Non Linear Problems of Second Order in Infinite Dimensional Manifolds

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Several problems arising in the study of an infinite dimensional manifold M are connected with non linear differential equations of second order. In this framework one may refer to interactions with jet fields, linear and non-linear connections, Lagrangians and Finsler structures (cf., for instance, [1], [2], [5]). A convenient environment for the study of such type of problems is the second order (acceleration) tangent bundle T^2M, consisting of all equivalent classes of curves on M that agree up to their second derivative. T^2M can be endowed with a vector bundle structure in the presence of a linear connection ∇ on the base M, in case of Banach modeled manifolds. This is also true for Fréchet manifolds under additional properties for the corresponding Christoffel symbols. The obtained vector bundle structure on T^2M remains invariant if ∇ is replaced by a conjugate connection with respect to any diffeomorphism of M. In this context, a wide class of second order differential equations on M can be realized as integral curves of second order vector fields. The proposed methodology is suitable for Banach and Fréchet modelled manifolds giving, for the latter, a new way of working in a framework where the lack of a general solvability theory inhibits the establishment of existence and uniqueness of solutions from initial conditions. Moreover, the techniques presented can be used for the description of a class of geodesic curves on infinite dimensional Riemannian manifolds. However, the potential applications of this work reach beyond classical Differential Geometry, having, for example, a central role in the theory of time-dependent Lagrangian particle systems.

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