Question Sheet 6

1) Define a subset A of $\mathbb{N}$ recursively by

a) $1 \in A$,
b) If $x \in A$ then $x + 3 \in A$ and $x^2 \in A$,
c) only elements arising from a) and b) are in A.

(i) Prove that $52 \in A$,
(ii)Prove that $259 \in A$,
(iii) Prove that $2 \notin A$. (Hint use RAA from logic)

2) Let $E = \{a, b\}$. Write down all elements of E^* which have length at most 3.

3) Let E be a finite set with k elements, $k \geq 1$. Find a formula for the number of elements of E^* which have length n. Explain why your formula is correct.

4) Let $E = \{0, 1\}$. Define $A \subseteq E^*$ inductively by

a) $0, 1 \in A$,
b) if $x \in A$ then $0x1 \in A$,
c) only elements arising from a) and b) are in A.

(i) Show that $0001111 \in A$.
(ii) Is $00001111 \in A$?

5) Let $E = \{a, b\}$. Define $R \subseteq E^*$ inductively by

a) $ba \in R$,
b) If $x = yaa$ for some $y \in E^*$ then $yaba \in R$,
If $x = yba$ for some $y \in E^*$ then $xa \in R$.
c) Only words arising from a) and b) are in R.

List at least 10 words from R.

6) Let $E = \{p, q, r, (,), \neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$. Define $L \subseteq E^*$ by

a) $p, q, r \in L$,
b) If $\alpha, \beta \in L$ then so are
$\neg(\alpha)$
$(\alpha) \vee (\beta)$

$(\alpha) \wedge (\beta)$
$(\alpha) \rightarrow (\beta)$
$(\alpha) \leftrightarrow (\beta)$

c) Only words arising from a) and b) are in L

Prove carefully that $(\neg((p) \wedge (q))) \rightarrow ((p) \rightarrow (r)) \in L$.