

Question Sheet 8

- 1) Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$. Find $|A|$, $|B|$, $|A \cup B|$ and $|A \cap B|$. Hence verify that $|A \cup B| = |A| + |B| - |A \cap B|$.
- 2) In a class of 50 students, 30 study Basic, 25 study Pascal and 10 study both. How many study neither?
- 3) A survey of 100 students revealed
 - 18 like to eat chicken,
 - 40 like to eat beef,
 - 20 like to eat lamb,
 - 12 like to eat both chicken and beef,
 - 5 like to eat both chicken and lamb,
 - 4 like to eat both beef and lamb,
 - 3 like to eat all three.

Classify a students who does not like to eat any of the three kinds of meat as a non-meat eater.

- (i) How many students like to eat at least one of the kinds of meat?
 - (ii) How many non-meat eaters are there?
 - (iii) How many students like to eat only lamb?
- 4) A 16-bit string is a sequence of 16 symbols, each symbol is a 0 or 1. How many 16-bit strings are there?
- 5) How many subsets of

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$$

are there?

Can you see how the answers to this question and question 4 are connected? If you cannot, perhaps you should look at smaller examples, i.e. calculate the number of 3-bit strings and the numbers of subsets of a set with three elements. Try to construct a correspondence between the strings and subsets.

6) (i) In the lectures it was calculated how many integers between 1 and 1000 (inclusive) either started or ended in 7. How many integers between 1 and 1000 have 7 as at least one of their digits?

Hint: Think of the complement.

(ii) How many integers between 1 and 1000 (inclusive) have at least one odd digit?

(0 is considered to be an even digit.)

7) How many integers between 1 and 567 are divisible by either 3 or 5?

8) Guess (or even prove) a formula for $|A \cup B \cup C \cup D|$ where A, B, C and D are four finite sets.

Use your result to calculate how many integers between 1 and 1000 are divisible by any of 2, 3, 5 or 7?

9) Let U be the set of letters of the alphabet. Let $A = \{a, b, c, d, e\}$, $B = \{a, b\}$, $C = \{B, \phi\}$ and $D = \{a, b, \{a, b\}\}$.

Find $A \cap B$, $C \cap D$, $A \cap D$, $C \cap \mathcal{P}(A)$, and $D \cap \mathcal{P}(A)$. (Here $\mathcal{P}(A)$ is the power set of A .)

Indicate whether each of the following is true or false.

(i) $A \in \mathcal{P}(A)$,

(ii) $C \subset \mathcal{P}(A)$,

(iii) $D \subset \mathcal{P}(A)$,

(iv) $B \subset D$,

(v) $B \in D$,

(vi) $\{a, b\} \in C$.

10) Let $A = \{1, 2\}$, $B = \{a, b, c\}$, and $C = \{x, y\}$. List all the elements in each of the following sets.

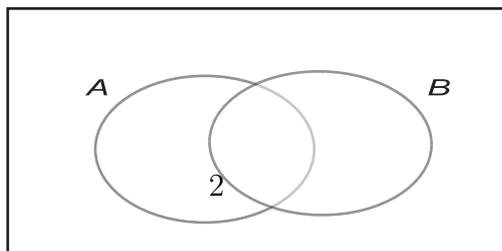
(i) $A \times C$,

(ii) $B \times B$,

(iii) $A \times B \times C$,

(iv) $(A \times A) \times (C \times C)$.

11) Draw the diagram



three times and shade the regions

$$(A \setminus B)^c, (B \setminus A)^c \text{ and } A \Delta B.$$

Assume you are told that

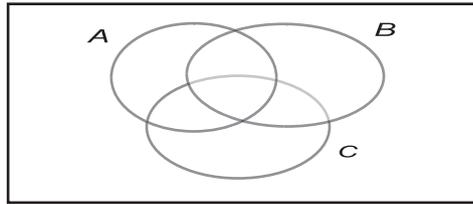
$$\begin{aligned} |U| = 14, & \quad |(A \setminus B)^c| = 12, \\ |A \cup B| = 9 & \quad \text{and } |A \Delta B| = 7. \end{aligned}$$

Find $|A|$ and $|B|$.

12) Out of 14 people, twelve said that it was not the case that they watched television but didn't listen to the radio. Also, for nine people it is not the case that they do not watch T.V. and do not listen to the radio. Finally, seven people either watch television or listen to the radio but do not do both.

How many people watch T.V.. and how many listen to the radio?

13) Draw the diagram



eight times and shade in the following regions.

$$\begin{aligned} C \cap B^c, & \quad (B \setminus C) \cap A, \\ A \Delta B, & \quad (A \cap C)^c, \\ (A \cap C) \Delta (A \cup C)^c, & \quad A \setminus (B \setminus C), \\ C^c \cap B, & \quad (B \cup C)^c. \end{aligned}$$

Assume you are told the following cardinalities.

$$\begin{aligned} |C \cap B^c| = 0, & \quad |(B \setminus C) \cap A| = 0, \\ |A \Delta B| = 6, & \quad |(A \cap C)^c| = 10, \\ |(A \cap C) \Delta (A \cup C)^c| = 7, & \quad |A \setminus (B \setminus C)| = 4, \\ |C^c \cap B| = 2, & \quad |(B \cup C)^c| = 7. \end{aligned}$$

Find $|A|$, $|B|$ and $|C|$.

14) For three sets $A, B, C \subseteq U$ you are told the following:

$$\begin{aligned} B &\subseteq A, & C^c \cup A &= U, \\ |C^c \cap A| &= 7, & |B \Delta C| &= 5, \\ |A \setminus (B \setminus C)| &= 6, & |A \setminus (C \setminus B)| &= 9, \\ |(B \cup C)^c| &= 5. \end{aligned}$$

Find $|A|$, $|B|$ and $|C|$.

15) A number of people were asked about their reading habits of national papers. The results were as follows.

- All readers of the Times read the Sun.
- Every person either reads the Sun or doesn't read the Mirror.
- 11 people read the Sun but don't read the Mirror.
- 8 people read either the Times or the Mirror but not both.
- 10 people read the Sun and either read the Mirror or do not read the Times.
- 14 people either read the Sun and not the Mirror or read both the Sun and Times.
- 9 people read neither the Times nor the Mirror.

Calculate how many people read each paper and how many people were surveyed.

16) If $|A| = 3$ and $|B| = 4$ what is $|\mathcal{P}(A \times B)|$?

17) About three sets $A, B, C \subseteq U$ you are told that

$$\begin{aligned} |U| &= 27, & |A \Delta (B \cap C)| &= 12, \\ |(A \cap B) \Delta (B \cap C)| &= 5, & |(B \cup C)^c| &= 5, \\ |C \Delta (B \Delta A)| &= 16, & |C \setminus (B \setminus A)| &= 12, \\ |A \cap (B \Delta C)| &= 8, & |C \cap (A \cup B)| &= 7. \end{aligned}$$

Find $|A|$, $|B|$ and $|C|$.