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Internal Representations and Operations in the Visual Comparison of Transformed Patterns: Effects of Pattern Point-Inversion, Positional Symmetry, and Separation*

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Abstract. A scheme for visual pattern recognition is described. It is supposed, amongst other things, that patterns are internally represented by the visual system in terms of local features, spatial-order relations between local features, and global spatial relations specifying approximate pattern position with respect to the point of fixation. It is further supposed that there are two distinct types of internal operation that may be applied to the components of internal representations in the process of pattern comparison: typically a discrete spatial-order-reversal operation and a continuous position-shift operation. Some general predictions of the scheme are tested against data obtained in an experiment using random-dot patterns that were subjected to rigid transformations and presented at various locations along the horizontal meridian. Patterns were presented sequentially, in pairs, to subjects in a "same-different" comparison task. Pattern pairs were to be responded to as "same" if they were identical or related by point-inversion (planar rotation through 180°) or responded to as "different". Extending earlier findings, the present results showed that "same"-detection performance for identical and point-inverted patterns depended differentially on the distance between the patterns and the symmetry of the pattern positions about the point of fixation in a manner consistent with the predictions of the scheme.

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1 Introduction

Visual recognition of patterns and figures is not generally invariant under rotation of the pattern or figure in the plane (Mach, 1897; Dearborn, 1899; Aulhorn, 1948; Arnoult, 1954; Kolers and Perkins, 1969, 1975; Rock, 1973, Chap. 3; Foster, 1978a; Kahn and Foster, 1981). For displays of limited duration, performance in discriminating rotated "same" patterns from "different" patterns falls off with rotation angle for angles up to or a little beyond 90°, and then increases again with rotation angle for angles up to 180° (Dearborn, 1899; Aulhorn, 1948; Rock, 1973, Chap. 3; Foster, 1978a; Kahn and Foster, 1981). This kind of performance may be contrasted with that obtained by R.N. Shepard and his colleagues in mental rotation experiments in which a monotonic dependence of reaction time for a correct response on angle of rotation has been obtained. In those studies (see e.g. Shepard and Metzler, 1971; Cooper and Shepard, 1973; Shepard, 1975; Shepard and Cooper, 1982), the experimental task typically involved the accurate discrimination of rotated patterns from mirror-image rotated patterns, and produced reaction times of the order of seconds.

The upturn in performance for discriminating "same-different" patterns at 180° angle of rotation obtained with limited-duration displays is not specific to particular types of pattern: it occurs with randomly contoured shapes (Dearborn, 1899; Rock, 1973), with random-dot patterns (Foster, 1978a; Kahn and Foster, 1981), and with alphabetic characters (Aulhorn, 1948). Randomly formed patterns used in some of these experiments were preferred because the patterns then had no special meaning or conventional orientation or handedness.

Such pattern-discrimination performance has implications for the form of the internal representations constructed by the visual system in its response to pattern stimuli and for the kinds of internal operation applied to these internal representations. The purpose of the present study is to set out a scheme for recognition of transformed patterns and to report a test of some general predictions of the scheme as they relate to the effects of planar 180° rotation of patterns, i.e. *point-inversion*, and positional symmetry and separation of patterns in the visual field.

2 A Recognition Scheme

2.1 Recognition of Point-Inverted Patterns in Symmetric Displays

An explanation of the non-monotonic dependence of "same-different" discrimination performance on rotation angle was offered by Foster and Mason (1979) in terms of a relational-structure scheme for visual pattern recognition. Set out in more formal detail, and with some changes in notation, it was as follows. It was assumed that a pattern A was represented internally by the visual system in terms of

(i) local features f_i, $i=1, 2, \ldots, m$, which for random-dot patterns could be dot clusters of a particular density and shape,

(ii) spatial-order relations r_x, r_y that specified how one local feature f_j was related to another f_k, $1 \leq j < k \leq m$, in a horizontal-vertical reference system, thus

$$r_x(f_j, f_k) = \begin{cases} 1, & \text{for } f_j \text{ "left of" } f_k, \\ -1, & \text{for } f_j \text{ "right of" } f_k, \\ 0, & \text{otherwise;} \end{cases}$$

$$r_y(f_j, f_k) = \begin{cases} 1, & \text{for } f_j \text{ "above" } f_k, \\ -1, & \text{for } f_j \text{ "below" } f_k, \\ 0, & \text{otherwise.} \end{cases}$$

It was further assumed that two patterns A_1, A_2 were judged to be the same if their internal representations could be brought into coincidence. In the special case of patterns related by a planar 180-deg rotation, i.e. a point-inversion, $A_2 = \iota A_1$, recognition was supposed to occur by a simple discrete internal global reversal $\sigma = (\sigma_x, \sigma_y)$ of the sign or sense of the spatial-order relations r_x, r_y, thus

$$\sigma_x \circ r_x = -r_x,$$

$$\sigma_y \circ r_y = -r_y.$$

Specifically, let R_1, R_2 be the internal representations of the patterns A_1, A_2, with

$$R_1 = \{f_i;\, r_x(f_j, f_k),\, r_y(f_j, f_k)\colon 1 \leq i \leq m,\, 1 \leq j < k \leq m\},$$

$$R_2 = \{\iota(f_i);\, r_x(\iota(f_j, f_k)),\, r_y(\iota(f_j, f_k))\colon \cdot\, 1 \leq i \leq m,\, 1 \leq j < k \leq m\},$$

where $\iota(f_i)$, $\iota(f_j, f_k)$ are the transformed local features and their transformed spatial relationships. Suppose that $\iota(f_i) = f_i$ or that any difference is incorporated in additional spatial-relation structure, if necessary by introducing spatial relations that refer to the orientation of the f_i. Then

$$R_2 = \{f_i;\, -r_x(f_j, f_k),\, -r_y(f_j, f_k)\colon \cdot\, 1 \leq i \leq m,\, 1 \leq j < k \leq m\}.$$

After application of the internal global sense-reversals σ to R_2,

$$\sigma(R_2) = \{f_i;\, \sigma_x(-r_x(f_j, f_k)),\, \sigma_y(-r_y(f_j, f_k))\colon \cdot\, 1 \leq i \leq m,\, 1 \leq j < k \leq m\},$$
$$= R_1.$$

The effects of point-inversion were thereby accommodated precisely. More generally, by counting the number of altered spatial-order relations for arbitrary planar pattern rotations and allowing for possible global sense-reversals, Foster and Mason (1979) were able to predict the detailed variation with rotation angle of recognition performance obtained with a large, fixed repertoire of random-dot patterns (Foster, 1978a).

The data reported by Foster (1978a) on the recognition of rotated patterns were obtained for pairs of patterns presented symmetrically about the fixation point. Other experiments showing the upturn in performance at 180° rotation also involved a symmetric presentation of the stimuli; typically patterns were viewed, one at a time, centrally in the visual field (Dearborn, 1899; Rock, 1973). It was found by Kahn and Foster (1981), however, that the upturn in performance for point-inverted patterns was dependent on the symmetry of the display; when symmetry of the pattern positions was disturbed, performance for point-inverted patterns was reduced.

2.2 Recognition of Point-Inverted Patterns in Asymmetric Displays

In the experiments by Kahn and Foster (1981) on the effects of display symmetry on recognition performance, subjects made "same-different" judgements on briefly presented pairs of random-dot patterns related by various transformations (as in Fig. 1) each with angular extent 0.5° visual angle.

In these experiments, "same" patterns were related by one of the following transformations: (*a*) identity transformation Id, (*b*) planar rotation $\varrho_{90°}$ through 90°, (*c*) reflection μ_y about a vertical axis, and (*d*) planar rotation through 180°, i.e. point-inversion ι; "different" patterns were unrelated in this way and were paired (*e*) at random. On each trial, each pattern

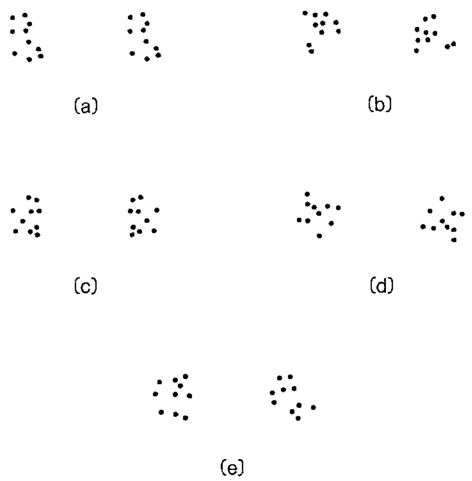

Fig. 1. Illustrations of pairs of transformed patterns used to determine effects of various rigid transformations and pattern positions on "same-different" judgements. Each pattern consisted of ten dots distributed pseudo-randomly within an imaginary circle of diameter 0.5° visual angle. "Same" patterns were related by one of the following transformations: (*a*) identity transformation Id, (*b*) planar rotation $\varrho_{90°}$ through 90°, (*c*) reflection μ_y about a vertical axis, and (*d*) point-inversion ι. "Different" patterns were obtained by pairing (*e*) at random

in each pair could be presented at the point of fixation, or at a fixed position 0.5° to the left or to the right of the point of fixation. The patterns were presented sequentially with 100-ms duration and 1-s interval. As a control for possible temporal effects related to memory matching and for eye-movement effects, a second experiment was performed with the pairs of random-dot patterns presented simultaneously and the range of position combinations modified to avoid spatially overlapping stimuli. In particular, the eccentric position 0.5° was increased to 1.0°.

Results from both experiments were similar. "Same"-detection performance for patterns related by the identity transformation Id was strongly affected by the distance between the patterns: the greater the separation, the worse the performance. Performance for point-inversion ι and reflection μ_y was maximum when the patterns were positioned symmetrically about the point of fixation, and the separation of the patterns had little effect. Performance for rotation $\varrho_{90°}$ was best described as a linear function of mean distance of the patterns from the fixation point.

To explain these findings, a modified scheme for internal pattern representations was proposed (Kahn and Foster, 1981). Patterns were assumed to be represented in terms of local features, the spatial relations between those local features, *and* the global positions of the patterns in the field with respect to the point of fixation. Set out in more formal detail and following the notation of Sect. 2.1, the scheme with some modifications was as follows. If p_0 was the point of fixation, it was assumed that the internal representation R of a pattern A had the form

$$R=\{f_i;\ r_x(f_j, f_k),\ r_y(f_j, f_k);\ \cdot\ d_x(A, p_0),\ d_y(A, p_0):\ 1\leq i\leq m,\ 1\leq j<k\leq m\},$$

where d_x, d_y are global spatial relations specifying the approximate position of the pattern in a horizontal-vertical coordinate system centred about the point of fixation. Two distinct kinds of internal operation on these representations were supposed to be possible.

(1) Spatial relations of a given kind may be relabelled with their opposites, in a single step, providing that the relabelling is applied uniformly to all the relations of that kind. This operation might typically be applied to the sense of the spatial relations such as "left of" and "above", as Foster and Mason (1979) had proposed. Thus, as in Sect. 2.1,

$$\sigma_x \circ r_x = -r_x\,,$$
$$\sigma_y \circ r_y = -r_y\,.$$

(2) Any individual component in a representation may be modified, but this modification can be effected only in a progressive continuous fashion. This operation might typically be applied to the components in the internal representation specifying the global position of the pattern with respect to the point of fixation. If $\alpha_t=(\alpha_x, \alpha_y)_t$, $0\leq t\leq 1$, is this sequence of operations, parameterized by time, then

$$(\alpha_x)_t \circ d_x = a_x(t)+d_x\,,\quad -\infty<a_x(t)<\infty\,,$$
$$(\alpha_y)_t \circ d_y = a_y(t)+d_y\,,\quad -\infty<a_y(t)<\infty\,.$$

The global sense-reversal operation σ also applies to d_x, d_y, thus

$$\sigma_x \circ d_x = -d_x\,,$$
$$\sigma_y \circ d_y = -d_y\,.$$

These operations σ and α_t, $0\leq t\leq 1$, may both be used in the internal comparison of two internal representations, but the efficiency of the operations was assumed to depend on the size of the modification needed to bring the internal representations into coincidence. Kahn and Foster (1981) described how for pairs of transformed patterns these operations could be used to explain the dependence of "same"-detection performance on positional symmetry and separation. Consider the following two examples.

(1) Suppose patterns A_1, A_2 differed only by a horizontal planar translation a, with A_1 to the left of A_2. Let R_1, R_2 be the corresponding internal repre-

sentations. Then

$$R_1=\{f_i;\ r_x(f_j,\ f_k),\ r_y(f_j,\ f_k);$$
$$\cdot\ d_x(A_1,\ p_0),\ d_y(A_1,\ p_0){:}\ 1\leq i\leq m,\ 1\leq j<k\leq m\},$$
$$R_2=\{f_i;\ r_x(f_j,\ f_k),\ r_y(f_j,\ f_k);$$
$$\cdot\ d_x(A_2,\ p_0),\ d_y(A_2,\ p_0){:}\ 1\leq i\leq m,\ 1\leq j<k\leq m\},$$
$$=\{f_i,\ r_x(f_j,\ f_k),\ r_y(f_j,\ f_k);$$
$$\cdot\ a+d_x(A_1,\ p_0),\ d_y(A_1,\ p_0){:}$$
$$\cdot\ 1\leq i\leq m,\ 1\leq j<k\leq m\}.$$

In this case, patterns were detected as "same" by continuous modification of the global-position components in the internal representations of the patterns until the representations coincided. Specifically, define

$$(\alpha_x)_t(d_x(A_2,\ p_0))=-at+d_x(A_2,\ p_0),\ 0\leq t\leq 1,$$
$$(\alpha_y)_t(d_y(A_2,\ p_0))=d_y(A_2,\ p_0),\qquad 0\leq t\leq 1.$$

After application of the sequence α_t, $0\leq t\leq 1$, to R_2

$$\alpha_1(R_2)=\{f_i;\ r_x(f_j,\ f_k),\ r_y(f_j,\ f_k);$$
$$\cdot\ -a+d_x(A_2,p_0),d_y(A_1,p_0):$$
$$\cdot\ 1\leq i\leq m,1\leq j<k\leq m\},$$
$$=R_1\,.$$

Increased pattern separation required more modification of the global-position component and as a result "same"-detection performance was reduced.

(2) Suppose patterns A_1, A_2 were related by point-inversion and positioned symmetrically about the point of fixation p_0. Then their internal representations

$$R_1=\{f_i;\ r_x(f_j,\ f_k),\ r_y(f_j,\ f_k);$$
$$\cdot\ d_x(A_1,\ p_0),\ d_y(A_1,\ p_0){:}\ 1\leq i\leq m,\ 1\leq j<k\leq m\},$$
$$R_2=\{f_i;\ -r_x(f_j,\ f_k),\ -r_y(f_j,\ f_k);$$
$$\cdot\ -d_x(A_1,\ p_0),\ -d_y(A_1,\ p_0){:}$$
$$\cdot\ 1\leq i\leq m,\ 1\leq j<k\leq m\}.$$

After application of the internal global sense-reversals σ to R_2

$$\sigma(R_2)=\{f_i,\ \sigma_x(-r_x(f_j,\ f_k)),\ \sigma_y(-r_y(f_j,\ f_k));$$
$$\cdot\ \sigma_x(-d_x(A_1,\ p_0)),\ \sigma_y(-d_y(A_1,\ p_0)){:}$$
$$\cdot\ 1\leq i\leq m,1\leq j<k\leq m\}.$$
$$=R_1\,.$$

In this case, patterns were detected as "same" by relabelling with the opposite term all those components in the internal representation that specified spatial sense. Thus the spatial relation "above" became "below", "left of" became "right of", and the global-position component "0.5° to the left of the point of fixation" became "0.5° to the right of the point of fixation". As a result, the two internal representations were brought into coincidence. If the two point-inverted patterns were not positioned symmetrically with respect to the point of fixation, the relabelling operation alone was not sufficient to bring the representations into coincidence; that is, either $\sigma_x(d_x(A_2,p_0))\neq d_x(A_1,p_0)$, or $\sigma_y(d_y(A_2,p_0))\neq d_y(A_1,p_0)$, or both. Because further modification of the global-position component was needed, for example, $(\alpha_x)_t(\sigma_x(d_x(A_2,p_0)))=(a_x)_t+d_x(A_1,p_0)$, $0\leq t\leq 1$, to achieve a match, "same"-detection performance was reduced.

This description of the hypothesized internal comparison process was based on the data obtained by Kahn and Foster (1981) with one of just three horizontal positions allowed for each member of the pattern pair: in the sequential-presentation experiment, these were the point of fixation and 0.5° to the left and to the right of the point of fixation; in the simultaneous presentation experiment, these were the point of fixation and 1.0° to the left and to the right of the point of fixation. In the present study, a more detailed experimental investigation was carried out into the effects of pattern position on "same"-detection performance for random-dot patterns related by the identity transformation Id and point-inversion ι.

3 Experimental Methods

Apparatus

The stimuli for the experiment were produced on the screen of an $X-Y$ display oscilloscope (Hewlett-Packard, Type 1300 A) with $P4$ sulfide phosphor (decay time 60 µs), controlled by a minicomputer (CAI Alpha LSI–2) with vector-graphics generator (Sigma Electronic Systems QVEC 2150). The screen was viewed binocularly at a distance of 1.7 m through a view-tunnel and optical system which produced a uniform white background field subtending 7.4° × 6.2° at the eye and of luminance approximately 60 cd · m^{-2}. The stimuli were white and appeared superimposed on the background field. The intensity of the stimuli was adjusted by each subject at the beginning of each experimental session to be ten-times luminance increment threshold. This setting was achieved by introducing a 1.0-log-unit neutral-density filter over the stimulus dots and the intensity of the dots adjusted to increment threshold on the unattenuated background.

Fixation was aided by two computer-generated white lines, approximately 0.9° long, positioned approximately 0.6° above and 0.6° below and in line with a computer-generated white fixation spot. The lines were displayed throughout each presentation; the fixation spot was extinguished at the start of each trial. The subject controlled the start of each trial and gave his responses on a hand-held push-button box connected to the computer.

Stimuli

Stimuli were random-dot patterns (as illustrated in Fig. 1) each consisting of ten dots distributed pseudo-randomly within an imaginary circle of diameter 0.5° visual angle. Each dot subtended about 0.03°. Fresh random-dot patterns were generated for every trial.

Pattern Positions

In each trial two patterns appeared sequentially. Each pattern was presented with its centre (defined by the constraining circle) in one of five locations: 1.0°, 0.5°, and 0.0° to the left of the fixation spot and 0.5° and 1.0° to the right of the fixation spot. Locations to the left will be indicated as negative and locations to right as positive.

In the subsequent analysis, position combinations that were mirror equivalents [e.g. the pair (1.0°, 0.5°) and the pair (−1.0°, −0.5°)] were considered as the same type of pair. Similarly, the sequence of positions in each trial (e.g. "1.0°" first, "0.5°" second) was not taken into account. There were thus nine types of position combination: (1.0°, 1.0°), (1.0°, 0.5°), (1.0°, 0.0°), (1.0°, −0.5°), (1.0°, −1.0°), (0.5°, 0.5°), (0.5°, 0.0°), (0.5°, −0.5°), (0.0°, 0.0°).

Within this system, all possible pair-types occurred equally often with each of the pattern transformations described below.

Pattern Transformations

There were two possible transformations (other than translations) relating the patterns in each "same" pair:

Id: the two patterns were identical (Fig. 1a);

ι: one pattern was obtained from the other by planar rotation through 180° about the centre of the circle constraining the pattern, i.e. point-inversion (Fig. 1d).

For "different" pairs, the two patterns were generated independently of each other (Fig. 1e).

Instructions

At the beginning of the experiment, subjects were informed of the nature of the stimuli and of the types of transformation involved. Subjects were instructed to indicate after the presentation of each pair of patterns whether they were "same" or "different" according to the above transformations. It was emphasized that steady fixation was to be maintained throughout each presentation period and that responses should be made as quickly as possible whilst preserving accuracy.

Presentation Sequence

Following initiation of the trial by the subject, the fixation spot was extinguished, and after a 1.0-s delay, the first stimulus pattern appeared for 100 ms; after a 1.0-s delay, the second stimulus pattern appeared for 100 ms. The subject's response was recorded by the computer. As a control, the time taken to make the response was also recorded. After a 1.0-s delay, the fixation spot was redisplayed indicating that the next trial could be started.

Experimental Design

There were 36 trials in each experimental run. In each run, every type of position combination occurred once with each of the "same" pattern transformations (Id and ι), and twice with "different" pattern transformations, so that a run consisted of 18 "sames" and 18 "differents". Each subject performed 36 runs over several days. The order of the pattern transformations and position combinations was chosen pseudo-randomly but balanced over runs to offset stimulus order and carry-over effects.

Subjects

Four subjects, three male and one female, aged 19 to 26 years, participated in the experiment. Each had normal or corrected-to-normal vision. All subjects except one (co-author JIK) were unaware of the purpose of the experiment.

4 Results

4.1 Discrimination Performance

Table 1 shows for identical and point-inverted patterns "same-different" discrimination as a function of pattern-pair location. Because discrimination was determined by responses to both "same" and "different" patterns, the discrimination index d' from signal detection theory (Green and Swets, 1966) was used to represent performance. The index d' is zero when performance is at chance level and increases monotonically with increasing performance. It has a number of advantages as a performance measure (Swets, 1973); it is, for example, bias-free and additive (Durlach and Braida, 1969).

The d' data in Table 1 are weighted by variances and averaged over subjects (Appendix). Chi-squared tests (Appendix) on individual subjects' data showed significant or close to significant differences between subjects' performances (for transformation Id, $\chi^2_{27}=51.0$, $p<0.01$; for transformation ι, $\chi^2_{27}=39.3$, $0.05<p<0.1$), but these differences disappeared after

Table 1. "Same"-detection performance for patterns related by identity transformation Id or point-inversion ι as a function of pattern position

	Positions of patterns in each pair (deg)								
1st pattern[a]:	1.0	1.0	1.0	1.0	1.0	0.5	0.5	0.5	0.0
2nd pattern:	1.0	0.5	0.0	−0.5	−1.0	0.5	0.0	−0.5	0.0
Transformation Id									
d':	2.446	2.219	1.978	1.761	1.573	2.865	2.395	2.297	2.898
SEM[b]:	0.039	0.031	0.024	0.023	0.022	0.047	0.032	0.032	0.048
Transformation ι									
d':	1.295	1.385	1.683	1.394	1.510	1.594	1.687	1.783	2.072
SEM[b]:	0.021	0.021	0.023	0.020	0.021	0.022	0.023	0.022	0.025

[a] Order has no significance
[b] No. of trials per position 144 "same", 288 "different"

allowance was made for each subject's overall performance level by normalizing with respect to each subject's mean level (Appendix) (for Id, $\chi^2_{23}=27.6$, $p>0.1$; for ι, $\chi^2_{23}=23.5$, $p>0.2$).

For the analysis, asymmetry of the pattern positions was defined as the separation of one pattern position from its image after reflection about the vertical midline (equivalent to twice the mean position of the patterns in the pair); both separation and asymmetry were thus expressed in degrees of visual angle. Contrasts (Lindman, 1974) were used to test for effects of separation and asymmetry on the averaged normalized d' data for transformations Id and ι. Results of this analysis are given below as standard normal z values. As a check on this procedure, a separate analysis was also carried out involving contrasts on individual subjects' performances without explicitly normalizing to individual performance levels. Results of this auxiliary analysis are given as t values after the results of the principal analysis. All tests were two-tailed tests.

For transformation Id, there was a highly significant effect of pattern separation ($z=6.73$, $p<0.001$; $t_3=6.27$, $p<0.01$) and no significant effect of positional asymmetry ($z=1.08$, $p>0.2$; $t_3=1.90$, $p>0.1$). For transformation ι, there was no significant effect of pattern separation ($z=0.97$, $p>0.2$; $t_3=0.80$, $p>0.2$) and a highly significant effect of positional asymmetry ($z=3.09$, $p<0.01$; $t_3=5.46$, $p<0.05$). The auxiliary analysis based on individual contrasts was consistent with the principal analysis based on contrasts on the averaged data, but, as expected (Lindman, 1974, p. 137), was less powerful.

4.2 Reaction Times

Averaged over subjects and conditions, correct responses (mean ± 1 SEM) were significantly faster than incorrect responses, 709 ± 65 ms vs. 1011 ± 123 ms ($t_{22}=2.17$, $p<0.05$), but correct "same" responses were not significantly faster than correct "different" responses, 698 ± 79 vs. 709 ± 65 ms ($t_{10}=0.09$, $p>0.5$). All tests were two-tailed tests.

There was no trade-off between performance (percent correct) and reaction time (RT). Averaged over subjects, RTs for correct "same" responses were significantly correlated with performance for transformation Id, gradient (mean ± 1 SEM) -7.55 ± 2.88 ms $\cdot$ %$^{-1}$ ($z=2.62$, $p<0.01$) and significantly correlated with performance for transformation ι, gradient -5.97 ± 1.95 ms $\cdot$ %$^{-1}$ ($z=3.06$, $p<0.01$); RTs for correct "different" responses were not significantly correlated with performance, gradient -0.58 ± 0.96 ms $\cdot$ %$^{-1}$ ($z=0.60$, $p>0.5$).

5 Discussion

The results of the analysis of the experiment have provided strong support for the scheme of internal representations and internal operations outlined in Sect. 2.2. Performance for identical patterns was strongly affected by the distance between the patterns and was not affected by the symmetry of the positions of the patterns with respect to the point of fixation. Conversely, performance for pairs of patterns related by point-inversion was strongly affected by the symmetry of the positions of the patterns with respect to the point of fixation and was not affected by the distance between the patterns. These results are consistent with previous findings by Kahn and Foster (1981) obtained with less extensive variation in pattern positions.

It is unlikely that the present findings are a simple artifact of variations over the visual field of acuity or distribution of attention. For pairs of point-inverted patterns presented at (1.0°, 1.0°) and at (1.0°, −1.0°), retinal eccentricity is identical, yet performance (Table 1) is highly significantly better in the second condition ($z=7.23$, $p<0.001$). More detailed arguments have been offered in Kahn and Foster (1981).

It was suggested earlier that the efficiency of the internal matching operation depended on the extent of the required modification to the internal representation. No particular mechanism or process was proposed by which this might occur. One obvious possibility is that the faithfulness or fidelity of the internal representation decays with time and the time taken to implement a particular modification to it increases monotonically with the size of the modification needed to bring the representations into coincidence (cf. Cooper and Shepard, 1973; Shepard, 1981; Shepard and Cooper, 1982). For discussion of the metric attached to the space in which these operations might occur, see Foster (1975, 1978b), Farrell and Shepard (1981), Shepard (1981), and Shepard and Cooper (1982).

6 Appendix

The scores for each subject were converted into the discrimination index d' using the false-alarm rate (that is, the proportion of incorrect "same" responses) from each of the position combinations to set the level for both transformations presented in that combination. Variances were estimated using the method described by Gourevitch and Galanter (1967).

(i) Chi-squared test for differences between subjects. The discrimination indices d'_{ij} and variances v_{ij}, where $i=1, \ldots, 4$ specifies the subject and $j=1, \ldots, 9$ specifies the position combination, were used to compute the quantity

$$\Sigma_{ij}(d'_{ij}-d'_{.j})^2/v_{ij},$$

where $d'_{.j}=(\Sigma_i d'_{ij}/v_{ij})/(\Sigma_i 1/v_{ij})$, which has variance $v_{.j}=(\Sigma_i 1/v_{ij})^{-1}$. Under the hypothesis that there are no differences between subjects' performances, the computed quantity should be distributed as chi-squared with 27 deg of freedom.

(ii) Chi-squared test for differences between subjects allowing for each subject's overall performance level. Let the notation be as in (i). The mean performance level $d'_{i.}=(\Sigma_j d'_{ij})/9$ for each subject $i=1, \ldots, 4$ was subtracted from his d' scores to give a normalized value $e_{ij}=d'_{ij}-d'_{i.}$. Under the hypothesis that there are no differences between subjects' performances when each of these is expressed relative to the subject's mean performance level, the quantity

$$\Sigma_{ij}(e_{ij}-e_{.j})^2/v_{ij},$$

where $e_{.j}=(\Sigma_i e_{ij}/v_{ij})/(\Sigma_i 1/v_{ij})$, should be distributed as chi-squared with 23 deg of freedom.

(iii) Contrasts for effects of separation and of asymmetry. Let the notation be as in (i). For the 9 types of position combination listed in order in Methods and in Table 1, set c_j, $j=1, \ldots, 9$, equal to -13, -4, 5, 14, 23, -13, -4, 5, -13, respectively, to test for separation, and equal to 23, 14, 5, -4, -13, 5, -4, -13, -13, respectively, to test for asymmetry. Under the hypothesis that there is no effect, the quantity

$$(\Sigma_j d'_{.j} c_j)/(\Sigma_j v_{.j} c_j^2)^{1/2}$$

should be distributed as the standard normal variable z. For the auxiliary test referred to in Results, set $y_i=\Sigma_j d'_{ij} c_j$. Then, under the hypothesis that there is no effect, the quantity

$$(\Sigma_i y_i)/((\Sigma_i (y_i - y_.)^2)/12)^{1/2}$$

should be distributed as t with 3 deg of freedom

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