Stochastic Flow Simulator

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Introduction

It is common for natural and industrially manufactured porous media to exhibit a distribution of void sizes. Classical models for the flow of fluids through and penetration of fluids into such porous media such as the Kozeny-Carman and Lucas-Washburn equations typically use average values of porosity and pore radius to characterise the geometry of the medium. Such models can, given suitable estimates of physical constants, provide useful predictions of mean flow rate but do not yield any insights into the distribution of flow rates from region to region. These distributions are important since they will affect, for example

- the penetration of coatings, inks, paints, *etc.*
- the passage or entrapment of materials by permeable membranes and filters
- the barrier properties of packaging materials
- the likelihood of extreme values, *e.g.* of local flow rates much greater than the mean.

The Stochastic Flow Simulator, *SFS*, is a software package that has been developed by the authors and allows the distribution of flow rates from region to region to be calculated for laminar, capillary, turbulent and molecular flow regimes. The inputs to the simulator are the mean porosity and the pore radius distribution of the porous medium of interest. The authors have developed models describing the pore structure and porosity distribution in stochastic materials. It turns out that the pore radius distribution for fibrous filter media, granular packings, sintered disks, paper and board and natural materials such as porous rock, *etc.* are all described well by the gamma distribution. A family of gamma distributions, all with unit mean, are illustrated in Figure 1.

Case Study 1

Four samples of industrially manufactured filters with similar porosities were provided for analysis. One filter was known to perform differently in a high flow-rate filtration application. Analysis of the pore radius distribution yielded the curves shown in Figure 2. The mean pore

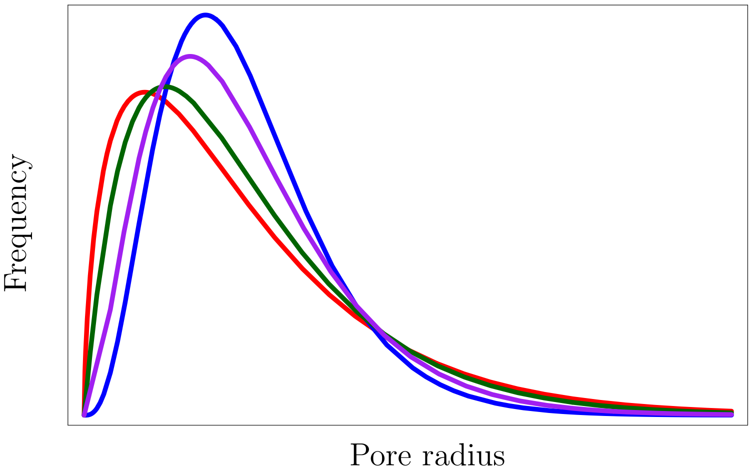

Figure 1: *Family of gamma distributions representing pore radius disributions. Each distribution plotted has same mean.*

radii for each of the samples were very similar, varying by only a few percent. Qualitatively, the pore radius distribution for the sample shown in red can be seen to be a little different from those of the other filters. The pore radius distributions for each filter were used as inputs to *SFS* and the outputs for turbulent flow are shown in Figure 3.

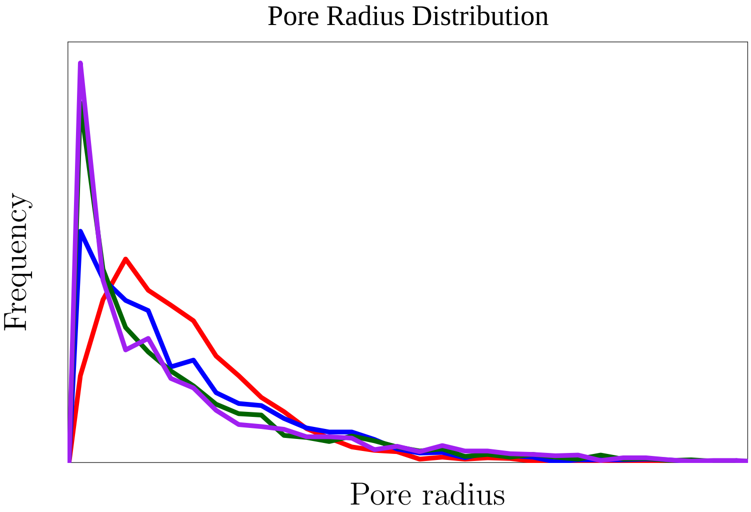

Figure 2: *Pore radius distributions for four industrially manufactured filters.*

It is immediately apparent from the flow distributions in Figure 3 that the sample depicted by the red curve exhibits a narrower turbulent flow distribution than the other samples.

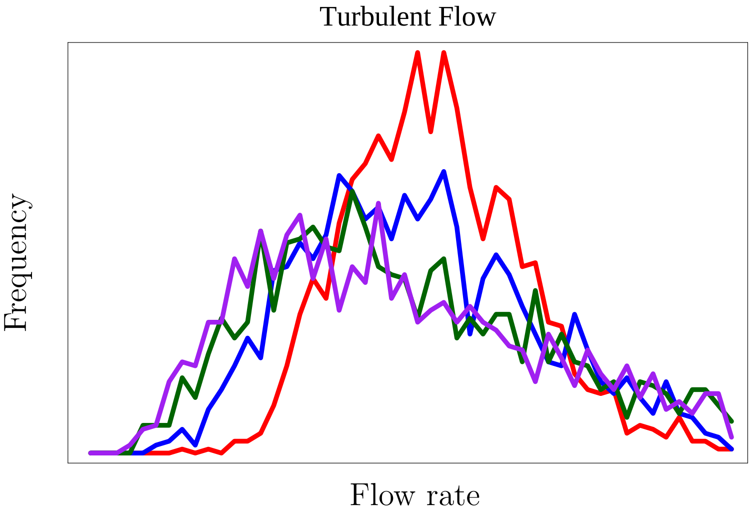

Figure 3: *Outputs from SFS showing distributions of turbulent flow for the samples shown in Figure 2.*

Case Study 2

Two paper samples were provided, each made from the same fibre type and on the same paper machine. One sample was known to have satisfactory printing performance, the other yielded a nonuniform print. Pore radius distributions for each sample are shown in Figure 4; the differences in the distributions were only slight. The outputs from *SFS*, using these pore radius distributions as an input are shown in Figure 5. Since the penetration of ink into a paper is a capillary process, this is the flow mode examined. It is immediately apparent that the difference in pore radius distribution has a dramatic effect on the distribution of local capillary flow rate. The broader distribution shown for the sample with the higher flow rate therefore had a broader distribution of ink penetration depths before ink curing and exhibited therefore a greater degree of print nonuniformity.

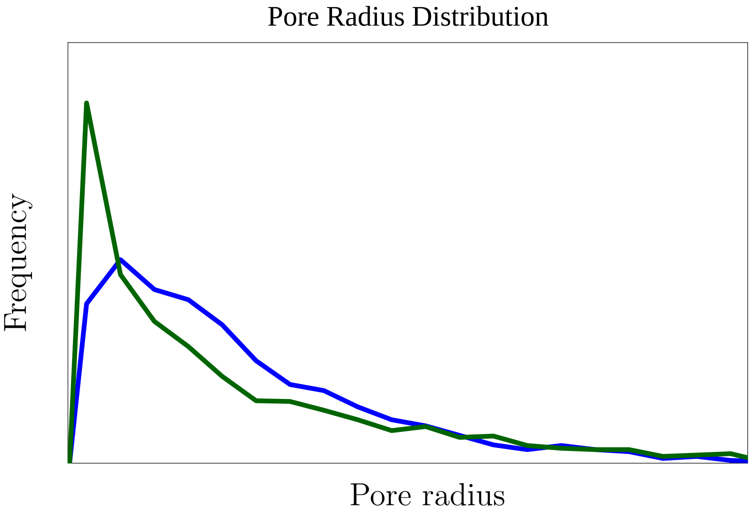

Figure 4: *Pore radius distributions for two paper samples.*

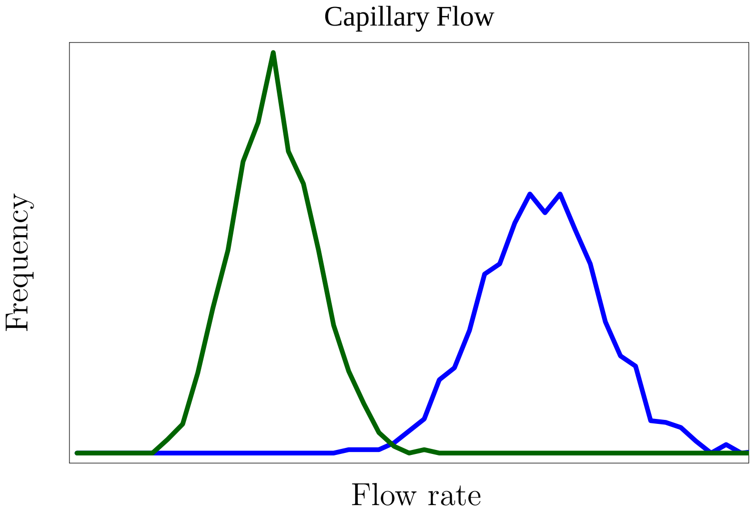

Figure 5: *Outputs from SFS showing distributions of capillary flow for the samples shown in Figure 4.*

Developments

We have also probed the distribution of porosity through experimental measurements and have derived models which faithfully describe the porosity distribution in stochastic fibrous materials including filter media and paper. Examples of the areal density, thickness and porosity distributions for a fibre network are illustrated in Figure 6. Work is ongoing to incorporate porosity distribution in *SFS*.

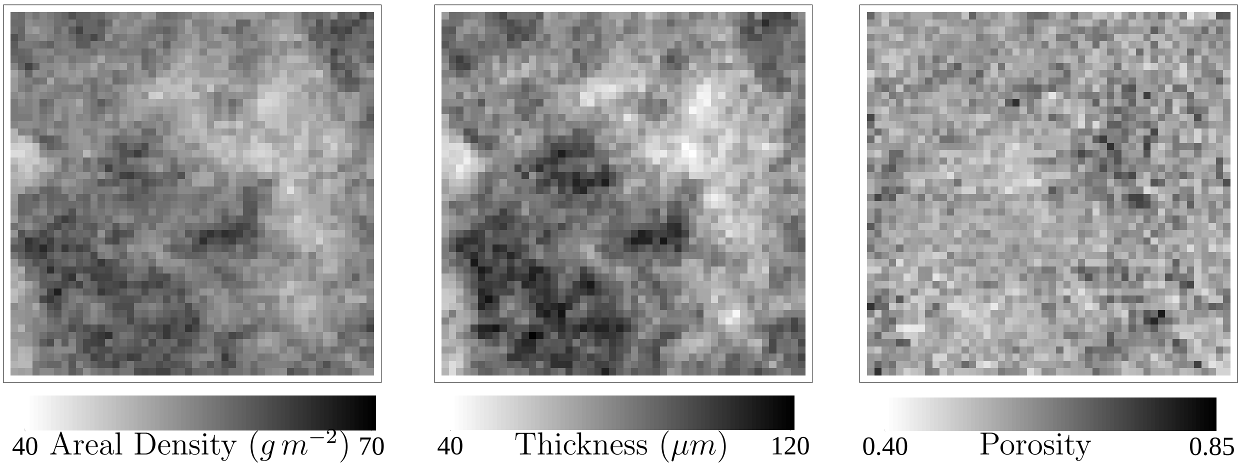

Figure 6: *In-plane distributions of areal density, thickness and porosity. Example shown is for a network formed from fibres with a mean areal density of 45* $g\,m^{-2}$. *Each image represents the same 50 mm* × 50 *mm zone.*