p < .0001$; $t(4) = 2.96$, $p < .05$, two-tailed test.

Relative levels. There were no decisive significant differences between mean peformance over display orientations for Id and Mi, $z = 1.92$, $p > .05$; $t(4) = 3.11$, $p < .05$, two-tailed tests; and for Id and Pi, $z = 2.16$, $p < .05$; $t(4) = 2.35$, $p > .05$, two-tailed tests. There were no significant differences between mean performances over −45° and 45° for Mi and over all orientations for Id and for Pi, $z \leqslant 1.18$, $p > .2$; $t(4) \leqslant .91$, $p > .2$, two-tailed tests.

These results confirmed Prediction 2 in the introduction, namely, that the ability to detect "sameness" of patterns related by a reflection should have shown an oblique effect and that such an effect should not have occurred for pattern pairs that were identical or were related by point-inversion. The fact that no oblique effect was obtained for identical patterns implies, inter alia, that the oblique effect shown for patterns related by a reflection was not attributable to the well-known reduction in spatial acuity associated with the oblique axes (Onley & Volkmann, 1958; Rochlin, 1955; Weene & Held, 1966).

Experiment 3: Selective Midline Effect

In this experiment we tested the effect of varying the vertical position of a pair of transformed patterns positioned symmetrically about the vertical midline. By Prediction 3 in the introduction, *same*-detection performance for identical patterns and for patterns related by reflection in a vertical axis should have been independent (within limits) of vertical position; for patterns related by point-inversion, *same*-detection performance should have been lower for patterns above or below the fixation point than for patterns in line with the fixation point.

Method

Subjects. Nine male subjects, from 21 to 27 years old, participated in the experiment. All were unpaid volunteers and were members of or visitors to the Department of Communication and Neuroscience. Each had normal or corrected-to-normal visual acuity. All except 1 subject (coauthor JIK) were unaware of the purpose of the experiment.

Apparatus. Apparatus and display were the same as in Experiment 1.

Stimuli. The stimuli were normalized random-dot patterns, as in Experiment 2 (Figure 3, inset).

Pattern positions. In each trial, two patterns appeared simultaneously. The distance of each of the patterns from the point of fixation was always 1.0°. There were two types of position combination used in this experiment: *in-line*—the patterns were positioned on an imaginary horizontal line through the fixation point, one pattern to the left, the other to the right; vertical offset was thus 0° (Figure 3, inset, Sections a, c, e); and *offset*—an imaginary line joining the fixation point to the pattern position was 45° to the horizontal; the patterns were either both above or both below the level of the fixation point, one pattern to the left, the other to the right; vertical offset was 0.7° (Figure 3, inset, Sections b, d, f).

Instructions and presentation sequence. The instructions to the subject and the time course of each presentation were as in the previous experiment.

Experimental design. There were 36 trials in each experimental run. In each run, both of the position combinations occurred three times with each *same* pattern transformation and nine times with each *different* pair, so that a run consisted of 18 *same* pairs and 18 *different* pairs. Each subject performed 12 runs in one session. In the offset position combination, the patterns were either both above or both below the horizontal midline. In every two runs, the offset position combination occurred six times with each pattern transformation, three times with the patterns above the fixation point and three times with the patterns below. The order of pattern transformations and position combinations was chosen pseudo-randomly, but balanced for order and carry-over effects over runs.

Results and Discussion

Figure 3 shows *same–different* pattern discrimination performance. Discrimination index d' is shown for the two pattern-position combinations, in-line (vertical offset 0°), and offset (vertical offset 0.7°), with each of the three pattern transformations: identity, Id (Sections a and b); reflection, Mi (Sections c and d); and point-inversion, Pi (Sections e and f).

The d' data were calculated from the pooled *same* and *different* scores. Differences between d' calculated thus and calculated as the weighted mean of individual subjects' d' values did not exceed 2% in any of the conditions. A chi-squared test (Appendix) on individual subjects' data showed no significant underlying differences between subjects in their variations over conditions, $\chi^2(39) = 38.8$, $p > .2$.

Reaction times. Correct *same* responses were significantly faster than incorrect *same* responses, 770 ms versus 995 ms, $F(1, 8) = 14.0$, $p < .01$. There was no significant interaction between correctness of *same* responses and pattern position, $F(1, 8) = 1.53$, $p > .2$; between response correctness and pattern transformation, $F(2, 16) = 2.62$, $p > .1$; or between response correctness, pattern position, and transformation, $F(2, 16) =$

0.96, $p > .2$. Correct *same* RTs were not significantly different from correct *different* RTs, 770 ms versus 810 ms, $t(8) = 1.02$, $p > .2$, two-tailed test. There was no trade-off between performance (percent correct) and RT. With all data considered as a single group, RTs for correct *same* responses were significantly negatively correlated with percent correct *same* responses, gradient -4.67 ± 1.94 ms·percent^{-1}, $t(52) = 2.41$, $p < .05$, two-tailed test; differences between slopes or intercepts over conditions were not significant, $F(10, 42) = 0.76$, $p > .5$. RTs for incorrect *same* responses were not significantly correlated with percent incorrect *same* responses, gradient -1.72 ± 3.17 ms·percent^{-1}, $t(52) = 0.54$, $p > .5$, two-tailed test; differences between slopes or intercepts over conditions were not significant, $F(10, 42) = 0.58$, $p > .5$.

Standardized contrasts were computed, and these showed that performance for patterns in the offset position combination relative to that for patterns in the in-line position was not significantly different for identity transformation, Id (Figure 3, Sections a–b)—$z = 0.69$, $p > .2$; $t(8) = 0.59$, $p > .5$, two-tailed tests; or for reflection, Mi (Figure 3, Sections c–d)—$z = 1.23$, $p > .2$; $t(8) = 1.53$, $p > .1$, two-tailed tests; but performance was significantly different for point-inversion, Pi (Figure 3, Sections e–f)—$z = 2.34$, $p < .05$; $t(8) = 2.36$, $p < .02$, two-tailed tests.

These results were consistent with Prediction 3 in the introduction, namely, that the ability to detect the sameness of pairs of patterns that were identical or related by reflection in a vertical axis should not have been affected by a (limited) vertical displacement of the patterns, whereas the ability to detect the sameness of pairs of patterns related by point-inversion should have been reduced by such a displacement.

General Discussion

Summary of Experiments 1–3

The principal experimental results were these.

1. For two patterns related by reflection and positioned symmetrically about the point of fixation, highest *same*-detection performance occurred when the axis of reflection was perpendicular to the imaginary line joining the pattern positions.
2. When the axis of reflection was perpendicular to this imaginary line, *same*-detection performance was higher when the line was horizontal or vertical than when the line was oblique.
3. No such orientation effects were found for *same*-detection performance with patterns that were identical or were related by point-inversion.
4. For patterns positioned symmetrically about the vertical midline, *same*-detection performance for identical patterns or patterns related by reflection in a vertical axis was independent of the vertical position of the patterns relative to the point of fixation, whereas performance for patterns related by point-inversion was reduced by vertical displacement relative to the point of fixation.

Residual High Performance With Obliquely Oriented Displays

Although *same*-detection performance for pairs of patterns related by reflection was much higher for horizontal or vertical reflection axes than for oblique axes (Figure 2, Sections e–h), performance in the latter case was still high. If the visual system is not equipped to respond specifically to patterns related by reflections in an oblique axis, then why was *same*-detection performance for such stimuli as high as that for patterns related by point-inversion (Figure 2, Sections i–l), to which the visual system *is* apparently equipped to respond? There are two possibilities. First, there may have been more than one way in which patterns related by reflection were detected as *same*. Thus, in addition to the operation of reversing the sense of horizontal or vertical relations before matching, there may have been a process of direct comparison of stimulus features located near to each other (compare Bruce & Morgan, 1975). Such a direct process would not have worked for identical patterns or patterns related by point-inversion, for, in the former case, identical local features would not have been near each other, and, in the latter case, the only identical local features near each other would have been those also near the imaginary line joining the patterns. For patterns related by reflection, direct feature comparison would have improved performance by some fixed increment independent of display orientation.

A second possible explanation for this overall high performance with oblique reflections is that the visual system makes use of display orientation cues to realign the horizontal–vertical internal coordinate system and subsequently to reencode the stimuli in terms of spatial relations that are (exclusively) oblique. Compensation for reflection in an oblique axis becomes equivalent to the simple operation hypothesized for patterns reflected in a horizontal or vertical axis. Such reorientations of an internal reference frame have been previously suggested by Rock and Leaman (1963), Attneave and Olson (1967), Attneave (1968), and Rock (1973). The hypothesized process of reorientation and reencoding would presumably have costs in terms of discrimination performance (for example, the time taken to effect the reorientation might allow deterioration in the fidelity of the internal representation), so that the sameness of patterns related by reflections in an oblique axis would have been more difficult to detect than the sameness of patterns related by reflection in a horizontal or vertical axis. The very high performance for patterns related by reflection in a horizontal or vertical axis might then be explained by the fact that the sense of only one set of relations (horizontal or vertical) needs to be reversed, compared with the senses of the two sets that need to be reversed for patterns related by point-inversion.

Normalization of Random-Dot Patterns

The random-dot patterns used in Experiments 2 and 3 were normalized in their horizontal and vertical extents. This modification was made on the basis of pilot studies using nonnormalized random-dot patterns. In those studies, performance for all display orientations was high, and introduction of normalization caused a decrease in performance for patterns related by reflection in an oblique axis and a small decrease for identical patterns; there was no change in performance for patterns related by point-inversion or by reflection in a horizontal or vertical axis. Notice that the observed oblique effect (Figure 2) could not have been an artifact of normalization or of the choice of axes of normalization: No oblique effect was found for identical or point-inverted patterns.

It seems plausible that normalization of the stimuli prevented subjects from making judgments about the sameness of patterns according to the following strategy: If patterns had the same width measured perpendicular to the line joining the pattern positions, then patterns should be reported as *same;* if they had different width, then they should be reported as *different.* After normalization, the widths of *different* patterns were identical to the widths of *same* patterns. (If there were no normalization, the strategy of reporting *different* when the perpendicular widths of the patterns were different would have modified only the false-alarm rate for a given display orientation, thus affecting measured performance for *all* pattern transformations for that orientation.) The existence of a differential effect on performance for, say, point-inversion, Pi, relative to reflection, Mi, implies that normalization must have inhibited the auxiliary strategy of reporting *same* if the perpendicular widths of the patterns were the same, and that strategy must have been useful for detecting only those patterns that were related by reflection in an oblique axis.

It might be hypothesized that the ability to detect sameness of patterns related by point-inversion was also the result of width-matching. This might have explained the result established in several preceding studies (see introduction) that, as *same* patterns are rotated, there is a worsening in performance for angles up to 90° and a subsequent improvement for angles up to 180°. This hypothesis can be rejected for the following reasons.

1. In Experiment 1 (Figure 1), *same*-detection performance for patterns related by reflection in a vertical axis or by point-inversion was higher than that for patterns related by reflection in a horizontal axis, although the perpendicular widths of the patterns in all three cases were the same.
2. Width-matching cannot explain the effects of positional symmetry and separation on *same-different* performance with transformed random-dot patterns (Foster & Kahn, 1985; Kahn & Foster, 1981).

Other Explanatory Schemes

The important result for a horizontal-vertical reference-structure scheme is that for pairs of patterns related by reflection there was an oblique effect but that for pairs of patterns that were identical or related by point-inversion there was no such effect. These and the other experimental results would not have been expected from object- or pattern-centered coordinate descriptions or from viewer-centered descriptions isotropic with angular position, particularly those using polar coordinate systems (Leibovic et al., 1971; Schwartz, 1980). A pattern-centered description assigned to each individual pattern would have predicted constant discrimination performance over all transformations and all pattern positions in every experiment. A less extreme pattern-centered description based on pattern pairs rather than on individual patterns would also not suffice, for such a scheme would have predicted constant performance over pattern positions in Experiments 2 and 3 for *all* transformations. A polar coordinate system could have predicted the results of Experiment 1, providing that the angle θ in the usual (r, θ) coordinate system was defined to be zero at the vertical meridian and that sense-reversal of θ was allowed. It could also have predicted the constant effect of position for transformation Pi in Experiment 2, providing that sense-reversal of the coordinate r was allowed, but not the constant effect for transformation Id nor the nonconstant effect for transformation Mi. It could have predicted the effect of position shown in Experiment 3 for transformation Pi and the constant effect for Mi, but not for Id.

Symmetry Perception

The present experiments entailed *same-different* judgments in which the *different* patterns were always patterns paired at random. A number of studies have examined specifically the question of the discriminability of symmetry, usually using reaction time as the dependent measure and, in some cases, a small, fixed repertoire of patterns. Corballis and Roldan (1974) examined the discrimination of identical and mirror-image patterns under two instructional conditions: the one requiring judgments *symmetrical* and *asymmetrical,* the other requiring judgments *mirror* and *same.* For random-dot patterns, instructions had no effect on reaction time. Separation of the patterns, however, was important, and for adjacent patterns, which were assumed to favor a holistic percept, it was found that symmetry was perceived more rapidly than repetition, whereas for separated patterns, which were assumed to favor the perception of distinct figures, there was no significant difference in RTs.

In a different study, Corballis and Roldan (1975) determined RTs to discriminate identical patterns from reflected patterns as a function of the orientation of a visible axis forming the perpendicular bisector of an imaginary line joining the adjacent patterns (similar to Figure 2, inset, Sections a–h, but with no gap between the patterns). They found no evidence of an oblique effect; RT increased strictly monotonically as orientation angle increased from 0° (vertical) to 90°, a finding reminiscent of the data on mental rotation obtained by Shepard and his colleagues. Moreover, when the experiment was performed with subjects' heads tilted at 45°, a shift in the orientation function occurred, suggesting some processing in terms of retinal coordinates.

Instead of using separate adjacent patterns, Palmer and Hemenway (1978) tested the perception of symmetry using single polygon figures displaying a variety of symmetries. Detection of symmetry was fastest for vertical, next fastest for horizontal, and slowest for diagonal axes, a result consistent with the majority of the literature on oblique effects and apparently contradicting the results of Corballis and Roldan (1975). Curiously, there was a small but not significant oblique effect for a symmetry equivalent to point-inversion.

The anatomically oriented theories of Mach (1897/1959) and Julesz (1971) offer an explanation only for the special case of symmetry about the (retinal) vertical midline. The relational-structural scheme outlined in the present study could provide the basis of a less specific theory of symmetry. A general rule might be formulated thus: Any pattern which gave rise to a representation that was *invariant under reversal of horizontal* (or vertical) relations should be classified perceptually as *symmetric.* Consistent with data obtained by Palmer and Hemenway (1978) on symmetry perception, such a scheme would explain the oblique effects for single (mirror) symmetry and for double symmetry, and the weak or absent effects for rotational (point-inversion) symmetry.

Preferred Axes and Transformational Schemes

It was noted in the introduction that the predictions of the relational-structure schemes of the kind considered here could also be generated by transformational schemes. The special action of sense-reversal operations defined along horizontal and vertical directions would be replaced by families of rotations in three dimensions defined, respectively, about the vertical and horizontal axes. Given the assumption of a special status for these rotation axes (Metzler & Shepard, 1974; Shepard & Cooper, 1982), a parallel account of the results obtained in the present experiments could be developed. Professor Shepard[1] has proposed the following: In Experiment 1, discrimination performance is high under identity transformation, Id (Figure 1, Section a), because it is a simple (horizontal), translation, under reflection, Mi_0, about the vertical axis (Figure 1, Section c) because it is a simple 180° rotation (in depth) about the vertical axis between the two patterns, and under point-inversion, Pi (Figure 1, Section f), because it is a simple 180° rotation (in the picture plane) about a point midway between the two patterns. Performance under all other transformations, that is, reflections about axes oriented at angles −45°, 45°, and 90° to the vertical, is low because the general screw displacement in three-dimensional space needed to take one representation into the other has both translational and rotational components and an axis (of the screw) that is less simply related to the pair of patterns.

In Experiment 2, the effects of position for transformation Id (Figure 2, Sections a–d) should be approximately equal because all the conditions require a simple translation (and because the axis of the degenerate screw displacement is in all four cases at infinity). The effects of position for transformation Pi (Figure 2, Sections i–l) should also be approximately equal because all the conditions require a 180° roation in the plane about a point midway between the two patterns. For reflection, Mi, the four position conditions all require a 180° rotation in depth about an axis that is the perpendicular bisector of the imaginary line connecting the centers of the two patterns; but for the combinations in Figure 2, Sections f and h, this axis has the preferred vertical or horizontal orientation, respectively, and hence yields better discrimination performance.

Finally, in Experiment 3, the offset should not affect performance under transformation Id (Figure 3, Sections a, b) because the axis, being at infinity, is not altered by the offset. The offset should also not affect performance under transformation Mi about the vertical axis (Figure 3, Sections c, d) because here, too, the axis is mapped into itself by the offset. For transformation Pi (Figure 3, Sections e, f), the fixed point, which is midway between the patterns, will be less readily picked up if it is offset from the fixation point.

Such an explanatory scheme cannot be rejected by the present discrimination data alone. Indeed, given the formal duality of transformational schemes (with preferred axes) and relational-structure schemes, any differences might be expected to be revealed here only at a secondary level, for example, in accounting for the magnitude and variation of reaction times. RT values were rather smaller than those usually obtained in mental rotation experiments, but the present *same–different* discrimination tasks were relatively simple. Moreover, RTs tended to increase as discrimination performance decreased, a finding consistent with the notion that operations about nonpreferred axes took longer. One piece of evidence from another study that might be offered against a transformational scheme concerns *same–different* discriminations of sequentially presented patterns differing by a rotation in the plane but each centered on the point of fixation. Performance was found not to be strictly monotonically decreasing with rotation angle (Kahn & Foster, 1981). The departure from strict monotonicity was not, however, sufficient to constitute a reliable upturn in performance for 180° rotation.

Generality of a Horizontal-Vertical Reference System

A special role for the horizontal and vertical in visual perception has been noted in other studies (Attneave, 1955, 1968; Attneave & Olson, 1967; Koffka, 1935; Mach, 1897/1959, chapter 6; Olson & Attneave, 1970; Rock, 1973). For example, Olson and Attneave (1970) showed that a pattern comprising horizontal and vertical lines gave better grouping effects than one comprising lines oriented at −45° and 45° to the vertical, despite the fact that the difference between the slopes of the lines in each of the patterns was 90° in both cases. Related effects were reported by Beck (1972) in peripheral form discrimination under conditions of stimulus uncertainty. Attneave and Curlee (1977) also showed that in the reproduction of dot patterns from immediate memory the order of dots on the horizontal and vertical axes was more accurately produced than their order on the diagonal axes.

The present findings showing a special role of the horizontal and vertical in *same–different* judgments on transformed patterns is in keeping with this consensus. Whether relational-structure representations with sense-reversal operations or transformational schemes with preferred axes are more appropriate, it seems likely that some system of orthogonal axes is intrinsic to the visual processing of spatial stimuli. What determines the actual direction of these orthogonal axes has been suggested variously as retinal, gravitational, and visual frames of reference (e.g., Attneave & Olson, 1967; Corballis & Roldan, 1975; Rock, 1973). What may be most germane is the natural framework defined by the display itself (Foster, 1980b; Metzler & Shepard, 1974; Shepard & Cooper, 1982), although for the stimuli considered here this framework must involve more than the pattern pairs themselves.

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[1] This account was offered by R. N. Shepard in a review of an earlier version of this article.

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Appendix

Statistical Tests

Scores for each subject and scores pooled over subjects were converted into the discrimination index d' using the false-alarm rate (that is, the proportion of incorrect *same* responses) appropriate for each condition. Variances v of the d' were estimated using the method described by Gourevitch and Galanter (1967). Significance tests were constructed in the manner described, for example, by Cox (1970, p. 80) and Marascuilo (1970). The resulting test statistics had asymptotic chi-squared or standard normal distributions.

(a) Chi-squared test for underlying differences between subjects in their variations over conditions. The discrimination indices d'_{ij} and variances v_{ij}, where $i = 1, \ldots, n_s$ specifies the subject and $j = 1, \ldots, n_c$ specifies the condition, were used to compute a mean performance level $d'_{i.} = \Sigma_j d'_{ij}/n_c$ for each subject $i = 1, \ldots, n_s$. This was subtracted from his or her d' scores to give a new variable $e_{ij} = d'_{ij} - d'_{i.}$. Under the null hypothesis that variations in underlying performances $\mu_{ij} = \mathrm{E}(d'_{ij})$ over conditions were the same for each subject, the quantity

$$\Sigma_{ij}(e_{ij} - e_{.j})^2/v_{ij},$$

where $e_{.j} = (\Sigma_i e_{ij}/v_{ij})/(\Sigma_i 1/v_{ij})$, should be distributed approximately as chi-squared with $n_s n_c - n_s - n_c$ degrees of freedom.

(b) Contrasts in d'. Let the notation be as in (a) and let c_j, $j = 1, \ldots, n_c$ be the contrast weights. Under the null hypothesis that $\Sigma_j c_j \mu_j = 0$ (scores pooled over subjects i), the quantity

$$(\Sigma_j d'_j c_j)/(\Sigma_j v_j c^2_j)^{1/2}$$

should be distributed approximately as the standard normal variable z. (In the applications of this test, false-alarm rates used in calculating d' values were either different in each of the conditions of interest [Experiments 2 and 3] or common [Experiment 1]; in the latter case, the contribution to each variance estimate from the false-alarm rate was omitted.) For a more robust but less powerful test, using subjects $i = 1, \ldots, n_s$, as the sampling unit, let $y_i = \Sigma_j d'_{ij} c_j$. Then under the same null hypothesis, the quantity

$$(\Sigma_i y_i)/(\Sigma_i (y_i - y_.)^2)/n_s(n_s - 1))^{1/2}$$

should be distributed approximately as t with $n_s - 1$ degrees of freedom.

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