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Four Issues Concerning Colour Constancy and Relational Colour Constancy

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Four issues concerning colour constancy and relational colour constancy are briefly considered: (1) the equivalence of colour constancy and relational colour constancy; (2) the dependence of relational colour constancy on ratios of cone excitations due to light from different reflecting surfaces, and the association of such ratios with von Kries' coefficient rule; (3) the contribution of chromatic edges to colour constancy and relational colour constancy; and (4) the effects of instruction and observer training. It is suggested that cognitive factors affect colour constancy more than relational colour constancy, which may be an inherently more robust phenomenon. © 1997 Elsevier Science Ltd. All rights reserved.

Chromatic induction Colour constancy Cone-excitation ratios von Kries scaling Relational colour constancy

In a previous article in this journal, Cornelissen and Brenner (1995) attributed to what has been called "relational colour constancy" certain properties that may seem at variance with earlier proposals by Craven and Foster (1992), Foster *et al*. (1992), and Foster and Nascimento (1994). Subsequent discussions between the two groups of authors concentrated on four issues:

1. the equivalence of colour constancy and relational colour constancy;
2. the dependence of relational colour constancy on ratios of cone excitations due to light from different reflecting surfaces, and the association of such ratios with von Kries' coefficient rule;
3. the contribution of chromatic edges to colour constancy and relational colour constancy; and
4. the effects of instruction and observer training.

The results of these discussions, which have also involved several other contributors, are summarized here since they may be of more general interest.

COLOUR CONSTANCY AND RELATIONAL COLOUR CONSTANCY

Are colour constancy and relational colour constancy equivalent, in that the existence of one implies the existence of the other and vice versa, in both theory and practice? By definition, colour constancy is the constancy of the perceived colours of surfaces under changes in the intensity and spectral composition of the illumination, whereas relational colour constancy is the constancy of the perceived relations between the colours of surfaces under such changes in illumination. Relational colour constancy may underlie the ability of observers to discriminate, reliably and effortlessly, illuminant changes from material changes in scenes (Craven & Foster, 1992; Foster *et al*., 1992), a task which provides an operational basis for defining colour constancy (as is explained later).††

An analysis of the connection between the two kinds of constancies might, in principle, be applied to arbitrarily

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††In addition to these two colour constancies defined with respect to illuminant changes, colour constancies have been defined with respect to other kinds of stimulus changes (Whittle & Challands, 1969; Brown, 1994, 1996); for example, "displacement colour constancy", that is, the constancy of perceived surface colours under changes in the background or surround field, and "atmo-spheric colour constancy", that is, the constancy of perceived surface colours under changes in the light-scattering properties of the viewing medium. None of these other kinds of constancies is considered here, except, indirectly, displacement colour constancy, in so far as it relates to spatial chromatic induction (see e.g. Walraven *et al*., 1987; Tiplitz Blackwell & Buchsbaum, 1988; Singer & D'Zmura, 1994; Zaidi *et al*., 1992; Jenness & Shevell, 1995), which, in turn, is relevant to some control measurements on colour constancy and relational colour constancy (see Section on Chromatic Edges).

complex scenes and illuminants; for the present purposes, however, assume that the illuminants are spatially uniform and that the scenes comprise a pattern of static, spatially disjoint, uniformly coloured, Lambertian reflecting surfaces; that is, they are laboratory "Mondrian patterns", after their superficial likeness to the paintings by Piet Mondriaan.*

In theory, the connection between colour constancy and relational colour constancy should be close; indeed, it can be shown that in a formal sense they are equivalent, for the illuminants and Mondrian patterns just described. Thus, discriminating illuminant changes from material changes in a scene partitions colour signals (the cone inputs from each surface in the scene) into classes that correspond one-to-one with constant colour percepts, in the most obvious sense of that term [for details, see Foster & Nascimento (1994) Appendix I]. And discriminating illuminant changes from material changes also corresponds to discriminating whether the relations between surface colours are unchanged. None of these discriminations actually requires that the illuminant be estimatable, although this is not to dismiss the importance of computational studies whose objective is the recovery of both reflectance and illuminant spectra (see e.g. Maloney, 1993; D'Zmura & Iverson, 1993).

It is emphasized that these correspondences—between classes of colour signals and constant colour percepts, and between classes of colour signals and constant colour relations—are all formal ones, derived for uniformly illuminated Mondrian patterns and an ideal visual system, rather than a representation of an empirical finding. Although such correspondences determine the equivalence of colour constancy and relational colour constancy in theory, there are several ways that they might not hold in practice, depending on the experimental method by which each constancy is assessed. For example, it might be assumed that the presence of relational colour constancy is a necessary condition for obtaining colour constancy (a failure in relational colour constancy implying that the colour appearance of a pair of surfaces is different under different illuminants). Such an assumption is sometimes made implicitly; thus, in some traditional colour-constancy tasks (e.g. Arend & Reeves, 1986; Arend *et al*., 1991; Cornelissen & Brenner, 1995), observers may be asked to attend to the colour relations in a Mondrian pattern to improve the match between a selected region of the pattern under one illuminant and the corresponding region of the same pattern under a different illuminant. Yet, with certain time-varying images, it is possible to measure colour constancy in the seeming absence of relational colour constancy (Cornelissen & Brenner, 1995), as is explained in the Section on Chromatic Edges. Some examples of practical difficulties which seem to affect the measurement of colour constancy more than the measurement of relational colour constancy are considered briefly in the Section on Instructional Effects.

CONE-EXCITATION RATIOS AND VON KRIES' COEFFICIENT RULE

Is relational colour constancy a simple consequence of the coefficient rule of von Kries? Both have been associated with assumptions about ratios of cone excitations, as the following makes clear. Recall that relational colour constancy is the constancy of the perceived relations between the colours of surfaces under changes in illuminant. It has been suggested (Foster & Nascimento, 1994) that the coding of these relations could be given by the ratios of cone excitations—or of some related quantities such as opponent-colour signals—generated in response to light reflected from different illuminated surfaces. These ratios refer to excitations within rather than between cone classes. Thus, suppose that $q_i(a)$ and $q_i(b)$ are the excitations in cone class i ($i = 1, 2, 3$ corresponding to short-, medium-, and long-wavelength-sensitive cones) produced by light from surfaces a and b under some illuminant e. Let r_i be the quotient $q_i(a)/q_i(b)$. It has been shown by computational simulation (Foster & Nascimento, 1994) that, for a large class of pigmented surfaces [the full Munsell set, which encompasses many natural spectra; see e.g. Jaaskelainen *et al*. (1990)] and for classes of surfaces with spectral reflectances that are random functions of wavelength, these ratios r_i are statistically almost invariant under changes in the illuminant e drawn from the sun and sky (correlated colour temperatures 4300–25,000 K) or from a Planckian radiator (temperatures 2000–100,000 K). Such spatial cone-excitation ratios therefore provide a possible, though not necessarily unique, basis for relational colour constancy.†

The invariance of spatial cone-excitation ratios has often been assumed, usually implicitly, in analyses of colour constancy that use von Kries' coefficient rule: since at least the time of Ives (1912) [see Brill (1995) for commentary], it has been common to interpret an insensitivity to illuminant changes as being due to a scaling of receptor responses. But this interpretation actually entails two assumptions. The scaling originally proposed by von Kries (1905) was a simplifying assumption in the analysis of the effects of light adaptation on colour appearance, the assumption being that these effects can be represented, for each cone class, by a single coefficient [the "coefficient rule", first tested quantitatively by Wright (1934)]. For von Kries' scaling

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*Unlike the common laboratory stimulus, the majority of Mondriaan's "Neo-Plasticist" paintings actually incorporated black gridlines and used a small gamut of colours: red, yellow, blue, black, grey, and white. As to the variation in the spelling of his name, from around 1912 when he moved from Amsterdam to Paris he signed his paintings with his surname spelt with either a single or double letter "a", depending partly on the subject of the work and on where it was destined.

†As A. Hurlbert has noted (at Trieste, 1995), it is not necessary that these ratios are computed at immediately post-receptoral levels; they could be computed at some higher level in the visual system, as part of a more general accommodation to the natural structure of the visual environment.

assumption to be generalized to the analysis of traditional colour-constancy phenomena requires a second assumption, namely, that exposure to illuminant changes, as revealed indirectly in reflected light, is equivalent to exposure to changes in a direct adapting light. This second assumption is necessary if colour constancy is to be achieved through receptor scaling; and it fails if spatial cone-excitation ratios are not invariant under illuminant changes (see the Appendix). Ultimately, the extent of the invariance is an empirical matter (Foster & Nascimento, 1994), which can only be addressed quantitatively by thorough sampling of environmental spectral reflectances and illuminants, as described earlier. Notice that in the explanation of relational colour constancy given earlier no assumption was made about receptor scaling; so the answer to the question posed at the beginning of this section is clearly "No".

Although von Kries' coefficient rule is not required for relational colour constancy, some general scaling of cone responses inevitably occurs with changes in illuminant. Providing that spatial cone-excitation ratios are invariant, instantaneous scaling should make changes in illuminant undetectable; but gradual scaling could make surface colours appear to change continuously over extended periods of time, both when the illuminant changes and when the gaze moves over different surfaces. It has been suggested (Brenner & Cornelissen, 1991) that a limited use of the information in spatial cone-excitation ratios could ensure that surface colours do not appear to change continuously in this way with changes in receptor sensitivity. An indication of the time course of one of the mechanisms involved in relational colour constancy can be obtained from measurements of the ability of observers to discriminate illuminant and material changes as a function of the time course of those changes: in an experiment with sequentially presented Mondrian images (Linnell & Foster, 1996a), performance was found to be best for almost instantaneous changes, falling smoothly to chance levels as durations increased to several seconds. [For some other measurements of constancy performance based on time-varying stimuli, see D'Zmura & Mangalick (1994) and Nascimento & Foster (1996).]

CHROMATIC EDGES

Does information in the region of chromatic edges in a scene contribute to colour constancy and to relational colour constancy? Although the possibility of such a contribution to relational colour constancy was not considered in Craven and Foster (1992) and Foster *et al*. (1992), it has been inferred from those studies (Cornelissen & Brenner, 1995) that edge information might be important. In fact, contrary to this inference, some control measurements in a relational colour-constancy task have suggested that edge information has little influence on performance. Thus, observers' ability to discriminate illuminant changes from material changes in Mondrian patterns was found (Nascimento, 1995, Section 5.1) to be only moderately impaired when black borders of width 1.5 deg visual angle were introduced between patches so that spatial chromatic-induction effects were largely eliminated (Tiplitz Blackwell & Buchsbaum, 1988; Brenner & Cornelissen, 1991); as the number of coloured patches decreased and their areas increased, the impairment diminished (Nascimento, 1995).

The possible contribution of edge information in traditional colour-constancy tasks has been assessed (Cornelissen & Brenner, 1995) by measuring observers' eye-movements while a selected region in a Mondrian pattern under one illuminant was matched against the corresponding region in the same pattern under a different illuminant (the colour changes defined by von Kries transformations). It was found that observers spent no more time looking at edges than at other regions of the images. Even so, these data do not in themselves imply that edge information is not used in traditional colour-constancy tasks, as observers may have relied on information derived from edges without looking at them. For example, in spatial chromatic induction, it is possible to detect effects on colour appearance when the nearest chromatic boundary is as far as 4 deg from the point of fixation (Brenner & Cornelissen, 1991) providing that the distance between the boundary of a target patch and that of the inducing surround is <1 deg.

In another version of this colour-constancy task [Cornelissen & Brenner (1995), Experiment III; see also Troost & de Weert (1991)], it was found that observers' colour-matching behaviour remained the same when the patches changed colour every 0.5 sec, so that edge contrasts varied with time (a condition in which relational colour constancy should be violated). Yet the difference in illuminant would have produced a systematic overall colour shift, which could have allowed some kind of time-averaged edge information to be computed.

INSTRUCTIONAL EFFECTS AND OBSERVER TRAINING

Do observers have to be specially instructed and trained to be colour constant? Large differences in levels of performance among observers have been reported in traditional colour-constancy tasks (e.g. Arend & Reeves, 1986; Arend *et al*., 1991; Cornelissen & Brenner, 1995). Although the reasons for the differences are not clear, some possible contributory factors have been identified. First, observers' viewing strategies may depend on whether they are instructed to make paper or hue matches: when making paper matches, they tend to spend more time looking at the surround than when making hue matches (Cornelissen & Brenner, 1995). Observers may differ in their patterns of eye movements as a result of experience in these kinds of colour-matching task. Second, in estimating surface reflectances, some observers may first try to estimate the illuminant's colour, a task that may in turn entail their taking account of or attending to regions with the highest luminance (see e.g. Hurlbert *et al*., 1990; McCann, 1992; Linnell & Foster, 1996b), as well as to the distribution of colours in the surround (see e.g. Arend *et al*., 1991; Brown, 1993, 1994; Singer & D'Zmura, 1994; Jenness & Shevell, 1995; Linnell & Foster, 1996b; Nascimento & Foster, 1996; Zaidi *et al*., 1996). Observers may differ in their knowledge of illuminants and of how surface cues should be combined in such tasks.

In contrast, relational colour constancy seems to depend less on cognitive factors, and in this respect may be an inherently more robust phenomenon. There is no advantage in estimating illuminant colour (see Section on Colour Constancy); and in practice observers can discriminate illuminant changes from material changes with just two coloured surfaces (Nascimento, 1995), too few for a reliable illuminant estimate to be formed. Nor is it necessary to offer observers advice on strategy, or feedback on their levels of performance. Contrary to one suggestion (Cornelissen & Brenner, 1995, p. 2446), feedback was not given to observers who were able to perform reliably in two relational colour-constancy tasks, the one using sequentially presented Mondrian patterns (Craven & Foster, 1992), the other using simultaneously presented Mondrian patterns (Foster *et al*., 1992). In a control experiment, however, feedback was introduced and an effect reported for one particular condition: in a task requiring the discrimination of Mondrian patterns presented side-by-side, receiving feedback enabled an experienced observer and an inexperienced observer each to improve their discrimination scores with images of unlimited duration, but not with images of short duration (Foster *et al*., 1992, p. 158). A possible explanation of this result is that comparing side-by-side images of unlimited duration has more in common with some traditional colour-constancy tasks (*cf* D'Zmura & Iverson, 1993, Section 4C).

In general, there is nothing in the nature of the task of discriminating illuminant changes from material changes that requires observer training. The illuminant changes that occur naturally in everyday visual scenes are easily recognised as such, as when a cloud moves over the sun or when an incandescent lamp is switched on inside a room already illuminated by daylight. As has been suggested (Foster & Nascimento, 1994), this ability may be based on the invariance of cone-excitation ratios that is in turn a consequence of the physical factors involved: the spectral power distribution of the illuminants, the spectral reflectances of the surfaces, and the spectral sensitivities of the receptors of the eye.

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APPENDIX

How does the generalized form of von Kries' scaling assumption—that is, that illuminant changes as revealed in reflected light can be represented by a single coefficient for each cone class—depend on the invariance of spatial cone-excitation ratios? Suppose that $q_i(a)$ and $q_i(b)$ are the stimulations of cone class i produced by light from surfaces a and b under some illuminant e, and that $\bar{q}_i(a)$ and $\bar{q}_i(b)$ are the corresponding adapted responses (von Kries' "modified effects"). Then, by von Kries' coefficient rule, $\bar{q}_i(a) = k_i q_i(a)$ and $\bar{q}_i(b) = k_i q_i(b)$, where k_i is the scaling coefficient for receptor class i under e. Suppose that the illuminant changes to e'. Then, analogously, $\bar{q}_i'(a) = k_i' q_i'(a)$ and $\bar{q}_i'(b) = k_i' q_i'(b)$. The generalized form of von Kries' scaling assumption asserts that $\bar{q}_i'(a)/\bar{q}_i(a) = \bar{q}_i'(b)/\bar{q}_i(b)$. This equality follows from the coefficient rule if spatial cone-excitation ratios are invariant; that is, $q_i(a)/q_i(b) = q_i'(a)/q_i'(b)$. Thus, $\bar{q}_i'(a)/\bar{q}_i(a) = (k_i' q_i'(a))/(k_i q_i(a)) = (k_i'/k_i)(q_i'(a)/q_i(a)) = (k_i'/k_i)(q_i'(b)/q_i(b)) = \bar{q}_i'(b)/\bar{q}_i(b)$; that is, $\bar{q}_i'(a)/\bar{q}_i(a) = \bar{q}_i'(b)/\bar{q}_i(b)$. The scaling of responses in cone class i, that is, $\bar{q}_i(a)$ to $\bar{q}_i'(a)$ and $\bar{q}_i(b)$ to $\bar{q}_i'(b)$, as the illuminant changes from e to e', can be represented by the action of a diagonal matrix relating tristimulus values in an asymmetric colour match; see Wyszecki and Stiles (1982) and, for some particular applications, West and Brill (1982), Brainard and Wandell (1992), and Finlayson *et al*. (1994). Notice that if $q_i(a)/q_i(b) \neq q_i'(a)/q_i'(b)$, that is, if spatial cone-excitation ratios are not invariant, then the generalized form of von Kries' scaling assumption fails.