A LaTeX Book Skeleton

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Preface

The book root file `bookex.tex` gives a basic example of how to use LaTeX for preparation of a book. Note that all LaTeX commands begin with a backslash.

Each Chapter, Appendix and the Index is made as a `*.tex` file and is called in by the `include` command—thus `ch1.tex` is the name here of the file containing Chapter 1. The inclusion of any particular file can be suppressed by prefixing the line by a percent sign.

Do not put an `enddocument` command at the end of chapter files; just one such command is needed at the end of the book.

Note the tag used to make an index entry. You may need to consult eg Lamport's book [1] for details of the procedure to make the index input file; LaTeX will create a pre-index by listing all the tagged items in the file `bookex.idx` then you edit this into a `theindex` environment, as `index.tex`.

In the source file `bookex.tex` the line:
`usepackage[colorlinks,citecolor=blue,linkcolor=blue]hyperref`
includes hyperlinks and chooses colours for them.
Also, the following lines just before `begin{document}` will put a watermark on every page:

```
usepackage{draftwatermark}                    %%%   These four lines
SetWatermarkText{Draft not for circulation}%%%  put a watermark on all
SetWatermarkScale{3}                          %%%  pages. Omit them if
SetWatermarkColor[rgb]{0.9,0,0}               %%%  it is not needed.
```

Just comment them out with % if you do not want a watermark.

Chapter 1

Basics of Extension and Lifting Problems

To boldly go where no map has gone before

1.1 Existence problems

We begin with some metamathematics. All problems about the existence of maps can be cast into one of the following two forms, which are in a sense mutually dual.

The Extension Problem Given an inclusion $A \overset{i}{\hookrightarrow} X$, and a map $A \overset{f}{\to} Y$, does there exist a map $f^\dagger : X \to Y$ such that $f^\dagger$ agrees with f on A?

Here the appropriate source category for maps should be clear from the context and, moreover, commutativity through a candidate $f^\dagger$ is precisely the restriction requirement; that is,

$$f^\dagger : f^\dagger \circ i = f^\dagger|_A = f\,. \tag{1.1}$$

If such an $f^\dagger$[1] exists as specified in (1.1), then it is called an **extension** of f and is said to **extend** f. In any diagrams, the presence of a dotted arrow or an arrow carrying a ? indicates a pious hope, in no way begging the question of its existence. Note that we shall usually omit $\circ$ from composite maps.

The Lifting Problem Given a pair of maps $E \overset{p}{\to} B$ and $X \overset{f}{\to} B$, does there exist a map

$$f^\circ : X \to E, \text{with } pf^\circ = f? \tag{1.2}$$

[1] $\dagger$ suggests striving for perfection, crusading

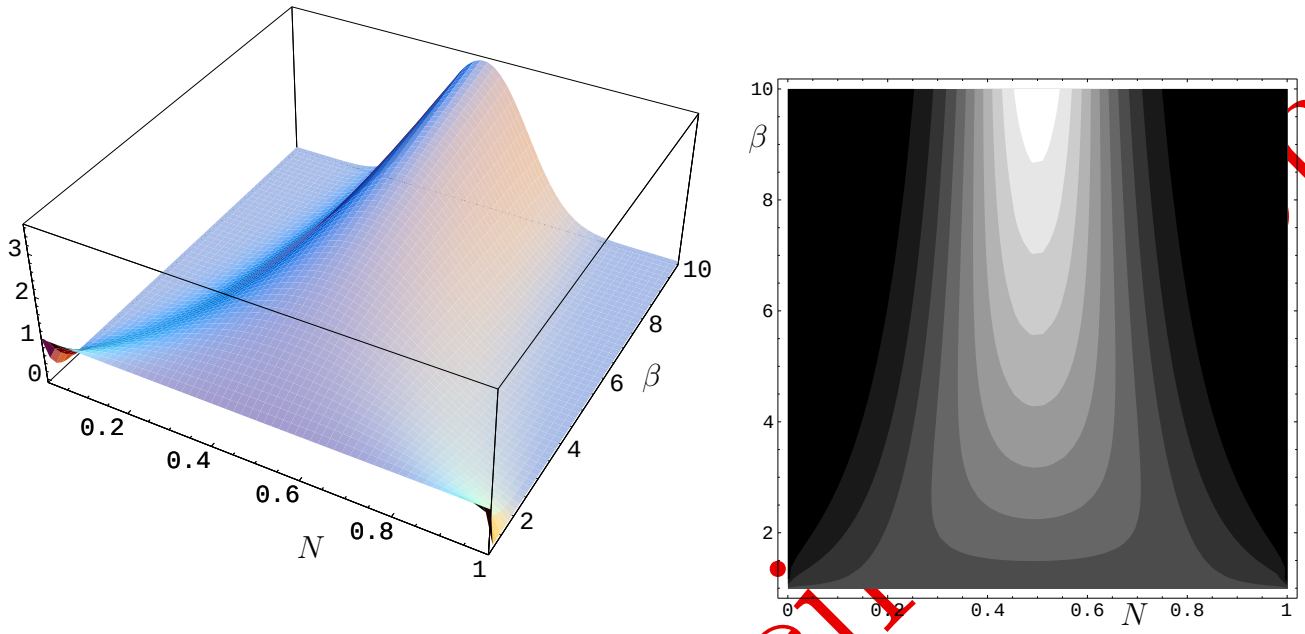
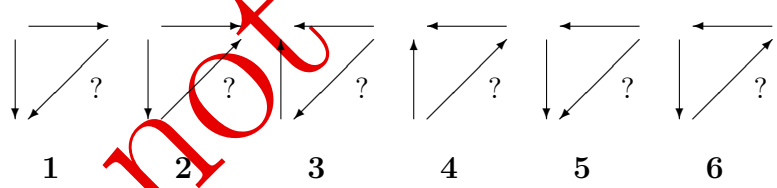


Figure 1.1: *The log-gamma family of densities with central mean* $< N >= \frac{1}{2}$ *as a surface and as a contour plot.*

That *all* existence problems about maps are essentially of one type or the other from these two is seen as follows. Evidently, all existence problems are representable by triangular diagrams and it is easily seen that there are only these six possibilities:



Chapter 2

Up to Homotopy is Good Enough

A log with nine holes— old Turkish riddle for a man

2.1 Introducing homotopy

In a topological category, a pair of maps $f, g : X \to Y$ which agree on $A \subseteq X$ is said to admit a **homotopy** H from f to g **relative to** A if there is a map

$$X \times \mathbb{I} \xrightarrow{H} Y : (x, t) \longmapsto H_t(x) \tag{2.1}$$

with $H_t(a) = H(a, t) = f(a) = g(a)$ for all $a \in A$, $H_0 = H(\ ,0) = f$, and $H_1 = H(\ ,1) = g$. Then we write $f \overset{H}{\sim} g\ (rel A)$.

If $A = \emptyset$ or A is clear from the context (such as $A = *$ for pointed spaces, *cf.* below), then we write $f \overset{H}{\sim} g$, or sometimes just $f \sim g$ and say that f and g are **homotopic**.

We can also think of H in (2.1) as either of:

- a 1-parameter family of maps

$$\{H_t : X \longrightarrow Y \ \mid\ t \in [0,1]\ \} \ \text{ with } \ H_0 = f \ \text{ and } \ H_1 = g\,; \tag{2.2}$$

- a curve c_H from f to g in the function space Y^X of maps from X to Y

$$c_H : [0,1] \longrightarrow Y^X : t \longmapsto H_t\,. \tag{2.3}$$

We call f **nullhomotopic** or **inessential** if it is homotopic to a constant map. Intuitively, we picture H as a continuous deformation of the *graph* of f into that of g. The following is an easy exercise.

n	S^n	R^n
1	1	1
2	1	1
3	1	1
4	1	∞
5	1	1
6	1	1
7	28	1
8	2	1
9	8	1
10	6	1
11	992	1
12	1	1
13	3	1
14	2	1
15	16256	1

Table 2.1: Numbers of distinct differentiable structures on real n-space and n-spheres

Proposition 2.1.1 *For all $A \subseteq X$, $\sim (relA)$ is an equivalence relation on the set of maps from X to Y which agree on A.*

Maps in the same equivalence class of $\sim (relA)$ are said to be **homotopic** $(relA)$.

Bibliography

[1] L. Lamport. **LaTeX A Document Preparation System** Addison-Wesley, California 1986.

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