On the structure of fibre flocs*

C.T.J. Dodson† and R.J. Kerekes

Pulp & Paper Centre,

University of British Columbia,

Vancouver B.C. Canada V6T 1Z4.

Abstract

Estimates are obtained for the volumetric concentration and the number of fibre contacts per fibre in a floc. The maximal concentration for flocs from typical rigid wood fibres increases from about four times to five times the sedimentation concentration, as aspect ratio increases from 20 to 40. The fibre aspect ratio appears as the principal fibre geometric influence on sedimentation concentration, maximal floc concentration, and relative rates of floc forming and breaking.

Introduction

On a fast papermachine, a jet of thickness about one centimetre is delivered at speeds in excess of twenty metres per second, with a fibre content of perhaps one percent by volume. It is still a matter of debate as to the extent to which fibres in a local region in a forming section of a papermachine have a memory of their prior organization before emerging in the jet from the headbox. A solution to this depends on sophisticated experiments and a better understanding of turbulent fibre suspensions than is available at present. Meanwhile, it is possible to make some estimates of average values and this is our objective here.

The methods we use are quite simple. For, until we have a good physical reason to believe that greater complexity yields more information than it demands as input, it seems pointless to make more detailed assumptions. We shall take as our state variable the number of fibre contacts per fibre and we shall suppose that our fibres are uniform cylinders. As always, we begin with a random assembly of fibres because it is the unique well-defined stochastic reference structure for the context. To create a structure that locally resembles a floc, we use a new result on densification by maximal packing of rigid rods into a random network.

*Presented at the COST Meeting on "Multi-Phase Flows in Papermaking", Grenoble 11-13 May 1997

†Work done on study leave from the University of Toronto. Present address: Department of Paper science, UMIST, Manchester M60 1QD, UK

Fibre Suspensions: The Random State

Consider a random fibre suspension with fibre mass density c_{mass}, fibre volume fraction c_{vol}, made from uniform cylindrical fibres having length λ, diameter ω, mass/length δ, and density ρ. We define two 'state numbers' for cylindrical fibres, of aspect ratio $A = \lambda/\omega$, in 3-dimensional random arrays:

Crowding number The expected number of fibres in a spherical region of diameter λ [12, 14, 15, 9]:

$$n_{crowd} = \frac{\pi}{6}\frac{c_{mass}\lambda^2}{\delta} = \frac{2}{3}A^2 c_{vol} \tag{1}$$

Contact number The expected number of fibre contacts per fibre [3]:

$$n_{contacts} = \frac{3\omega n_{crowd}}{\lambda} = \frac{3n_{crowd}}{A} = 2Ac_{vol} \tag{2}$$

These suggest considering also the
Number of contacts per unit mass of fibre:

$$\frac{n_{contacts}}{\lambda\delta} = \frac{\pi\omega}{2\delta^2}c_{mass} \tag{3}$$

which expression emphasises the importance of δ.

Non-Random Fibre Suspensions: Flocs

Various non-random packings of cylinders have been discussed in the literature, but it turns out that the expected number of contacts per fibre, $n_{contacts}$, is always rather close to that for the random case [3]. The particular case of sedimented fibre suspensions is also discussed in [3], and corresponds in practice to the condition $n_{contacts} = n^{sed}_{contacts} \approx 3$; hence, $c^{sed}_{vol} = \frac{3}{2A}$.

In flocculated fibre suspensions, there is evidence that, like in commercial paper [4], the floc size distribution is approximately lognormal [6, 7]. In paper, the mean floc density increases with mean floc diameter [3] and the standard deviation of floc diameter increases with mean floc diameter [4]. Since we are interested in modelling a floc as a stochastic clump of fibres created by elastic, frictional and hydrodynamic forces, we expect the structure to be a 'densification' from the random state. The question is then: how do we pack more fibres into a given initial random assembly? This seems to have been answered recently in a convenient form by Parkhouse and Kelly [16]. These authors devised a criterion of a limiting packing state beyond random that corresponds reasonably with the notion of pushing fibres into all available spaces that could receive them. Their result estimates an upper bound, c^{max}_{vol}, on the volumetric concentration of stochastic rigid cylinder assemblies, in terms of the fibre aspect ratio A only:

$$c^{max}_{vol} = \frac{2\log A}{A} \tag{4}$$

It then follows from above that we have an estimate of the limiting number of contacts per fibre (cf. Figure 1):

$$n_{contacts}^{max} = 2Ac_{vol}^{max} = 4\log A \tag{5}$$

which corresponds quite closely with estimates from experiments with synthetic fibre flocs reported by Kropholler et al. [11]. Now, Equations (4), (5) are for *maximal* infilling from a random initial state, using *rigid* cylindrical fibres. Such a network would have no intrinsic strength and could be made denser by using *flexible* fibres to create a coherent structure. However, flexible fibres in turbulent mixing are unlikely to develop maximally dense flocs. Accordingly, we propose to use Equation (4) and Equation (5) to provide estimates for the volumetric concentration and number of contacts per fibre in a woodpulp fibre floc; these depend on fibre aspect ratio only:

$$c_{vol}^{floc} \approx \frac{2\log A}{A} \tag{6}$$

$$n_{contacts}^{floc} \approx 4\log A \tag{7}$$

Now we have an interesting way to view the geometrical effect of fibre aspect ratio, by expressing this floc concentration in ratio to the sedimentation concentration:

$$\frac{c_{vol}^{floc}}{c_{vol}^{sed}} = \frac{4}{3}\log A \tag{8}$$

Equation (8) is plotted in Figure 1 and we see that, over the normal range of interest for papermaking fibres, the floc concentration increases slowly from about four to five times the sedimentation concentration. This emphasises the usefulness of the sedimentation concentration and the importance of good estimates of fibre length and width distributions.

Equation (6) gives $c_{vol}^{floc} \approx 0.07$ for $A = 140$ and Soszynski [19] page 96 measured $c_{vol}^{floc} = 0.06$. See also the work of Soszynski on the apparent volumetric concentration of cellulose fibres [20].

Flocculation Dynamics

Hydrodynamic forces cause fibres to make and break contacts and, in decaying turbulence, coherent flocs of fibres can grow in milliseconds [9, 10]. Longer fibres make larger flocs and at a fixed fibre length stiffer fibres make more stable and larger flocs [8, 2, 5].

We represent the local state of flocculation by the local fractional density of fibre-fibre contacts v. Now, the maximum number of fibre contacts per fibre is $2A$, consisting of A above and A below a given fibre, so v reduces to the volumetric concentration because we have

$$v = \frac{n_{contacts}}{2A} = c_{vol}^{floc}. \tag{9}$$

Then, a simple rate equation which applies to blood clotting [17, 1], among other things, becomes in our case:

$$\frac{dv}{dt} = \frac{1-v}{\tau_+} - \frac{v}{\tau_-} + \epsilon(t) \qquad \text{for } 0 \leq v \leq 1 \tag{10}$$

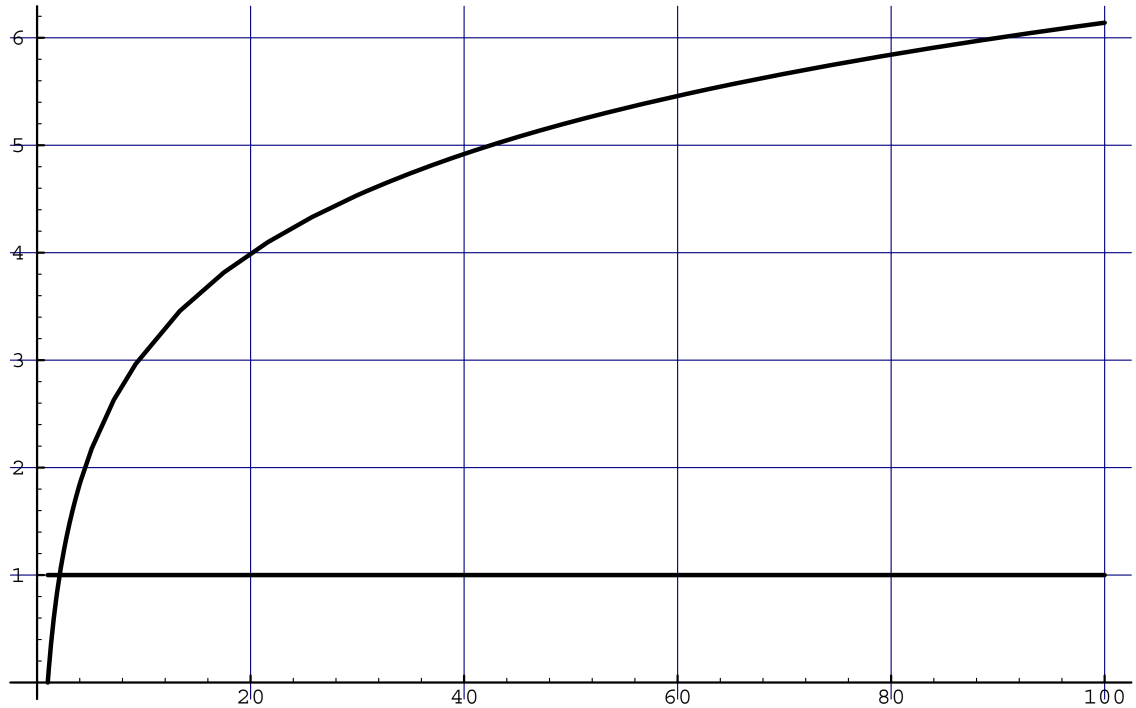

Figure 1: *Plot of* $\frac{4}{3}\log A$, *the floc concentration in ratio to sedimentation concentration as a function of fibre aspect ratio* A. *To obtain the number of fibre contacts per fibre,* $4\log A$, *multiply the ordinate by 3.*

where τ_+ and τ_- are the rate constants for making and breaking contacts, respectively and $\epsilon(t)$ is a zero-mean noise function which represents the ambient turbulent conditions. For the random case, we showed that the mean value of v is the mean volumetric concentration: $\bar{v} = c_{vol}$, which indicates how the rate equation could be deployed in an experiment that could measure time averages over finite spatial zones.

Taking $\epsilon(t) = 0$ in equation (10) converts it into a deterministic ordinary differential equation which has solution

$$v(t) = \frac{1}{a} - (\frac{1}{a} - v_0)e^{-at} \quad \text{for } 0 \leq v_0 \leq 1 \tag{11}$$

where $v_0 = v(0)$ is the initial value and $a = 1 + \frac{\tau_+}{\tau_-}$ controls the time scale. There is a unique critical solution which corresponds to $v'(t) = 0$ and is given by $v_0 = \frac{1}{a}$ by

$$v(t) = v_\infty = \frac{1}{a} = \frac{1}{1 + \frac{\tau_+}{\tau_-}}. \tag{12}$$

Evidently, this constant solution is a stable attractor for all other solutions, as illustrated in Figure 2 for the case $v_\infty = 0.2$ and a range of initial values v_0 in equation (11). In practice, there would be simultaneous occurrences of making and breaking of flocs, so observation of a given region of suspension would be expected to reveal a noisy version of Figure 2.

If we interpret v_∞ as the stable state for flocs and use the estimate for $n^{floc}_{contacts}$ from

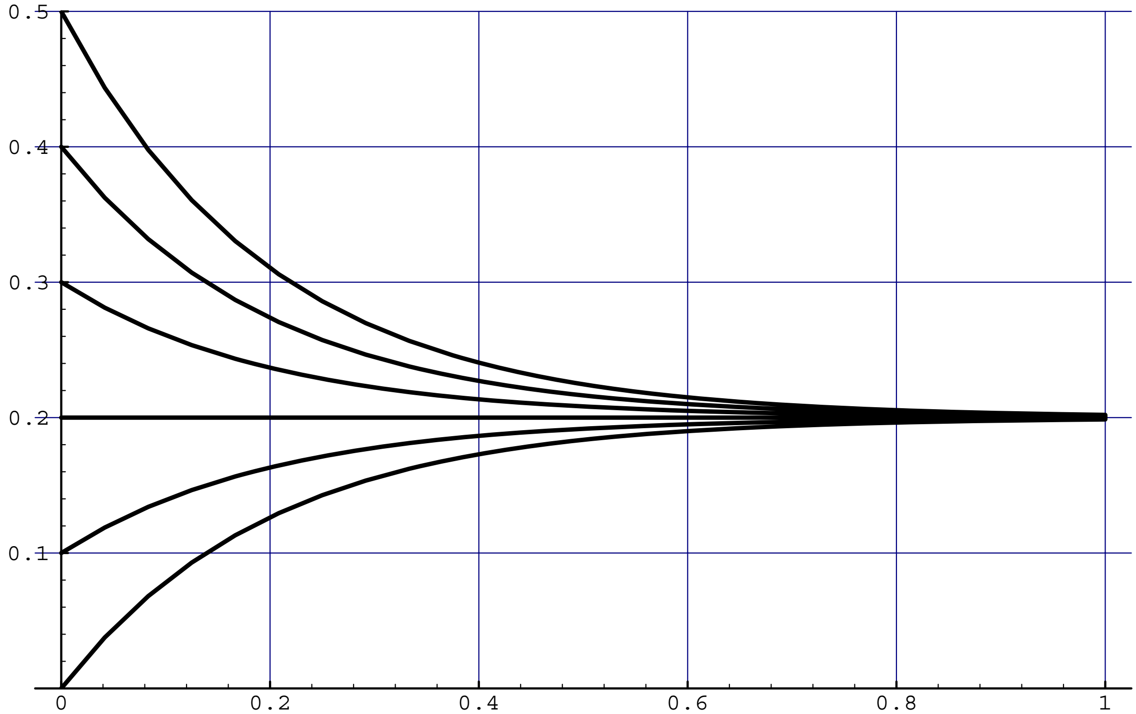

Figure 2: *Plot of deterministic solutions, $v(t)$, to the rate equation for making and breaking fibre contacts, with a range of initial values $v_0 = 0.1, 0.2, 0.3, 0.4$ and asymptotic value $v_\infty = 0.2$.*

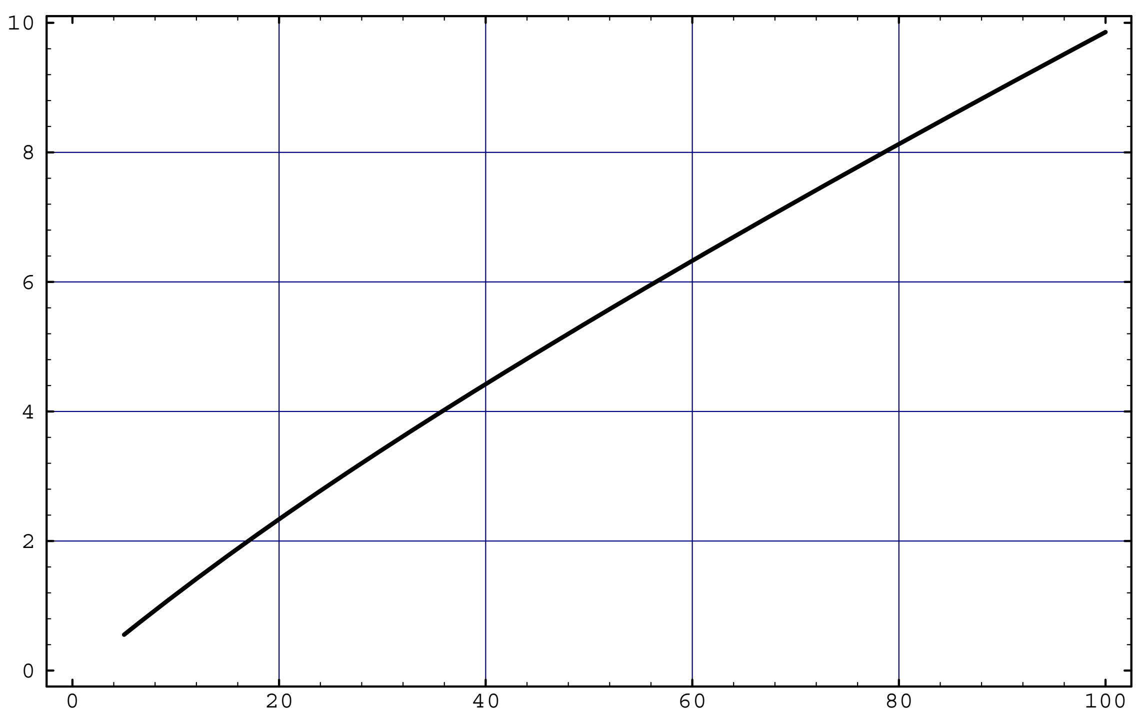

Figure 3: *Plot of ratio of rate constant for making to rate constant for breaking fibre contacts, $\frac{\tau_+}{\tau_-} = \frac{A}{2 \log A} - 1$, as a function of fibre aspect ratio A.*

equation (5), then we have

$$v_\infty = \frac{n^{floc}_{contacts}}{2A} = c^{floc}_{vol} \approx \frac{2\log A}{A} = \frac{1}{1+\frac{\tau_+}{\tau_-}}. \tag{13}$$

Rearranging this we have a prediction of the *relative* rate constants for making and breaking contacts, in terms of the aspect ratio of fibres

$$\frac{\tau_+}{\tau_-} = \frac{A}{2\log A} - 1 \approx \frac{A}{10} \quad \text{for } 10 \leq A \leq 100 \tag{14}$$

as shown in Figure 3. The *absolute* rate constants will depend on other factors than the purely geometrical aspect ratio, such as the ambient hydrodynamic conditions and fibre rigidity. Kerekes et al. [9, 10] found that, in decaying turbulence, flocs formed in a few tens of milliseconds. Our estimate above suggests that in a flow field with flocs forming and breaking up at different places, they break up at three or four times the rate of forming under comparable gradients of turbulent conditions.

Acknowledgements

The authors wish to thank the University of Toronto, the University of British Columbia, NSERC and the National Network of Centres of Excellence for Chemi-Mechanical Pulps for support during the progress of this work. They are grateful also to Dr. R.S. Seth for drawing their attention to the article by Parkhouse and Kelly [16] and to Dr. S.R. Andersson for providing a copy of a Departmental Report [1].

References

[1] S.R. Andersson. Hydrodynamics of fibre suspensions. Report, Department of Chemical Engineering Design, Chalmers University of Technology, Göteborg, Sweden 1994.

[2] M. Deng and C.T.J. Dodson. **Paper: An Engineered Stochastic Structure**. Tappi Press, Atlanta 1994.

[3] C.T.J. Dodson. Fibre crowding, fibre contacts and fibre flocculation. *Tappi J.* 79, 9 (1996) 211-216.

[4] R.R. Farnood and C.T.J. Dodson. The similarity law of formation. Presentation at the International Paper Physics Conference, Niagara-on-the-Lake, 11-14 September 1995.

[5] R.R. Farnood, C.T.J. Dodson and S.R. Loewen Estimation of intra-floc forces. *Appita J.* 47, 5 (1994) 391-396.

[6] M.J. Hourani. Fibre flocculation in pulp suspension flow. Part 1: theoretical model. *Tappi J.* 71(5) (1988) 115-118.

[7] M.J. Hourani. Fibre flocculation in pulp suspension flow. Part 2: experimental results. *Tappi J.* 71(6) (1988) 186-189.

[8] H. Kaji, K. Monma and T. Katsura. Fractal analysis of flocculation in pulp suspension. In **Proc. Tappi International Paper Physics Conference**, Kona Hawaii 22-26 September 1991. pp 291-297.

[9] R.J. Kerekes and C.J. Schell. Characterization of fibre flocculation by a crowding factor. *J. Pulp Paper Sc.* 18, 1 (1992) 32-38.

[10] R.J. Kerekes and C.J. Schell. Effects of fiber length and coarseness on pulp flocculation. *Tappi J.* 78, 2 (1995) 133-139.

[11] H.W. Kropholler, C.T.J. Dodson, C. Baxter and W.W. Sampson. Experiments for estimating fibre flocculation tendency. Presented at the Tappi Paper Physics Seminar, June 10-13 1996, STFI, Stockholm.

[12] S.G. Mason. The motion of fibres in flowing fluids. *Pulp and Paper Mag. Can.* 51(5) (1950) 93-100.

[13] S.G. Mason. Some factors involved in the flocculation of pulp suspensions and the formation of paper. *Pulp and Paper Mag. of Can.* 51(10) (1950) 94-98.

[14] S.G. Mason. Fibre motion and flocculation. *Tappi J.* 37(11) (1954) 494-501.

[15] S.G. Mason. Fibre motion and flocculation. *Pulp and Paper Mag. of Can.* 55(13) (1954) 96-102.

[16] J.G. Parkhouse and A. Kelly. The random packing of fibres in three dimensions. *Proc. Roy. Soc. Lond. A* 451 (1995) 737-746.

[17] D. Quemada. **Biofluids as structured media: Rheology and flow properties of blood**. Lecture Notes in Physics 381, Springer-Verlag Berlin-Heidelberg New York 1990.

[18] J. Ringner. **The influence of fibre length distribution on the network strength of fibre suspensions**. MSc thesis, Dept. Chemical Engineering, Chalmers University, Göteborg 1995.

[19] R.M. Soszynski. **The formation and properties of coherent flocs in fibre suspensions** Ph.D. Thesis, Department of Chemical Engineering, University of British Columbia, 1987.

[20] R.M. Soszynski. The apparent volumetric concentration of wood pulp fibres in suspension. *Appita J.* 42, 5 (1989) 362-363.