Question Sheet 5

1) Let $U = \{1, 2, 3, 4, 5, 6\}$. In each of the following cases give examples of sets $A, B, ... \subseteq U$ such that the equality does **not** hold.

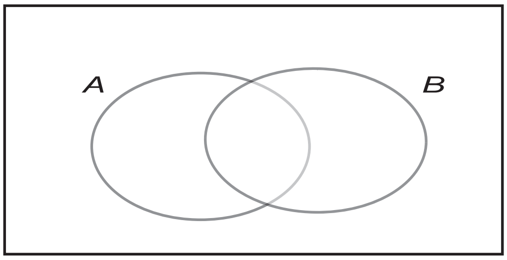
(i) $(A \cup B) \cap C^c = A \cup (B \cap C^c)$,

(ii) $A \cap B \cap C = A \cap B \cap (C \cup B)$,

(iii) $(A \cup B) \cap A^c = B$,

(iv) $(A \cup B)^c \cap C = (A^c \cap C) \cup (B^c \cap C)$.

2) Draw the diagram



six times and shade the regions

(i) $A \cup B^c$,

(ii) $A^c \cup B^c$,

(iii) $(A \cap B)^c$,

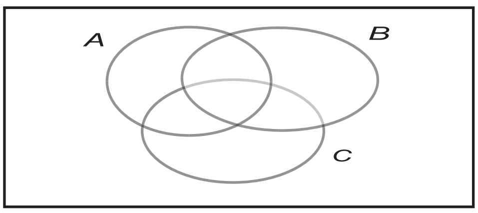
(iv) $A^c \cap B$,

(v) $A^c \cap B^c$,

(vi) $(A \cup B)^c$.

What equalities do you find?

3) Draw the diagram



six times and shade the regions

(i) $A \cup (B \cap C)$,
(ii) $A \cap (B \cup C)$,
(iii) $(A \backslash B) \backslash C$,
(iv) $(A \triangle B) \triangle C$,
(v) $(A \cap B) \cup (A \cap C)$,
(vi) $(B \cup A) \cap (C \cup A)$.

What equalities do you find?

4) Let $U = \mathbb{Z}$. Recall that, for a real number x, the notation $|x|$ denotes the size or magnitude of x and is given by

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

Consider the predicates

$$\begin{aligned} p_1(x) &: \ | x - 2| < 5, \quad \text{(which means } -5 < x - 2 < 5\text{)}, \\ p_2(x) &: \ | x + 2| > 4, \quad \text{(which means either } x + 2 > 4 \text{ or } x + 2 < -4\text{)}, \\ p_3(x) &: \ (x - 1)^2 \leq 16. \end{aligned}$$

Let A be the solution set of $p_1(x)$, so $A = \{x \mid p_1(x)\}$, B the solution set of $p_2(x)$, so $B = \{x \mid p_2(x)\}$, and C the solution set of $p_3(x)$, so $C = \{x \mid p_3(x)\}$.

(i) Find A, B and C in list form,

(ii) Find the solution set of $p_1(x) \wedge (\neg p_2(x))$ in list form and express this set in terms of A and B and the set operations $\cap, \cup$ and ^{c},

(iii) Find the solution set of $p_1(x) \vee p_3(x)$ in list form and express this set in terms of A and C and the set operations $\cap, \cup$ and ^{c}.

5) Let $A = \{x \in \mathbb{R} : x - 1 > 2 \text{ and } x < 4\}$ and let $B = \{x \in \mathbb{R} : 5 \leq x^2 \leq 20\}$. Show that if $x \in A$ then we have $x \in B$. Hence deduce that $A \subseteq B$.

6) Let $A = \{x \in \mathbb{R} : x^2 - 3x + 2 < 0\}$ and let $B = \{x \in \mathbb{R} : 1 < x < 2\}$. In the same way as in Question 6 show that $A \subseteq B$. Also show that $B \subseteq A$. Deduce that $A = B$.

7) Let A, B and C be subsets of a universal set U. Use **only** the Boolean laws of *Logic* to prove

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,

(ii) $A \cup (B^c \cup A^c) = (A^c \cup B)^c$,

Compare with Sheet 1 Question 8(i)

(iii) $A = A \cup A$,

Compare with Sheet 1 Question 8(iv)

(iv) $(A \cap B)^c = A^c \cup B^c$.

So do *not* use the Boolean laws of Set Theory.

8) Let A, and B be subsets of a universal set U. Using **only** the Boolean laws of *Set Theory* simplify the following expressions

(i) $(A^c \cap B^c)^c$,

(ii) $A \cup (A^c \cap B)$,

(iii) $A^c \cup (A^c \cup B)^c$,

(iv) $A \cap A$.

Hint: Start with $A = A \cap U$ and use another law on U. Also, look back at your solution to Sheet 1, Question 8(iii).

9) Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Determine the number of

(i) subsets of A,

(ii) subsets containing three elements,

(iii) subsets containing the elements 1 and 2,

(iv) subsets containing an even number of elements. (Consider 0 to be an even number.)