THE PERCEPTION AND APPLICATION OF FLASHING LIGHTS

AN INTERNATIONAL SYMPOSIUM
HELD AT IMPERIAL COLLEGE, LONDON
UNDER THE JOINT AUSPICES OF
THE NATIONAL ILLUMINATION COMMITTEE
OF GREAT BRITAIN
AND THE APPLIED OPTICS SECTION
IMPERIAL COLLEGE
ON 19–22 APRIL 1971

ADAM HILGER LTD
LONDON

SOME THEORETICAL ASPECTS OF AN APPARENT MOTION PHENOMENON ASSOCIATED WITH CERTAIN CONFIGUATIONS OF FLASHING LIGHTS

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1. INTRODUCTION

If two discrete stationary sources of light are flashed on and off with one source lagging the other, then under certain conditions a sensation of motion appears to take place between the two sources.[1]

This apparent motion phenomenon has been the subject of numerous psychological studies (see Graham, 1965, for a comprehensive review[2]), and several authors (Wertheimer,[1] Brown,[3] Gibson,[4] Kolers[5]) have discussed the relationship between apparent motion and real motion. It has been proposed by Wertheimer,[1] Brown,[2] and others, that the physical processes underlying real and apparent motion are identical, and physiological evidence tends to support this viewpoint (Grusser *et al.*[6]). (Kolers[5] has argued against this proposition, but see Foster.[7])

The purpose of this present study is to demonstrate that certain classes of real and apparent motion phenomena may be described by the same functional processes. Specifically, we intend to show the following:

(i) The existence of an apparent motion effect, associated with stationary flashing light sources, is a natural consequence of a model designed to describe certain real motion effects.
(ii) The frequency response characteristics of this apparent motion effect are the same as the frequency response characteristics of the equivalent real motion effect.

Since extensive use will be made of the real motion model referred to above, we start with a brief outline of its construction and then derive an expression for the associated general motion response. (For a fuller account, see Foster.[7])

2. A MODEL FOR REAL MOTION STIMULI

The model described below is based upon experimental data obtained with a particular set of input stimuli. In order to provide a complete

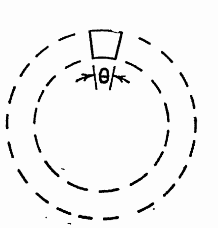

FIG. 1. The primary stimulus field: The annular aperture is restricted to a sector, the area of which is specified by the angular coordinate θ.

system description, it is necessary to specify this set. This we now do.

A rotating radial grating of arbitrary waveform forms the primary stimulus. The view of this grating is restricted to foveal annulus, portions of which may be sectioned off. (The remaining area is specified by the angular coordinate θ; see Fig. 1.) The sectored annulus is superimposed upon a uniform white background field, and the subject fixates, monocularly, the centre of the annulus. The amplitude of the primary stimulus waveform is restricted to 10 per cent of the background field level. Illumination is photopic. (For a fuller account of the experimental approach and method, see Foster.[7])

In Fig. 2 we show the model. The R represent receptors (or groups of receptors) separated by the angular distance $\Delta\theta$ (measured angularly around the annular field); the V represent de Lange low pass filters (see de Lange[8]), and the H represent modified Reichardt multipliers (see Reichardt and Varju,[9] and Foster[7]). Information about the local time-varying course of the signal is carried in the channels marked 01, and information about the spatial organization of these local temporal fluctuations is carried in the channels marked 02. Provided the signals from both 01 and 02 surmount their respective threshold mechanisms, a sensation of well-defined directed motion is induced (if the spatial period of the stimulus is not too small).

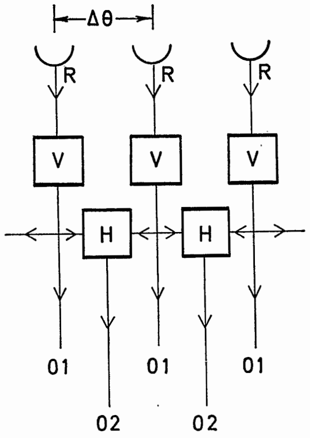

FIG. 2. A model of the human visual system for real motion stimuli (see text). The R are receptors (or groups of receptors), the V are de Lange low pass filters, and the H are modified Reichardt multipliers.

We now examine the general motion response of this network.

Let $r_{\omega\lambda}(\Delta\theta)$ be the response of a single H-unit to a moving sinusoid of spatial period λ and temporal (angular) frequency ω. It may be shown that in the steady state

$$r_{\omega\lambda}(\Delta\theta) = \frac{\omega}{a^2+\omega^2}.\sin\left(\frac{2\pi.\Delta\theta}{\lambda}\right) \tag{1}$$

where a is the rate constant of the cross filters (*q.v.*, in Foster[7]). Let $P_{\mathrm{H}}(\theta)$ describe the sensitivity of the system to variations in the total H-unit population with variations in area θ. Similarly, let $p_{\mathrm{H}}(\Delta\theta)$ describe the sensitivity of the system to variations in the relative H-unit population with variations in the input-pair separation $\Delta\theta$. Then, the final *weighted* H-unit output, $s_{\theta\omega\lambda}(\Delta\theta)$ say, is given by the following:

$$s_{\theta\omega\lambda}(\Delta\theta) = P_{\mathrm{H}}(\theta).p_{\mathrm{H}}(\theta).r_{\omega\lambda}(\Delta\theta) \tag{2}$$

We define the general motion response of the network, $\mathscr{R}$, to be the set of all outputs $s_{\theta\omega\lambda}(\Delta\theta)$ which are each large enough to surmount some threshold T_{H}. That is, using conventional set-theoretic notation:

$$\mathscr{R} = \{s_{\theta\omega\lambda}(\Delta\theta): \Delta\theta\ \varepsilon\ [\Delta\theta_{\min}, \Delta\theta_{\max}], |s_{\theta\omega\lambda}(\Delta\theta)| > T_{\mathrm{H}}\} \tag{3}$$

where $s_{\theta\omega\lambda}(\Delta\theta)$ is defined by equation 2, and $[\Delta\theta_{\min}, \Delta\theta_{\max}]$ is the set of all possible input-pair separations.

In order that $\mathscr{R}$ may give rise to a sensation of motion with well-defined direction, we require $\mathscr{R}$ to be non-empty and all the $s_{\theta\omega\lambda}(\Delta\theta)$ to have the same sign (which we choose positive).

3. THE EXISTENCE OF AN APPARENT MOTION EFFECT

In the previous section we presented the elements of a model for real motion stimuli. We now show that such a scheme implies the existence of an apparent motion effect associated with certain configurations of flashing light stimuli. (Note: The comments made in this and in subsequent sections are valid only for the set of input stimuli defined in Section 2.)

Consider two distinct points in the stimulus field (see Fig. 1) such that their angular separation $\Delta\theta\ \varepsilon\ [\Delta\theta_{\min}, \Delta\theta_{\max}]$. Suppose we apply to each of these points a time-varying sinusoid of temporal frequency

ω and phase difference $2\pi\Delta\theta/\lambda$. The response $r_{\omega\lambda}(\Delta\theta)$ then generated by the relevant H-unit is clearly indistinguishable from the equivalent real motion response. This does not necessarily mean that the set $\mathscr{R}$, defining the general motion response of the system, is non-empty (see equation 3). However, since $P_{\mathrm{H}}(\theta)$ and $p_{\mathrm{H}}(\Delta\theta)$ are simply weighting factors, the condition $|s_{\theta\omega\lambda}(\Delta\theta)| > T_{\mathrm{H}}$ may be satisfied by choosing the amplitude of the input sinusoids sufficiently large. The non-emptiness of $\mathscr{R}$ may thus be assured.

We therefore have the implication that under suitable conditions, two discrete stationary sources of light, flashing with a certain temporal frequency and separated by an appropriate time-lag, will give rise to a sensation of real motion. Proposition (i) of Section 1 is consequently verified.

4. FREQUENCY RESPONSE CHARACTERISTICS

We now examine two specific stimulus configurations: one a stationary flashing light stimulus, and the other a real motion stimulus. Both have the same constant angular area $\theta_0 \varepsilon [\Delta\theta_{\min}, \Delta\theta_{\max}]$. For each configuration we obtain an explicit representation of the general motion response $\mathscr{R}$. The frequency response characteristics of the two sets are then compared.

In the first case (Case 1), let θ_0 refer to an intact aperture (as in Fig. 1). We arrange for a sinusoidal waveform of spatial period λ_0 and temporal frequency ω_0 to rotate behind this aperture. From equations 1, 2 and 3, the general real motion response of the model is then given by the following:

$$\mathscr{R}_{\theta_0} = \{s_{\theta_0\omega_0\lambda_0}(\Delta\theta) \colon \Delta\theta \;\varepsilon\; [\Delta\theta_{\min}, \theta_0], \; s_{\theta_0\omega_0\lambda_0}(\Delta\theta) > T_{\mathrm{H}}\} \tag{4}$$

where

$$s_{\theta_0\omega_0\lambda_0}(\Delta\theta) = P_{\mathrm{H}}(\theta_0) . p_{\mathrm{H}}(\Delta\theta) . \frac{\omega_0}{a^2+\omega_0^2} . \sin\left(\frac{2\pi . \Delta\theta}{\lambda_0}\right)$$

The mod signs on $s_{\theta_0\omega_0\lambda_0}(\Delta\theta)$ have been dropped so that $\mathscr{R}$ corresponds to motion with well-defined direction (*q.v.*, Section 2).

In the second case (Case 2), let θ_0 refer to the aperture shown in Fig. 3, in which the two equal half-sectors of areas $\frac{1}{2}\theta_0$ are separated by $\frac{1}{2}\theta_0$. Suppose each of these half-sectors is spatially uniform and each varies sinusoidally in time. Let the temporal frequency be ω_0 and the phase lag between the sources be $2\pi\Delta\phi$. The general motion response of the model to this stimulus is then:

$$\mathscr{R}'_{\theta_0} = \{s'_{\theta_0\omega_0\Delta\phi}(\Delta\theta) \colon \Delta\theta \;\varepsilon\; [\max\{\tfrac{1}{2}\theta_0, \Delta\theta_{\min}\}, \tfrac{3}{2}\theta_0], \; s'_{\theta_0\omega_0\Delta\phi} > T_{\mathrm{H}}\} \tag{5}$$

where

$$s'_{\theta_0\omega_0\Delta\phi}(\Delta\theta) = P'_{\mathrm{H}}(\theta_0) . p_{\mathrm{H}}(\Delta\theta) . \frac{\omega_0}{a^2+\omega_0^2} . \sin(2\pi . \Delta\phi)$$

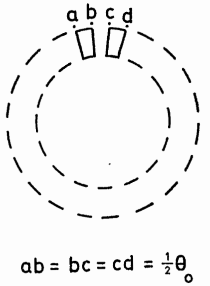

FIG. 3. The primary stimulus field of Case 2 (see text). The annular aperture is restricted to two sectors, each of area $\frac{1}{2}\theta_0$, separated by $\frac{1}{2}\theta_0$.

The original areal weighting factor $P_{\mathrm{H}}(\theta)$ has been replaced by the factor $P'_{\mathrm{H}}(\theta)$ defined for the modified stimulus configuration.

If equations 4 and 5 are now compared, it is seen that providing the rate constant a is independent of $\Delta\theta$, the sets $\mathscr{R}_{\theta_0}$ and $\mathscr{R}'_{\theta_0}$ have the same dependence upon temporal frequency ω. In particular, both maximize with respect to number and magnitude of elements at $\omega = a$. Since Case 1 constitutes a real motion stimulus and Case 2 a stationary flashing light stimulus, we have the following prediction.

> *Prediction A:* The real motion stimulus of Case 1 and the stationary flashing light stimulus of Case 2 should give rise to visual responses which maximize, with respect to temporal frequency, at the same value.

We now show that if certain other assumptions are made, the above prediction may be made more specific.

Let us consider the general case described in Section 2, with $\mathscr{R}$ given by equation 3. Suppose θ is reduced and λ is increased so that a sensation of well-defined directed motion can only be obtained over a small range of ω. The behaviour of $\mathscr{R}$ is then almost completely determined by the element max $\mathscr{R}$, defined thus:

$$\max \mathscr{R} = \max\{s_{\theta\omega\lambda}(\Delta\theta) \colon \Delta\theta \;\varepsilon\; [\Delta\theta_{\min}, \min\{\theta, \Delta\theta_{\max}\}], \; s_{\theta\omega\lambda}(\Delta\theta) > 0\} \tag{6}$$

where

$$s_{\theta\omega\lambda}(\Delta\theta) = P_{\mathrm{H}}(\theta) . p_{\mathrm{H}}(\Delta\theta) . \frac{\omega}{a^2+\omega^2} . \sin\left(\frac{2\pi . \Delta\theta}{\lambda}\right)$$

If $p_{\mathrm{H}}(\Delta\theta)$ falls of sufficiently rapidly (see Foster[7]), then

$$\max \mathscr{R} = s_{\theta\omega\lambda}(\Delta\theta_{\min}) \tag{7}$$

Lemma: If $\lambda = 180°$, and θ_{min} is the associated value of θ such that a sensation of well-defined directed motion can only be obtained over a small range of ω, then by experiment[7]

$$\theta_{\text{min}} \doteqdot 2\,\Delta\theta_{\text{min}} \tag{8}$$

We return to the stimulus configurations of Case 1 and Case 2, with $\mathscr{R}$ defined by equations 4 and 5 respectively. Suppose we fix $\lambda_0 = 180°$, and $\theta_0 = \theta_{\text{min}}$. Then, using the above, we have for Case 1:

$$\max \mathscr{R}'_{\theta_{\text{min}}} = P_{\text{H}}(\theta_{\text{min}})\,.\,p_{\text{H}}(\tfrac{1}{2}\theta_{\text{min}})\,.\,\frac{\omega_0}{a^2+\omega_0^2}\,.\,\sin(\tfrac{1}{2}\theta_{\text{min}}) \tag{9}$$

and for Case 2:

$$\max \mathscr{R}'_{\theta_{\text{min}}} = P'_{\text{H}}(\theta)_{\text{min}}\,.\,p_{\text{H}}(\tfrac{1}{2}\theta_{\text{min}})\,.\,\frac{\omega_0}{a^2+\omega_0^2}\,.\,\sin(2\pi\,.\,\Delta\phi) \tag{10}$$

It is reasonable to suppose that $P_{\text{H}}(\theta_{\text{min}})$ and $P'_{\text{H}}(\theta_{\text{min}})$ do not differ too greatly. If we select $2\pi\Delta\phi = \frac{1}{2}\theta_{\text{min}}$, it then follows that

$$\max \mathscr{R}_{\theta_{\text{min}}} \doteqdot \max \mathscr{R}'_{\theta_{\text{min}}} \tag{11}$$

which holds whether a is a function of $\Delta\theta$ or not (see Prediction A, above). Since at motion threshold $\mathscr{R}$ is determined by max $\mathscr{R}$, we also have

$$\mathscr{R}_{\theta_{\text{min}}} \doteqdot \mathscr{R}'_{\theta_{\text{min}}} \tag{12}$$

This implies the following stronger form of Prediction A:

> *Prediction B:* With $\lambda = 180°$, $\theta_0 = \theta_{\text{min}}$, and $2\pi.\Delta\phi = \frac{1}{2}\theta_{\text{min}}$, the real motion stimulus of Case 1 and the stationary flashing light stimulus of Case 2 should give rise to visual responses which are identical in their temporal frequency dependence.

In order to test Prediction B, the experiment described below was carried out.

5. EXPERIMENT TO TEST PREDICTION B

Using the experimental method outlined in Section 2, the subject was presented with two stimulus configurations:

1. A real motion stimulus consisting of a 180° sine grating rotating behind an aperture with area $\theta = \theta_{\text{min}}$ (as in Fig. 1).
2. A stationary flashing light stimulus consisting of two uniformly illuminated apertures, with areas $\frac{1}{2}\theta_{\text{min}}$ and separation $\frac{1}{2}\theta_{\text{min}}$ (as in Fig. 3), varying sinusoidally in time with phase difference $\frac{1}{2}\theta_{\text{min}}$.

In both cases, the temporal frequency of the stimulus was set at

random values, and the subject indicated whether the induced sensation was that of well-defined directed motion or not. A histogram was then constructed of number of positive responses plotted against frequency ω. The results for the two subjects are shown in Fig. 4. The total area θ_{min} is marked in each case.

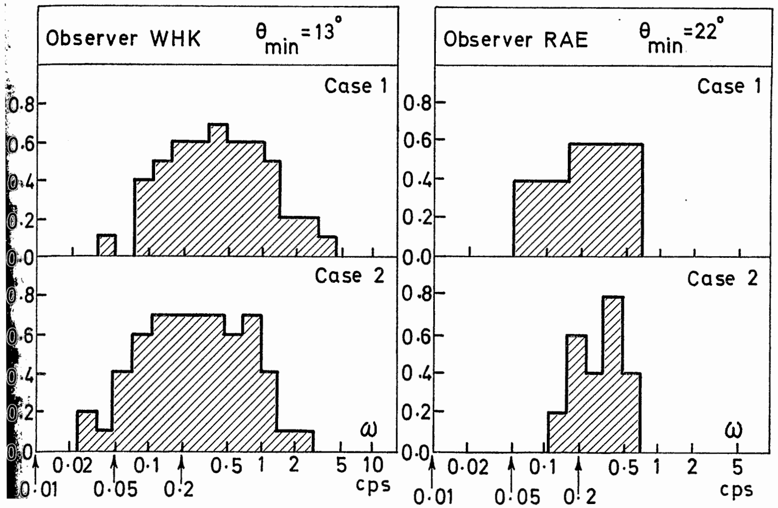

FIG. 4. A comparison of the motion responses induced by the real motion stimulus of Case 1 and the flashing light stimulus of Case 2 (see text). The number of positive responses (in arbitrary units) is plotted against temporal frequency ω (in cycles per seconds).

It is seen that for each subject the real and apparent motion curves are very similar, both maximizing at the same value of ω. If we assume that the number of positive responses occurring depends upon the strength of the induced motion sensation, then Predictions A and B are verified.

6. CONCLUSION

It has been shown that the existence of an apparent motion effect, associated with certain flashing-light sources, is a natural consequence of a real motion model. It has also been shown that the temporal frequency response characteristics of this apparent motion effect are the same as those of the equivalent real motion effect.

We therefore have the implication that, for a restricted class of stimuli, the same functional mechanisms may be used to represent both the processes underlying real motion phenomena and the processes underlying apparent motion phenomena.

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DETECTION, DISCRIMINATION AND BRIGHTNESS OF SUCCESSIVELY PRESENTED FLASHING LIGHTS*

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The research reported here is concerned with the detection, discrimination and brightness of two flashes of *equal* area presented to the *same* foveal location in succession. In all measurements to be reported the interval separating the two flashes was greater than that required to just perceive two flashes.

APPARATUS

The apparatus consisted of a specially constructed four-channel binocular maxwellian viewing system, the complete details of which have previously been published.[1] Before the beginning of each experimental session the subject was dark adapted for 15 minutes and then looked at the prevailing luminance of the 12° 15′ adapting field for 5 more minutes with either the right eye alone or with both eyes, depending on the measurements being made. Test fields subtended 2°; adaptation fields subtended 12° 15′. Test fields were always superimposed on the adapting field, and viewing was foveal, through a 2-mm artificial pupil, controlled by two fixation points located just above and below the 2° square test field. Viewing could be either monocular or binocular.

SUCCESSIVE FLASH DETECTION

While the influence of spatial variations in test and background fields using increment threshold detection has received extensive previous investigation,[2-12] little work has been done on the detectability of one of two equal size fields successively presented to the same retinal location without the presence of spatial transients between the fields. Sperling[11] has suggested that spatial transients may mediate detection when fields are simultaneously presented (simultaneous contrast) while

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* The majority of the research reported in this paper has been supported by grants NB 7340 from the National Institute for Neurological Disease and Blindness, National Institute of Health, and EY-00023 from the Eye Research Institute, National Institute of Health, and in part from a National Institute of Health Biomedical Institutional Grant.