

Question Sheet 4

1) Let $A = \{1, 2, 3, 4\}$. Which of the following statements are true?

- (i) $\{1\} \subseteq A$,
- (ii) $6 \in A$,
- (iii) $\{2\} \in A$,
- (iv) $\{1, 4, 4\} \subseteq A$,
- (v) $4 \subseteq A$,
- (vi) $A \subseteq A$,
- (vii) $A \subset A$,
- (viii) $\{4, 3, 2\} \subset A$,
- (ix) $\phi \in A$,
- (x) $\phi \subseteq A$.

If a statement is false, give a reason.

2) Let $B = \{1, \{2, 3\}, 4\}$. Which of the following statements are true?

- (i) $4 \in B$,
- (ii) $2 \in B$,
- (iii) $\{2, 3\} \subseteq B$,
- (iv) $\{\{2, 3\}\} \subseteq B$,
- (v) $B = A$ (where A is as in question 1),
- (vi) The number of elements in B is 3.

If a statement is false, give a reason.

3) Write out in list form the following sets

- (i) ($U =$ set of letters of the English alphabet.)

$$\{x \mid x \text{ is the letter following a vowel}\},$$

- (ii) ($U = \mathbb{Z}$)

$$\{x \mid x^2 \leq 10\},$$

- (iii) ($U = \mathbb{Q}$)

$$\left\{ x \mid x = \frac{p}{q} \text{ where } p, q \in \mathbb{Z}, |p| \leq 2, \text{ and } 2 \leq q \leq 4 \right\}.$$

(Here $|p|$ means the magnitude of p so, for example, $|3| = 3$ while $|-3| = 3$.)

4) Write in predicate form the following sets, not forgetting to specify the Universal set U .

(i) $\{-3, -2, -1, 1, 2, 3\}$,

(ii) $\{0, 3, 8, 15, 24, \dots\}$,

(iii) $\{-\sqrt{2}, \sqrt{2}\}$,

(iv) $\{c, g, k, q, w\}$,

(v) $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$.

5) List all the subsets of $\{a, b, c, d\}$.

6) In the following predicates find examples of values for the variables that make the resulting proposition TRUE.

(i) ($U =$ set of letters of the alphabet.)

$$p(x) : x \text{ occurs in the phrase SET THEORY,}$$

(ii) ($U = \mathbb{R}$)

$$p(x, y) : x + y = 0,$$

(iii) ($U = \{1, 2, 3, 4, 5\}$)

$$p(x, y, z) : x^2 + y^2 = z^2,$$

(iv) ($U = \mathbb{Z}$)

$$p(x, y) : (x > y) \wedge (x^2 < y^2),$$

(v) ($U = \mathbb{N}$)

$$p(x, y) : (x + y = 0) \vee (xy \leq 1).$$

7) Repeat Question 6 but finding values for which the resulting propositions are FALSE.

8) Which of the following sets are finite and which are infinite?

(i) The set of all even negative integers,

(ii) $\{x : x \in \mathbb{Q} \text{ and } 0 < x < 1\}$,

- (iii) $\{x : x \in \mathbb{Z} \text{ and } x^2 < 9\}$,
- (iv) $\{x : x \in \mathbb{R} \text{ and } |x + 1| < 3\}$,
- (v) $\{x : x \in \mathbb{R} \text{ and } x^2 - 3x + 2 = 0\}$.

9) Let $U = \{1, 2, 3, 4, a, b, c\}$, $A = \{1, 3, a, c\}$, $B = \{1, 3, 4\}$ and $C = \{4, b, c\}$. Find the following sets in list form.

- (i) $B \cap C$,
- (ii) $A \cup B$,
- (iii) $(A \cup B)^c$,
- (iv) $A \cup (B \cap C)$,
- (v) $A^c \cap B^c$,
- (vi) $(B^c \cap A) \cup (B \cap A^c)$.