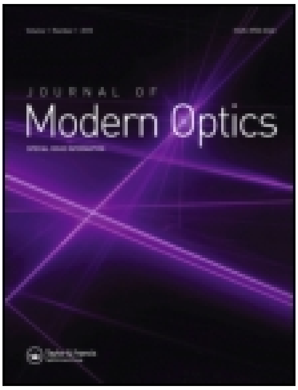


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# A Note on Whether a Non-linearity Precedes the de Lange Filter in the Human Visual System

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## Correspondence

# A note on whether a non-linearity precedes the De Lange filter in the human visual system

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De Lange has shown [1–4] that the human visual system behaves essentially like a low-pass filter in its temporal-frequency response at flicker-fusion threshold. The actual response characteristics and the filter giving rise to these characteristics (for a given stimulus configuration and fixed temporal-mean luminance level) are customarily referred to as the *De Lange attenuation characteristics* and the *De Lange filter*, respectively.

De Lange claimed [2] that this filter could not be preceded in the visual system by a non-linear element. By way of support for this assertion, he presented a computation that considered, as an example, the special case of a logarithmic non-linearity.

The purpose of this note is to show that De Lange's assertion is true for a certain very large class of non-linear functions. We use an argument based on De Lange's original example: the assumption is made that a specified non-linearity comes before the De Lange filter, and this is then shown to lead to a contradiction. (Work by Levinson and Harmon [5] and others suggests that there are separate low-pass and high-pass components of the De Lange filter, and so if the filter is not considered as a single unit, the argument should be taken to apply to the low-pass part only.)

We start by introducing the notion of a tolerance (see [6]) on the luminance-information channel and defining the class of non-linear functions to which the argument is to apply.

Consider at some fixed point along the luminance-information channel, luminance information regarded as a 1-dimensional quantity, some signal value $x$. Because of the limited sensitivity of the system, we can change $x$ by a small amount $\delta x > 0$ to $x + \delta x$ without changing the final percept. The least upper bound on the allowable changes $\delta x$ defines a number $\epsilon > 0$, the *tolerance* at this point, such that two signal values $x$ and $y$ give identical final percepts if (and only if) $|x - y| < \epsilon$. We refer to such $x$ and $y$ as being *indistinguishable*. $\epsilon$ is assumed here to be fixed at any given location; in particular, $\epsilon = \epsilon_0 > 0$, say, both immediately before and immediately after the De Lange filter. (We suppose that the attenuation of the filter at zero frequency is unity, so two signal values that are indistinguishable (or distinguishable) just after the filter ought to be indistinguishable (or distinguishable) just before).

Let the closed interval $[c_1, c_2]$, $c_1 < c_2$, be the range of signal values that are encountered at the input of the filter. Suppose we introduce a continuous function $f$ with domain $[c_1, c_2]$ and range $[c_1, c_2]$ into the luminance-information channel just before the filter. (Requiring $f$ to be continuous causes no real loss in generality.) By the Weierstrass approximation theorem, there exists a polynomial $p$ with real coefficients such that $f(x)$ is indistinguishable from $p(x)$ for all $x$ in $[c_1, c_2]$. In view of this, we will assume in the ensuing that all such functions $f$ are polynomials.

Suppose, now, $f$ is non-linear, here taken in the sense that $f$ cannot be expressed in the form

$$f(x) = a + bx, \quad c_1 \leqq x \leqq c_2,$$

for real constants $a$ and $b$. This implies that the value of the second derivative $f^{(2)}$ of $f$ ($f^{(k)}$ denotes the $k$th derivative) is non-zero at some $x_0 \in [c_1, c_2]$ and $f$ is locally either convex or concave with $f(x_0+z) - (f(x_0) + zf^{(1)}(x_0))$ being either positive for all $z \neq 0$ or negative for all $z \neq 0$ in some suitably small open interval $(-\delta, \delta)$, $\delta > 0$. If we want this non-linearity to be distinguishable (in the sense defined earlier) from a linear function, then we might introduce the requirement that for some $z_1$, $z_2 \in (0, \delta)$ both

$$|f(x_0+z_1) - (f(x_0) + z_1 f^{(1)}(x_0))| \quad \text{and} \quad |f(x_0 - z_2) - (f(x_0) - z_2 f^{(1)}(x_0))|$$

are greater than $\epsilon_0$. The non-linear functions for which the assertion is to be proved will satisfy a condition of this kind.

Let $\mathscr{F}[c_1, c_2]$ be the class of non-linear functions taking $[c_1, c_2]$ into $[c_1, c_2]$ that have the following property. There exists in $[c_1, c_2]$ at least one point, which we denote by $x_1$, for which a number $\delta_0$, $0 < \delta_0 < \min\{x_1 - c_1, c_2 - x_1\}$, can be found such that

$$f(x_1 + \delta_0) - (f(x_1) + \delta_0 f^{(1)}(x_1)) \geqq 2\epsilon_0, \tag{1}$$

and

$$f(x_1 - \delta_0) - (f(x_1) - \delta_0 f^{(1)}(x_1)) \geqq 2\epsilon_0, \tag{2}$$

or, for which

$$f(x_1) + \delta_0 f^{(1)}(x_1) - f(x_1 + \delta_0) \geqq 2\epsilon_0, \tag{1'}$$

and

$$f(x_1) - \delta_0 f^{(1)}(x_1) - f(x_1 - \delta_0) \geqq 2\epsilon_0. \tag{2'}$$

In the following, equations (1) and (2) will be assumed to hold. The discussion proceeds in a similar fashion when, instead, equations (1′) and (2′) hold.

We now start the argument proper.

Assume that a non-linear element $f \in \mathscr{F}[c_1, c_2]$ precedes the De Lange filter, all other elements preceding the filter being understood linear. Present to the visual system a stimulus consisting of a spatially uniform field with luminance varying periodically with time $t$ such that the signal $s_1(t)$, $-\infty < t < \infty$, at the input to the non-linearity varies thus,

$$s_1(t) = \begin{cases} x_1 + \delta_0 & \text{if} \quad nT \leqq t < (n+\frac{1}{2})T \\ x_1 - \delta_0 & \text{if} \quad (n+\frac{1}{2})T \leqq t < (n+1)T \end{cases}, \tag{3}$$

where $n \in \mathbf{Z}$ ($\mathbf{Z}$ the set of integers) and $T > 0$ is a constant. (We are considering the steady-state situation, and therefore allow signals to be defined over all $t$.)

After transmission through the non-linearity $f$, the signal $s_1(t)$, $-\infty < t < \infty$, is modified to $s_2(t)$, $-\infty < t < \infty$, where, making use of the fact that $f$ is a polynomial,

$$s_2(t) = f(x_1) + (-1)^{p(t)}\delta_0 f^{(1)}(x_1) + \frac{\delta_0^2}{2!} f^{(2)}(x_1) + (-1)^{p(t)} \frac{\delta_0^3}{3!} f^{(3)}(x_1)$$

$$+ \frac{\delta_0^4}{4!} f^{(4)}(x_1) + \ldots + \frac{\delta_0^m}{m!} f^{(m)}(x_1), \quad -\infty < t < \infty, \tag{4}$$

where $p(t) = 0$ if $nT \leqq t < (n + \frac{1}{2})T$ and $p(t) = 1$ if $(n + \frac{1}{2})T \leqq t < (n+1)T$, $n \in \mathbf{Z}$; $m$ is assumed even. Denoting

$$f(x_1) + \frac{\delta_0^2}{2!} f^{(2)}(x_1) + \ldots + \frac{\delta_0^m}{m!} f^{(m)}(x_1)$$

by $k_1$, and

$$\delta_0 f^{(1)}(x_1) + \frac{\delta_0^3}{3!} f^{(3)}(x_1) + \ldots + \frac{\delta_0^{m-1} f^{(m-1)}}{(m-1)!}(x_1)$$

by $k_2$, equation (4) may be rewritten

$$s_2(t) = k_1 + (-1)^{p(t)} k_2, \quad -\infty < t < \infty. \tag{4'}$$

Observe that by adding equations (1) and (2), we obtain $k_1 \geqq f(x_1) + 2\epsilon_0$.

Now, suppose the temporal frequency $\nu = 1/T$ (of the fundamental) of the stimulus is increased so that the time-varying nature of the stimulus is no longer detectable by the system, i.e. so that $\nu$ is greater than flicker-fusion frequency *FFF*. By expanding $(-1)^{p(t)}$, $-\infty < t < \infty$, in a Fourier series, we see that the response signal $s_3(t)$, $-\infty < t < \infty$, of the De Lange filter to the signal $s_2(t)$, $-\infty < t < \infty$, is given by

$$s_3(t) = k_1 H(0) + \frac{4}{\pi} k_2 [H(\nu) \sin(2\pi\nu t + \phi(\nu))$$

$$+ \tfrac{1}{3} H(3\nu) \sin(6\pi\nu t + \phi(3\nu) + \ldots], \quad -\infty < t < \infty, \tag{5}$$

where $H(\nu)$ is the attenuation of the De Lange filter at frequency $\nu (H(0) = 1)$, and $\phi(\nu)$ is the corresponding phase shift. The modulus of the second term in equation (5), which converges since $H(\nu)$ falls off rapidly with $\nu$, is necessarily less than $\epsilon_0$. We hence have the result that for $\nu > FFF$, the response $s_3(t)$ of the De Lange filter is indistinguishable from $k_1$ for all $t$.

By the Talbot–Plateau law, the given stimulus can be matched, visually, by a stimulus with constant luminance equal to the former's time-average. Since

$$\frac{1}{T} \int_0^T s_1(t)\, dt = x_1$$

(equation (3)), this means that the response $s_3(t)$ of the De Lange filter is indistinguishable for all $t$ from $f(x_1)$ also.

Recalling the $\epsilon_0$ definition of indistinguishability, we recognize that a number, e.g. $s_3(t)$, cannot be indistinguishable from two numbers, e.g. $k_1$ and $f(x_1)$, whose difference is greater than or equal to $2\epsilon_0$. Since $k_1 \geqq f(x_1) + 2\epsilon_0$, we therefore have a contradiction. The hypothesis that a non-linearity $f \in \mathscr{F}[c_1, c_2]$ precedes the

De Lange filter is consequently false, and we conclude that any such non-linearity, if present, must follow the filter.

Note that we have implicitly assumed all functions $f \in \mathscr{F}[c_1, c_2]$ to be zero-memory. A pure delay-element can be attached to each non-linearity without affecting the argument.

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