

Question Sheet 3

1) Using truth tables determine whether the following arguments are valid or invalid.

- (i) $p \rightarrow q, \neg q \vdash \neg p$
- (ii) $p \rightarrow q, q \vdash p,$
- (iii) $p \vee q, \neg p \vdash q,$
- (iv) $p \wedge q, q \vee r \vdash p \wedge r,$
- (v) $p \rightarrow q, q \rightarrow r \vdash p \rightarrow r,$
- (vi) $p \rightarrow q, q \rightarrow r \vdash (q \vee r) \rightarrow p,$
- (vii) $p \vee q, p \rightarrow r, q \rightarrow r \vdash r.$

2) Consider the argument

$$(p \rightarrow q) \wedge ((\neg q) \rightarrow (\neg s)), p \vee s, (\neg s) \rightarrow (q \wedge r) \vdash q \rightarrow s.$$

Find the truth-values for p, q, r and s such that the premises of this argument are true yet the conclusion false. Hence deduce that the argument is invalid.

3) Prove, using the rules of inference from A to D.S., that the following arguments are valid.

- (i) $s \rightarrow (p \rightarrow \neg s), s \vdash \neg p,$
- (ii) $A, A \rightarrow B, A \rightarrow C \vdash B \wedge C,$
- (iii) $(s \rightarrow p) \rightarrow (t \rightarrow s), t \rightarrow (s \rightarrow p), t \vdash p,$
- (iv) $(\neg A) \rightarrow C, (\neg C) \wedge D \vdash A \vee B,$
- (v) $p \rightarrow s, (\neg p) \rightarrow r, \neg r \vdash s,$
- (vi) $p, s \vee (\neg p), (\neg s) \vee t \vdash t,$
- (vii) $A, (A \vee B) \rightarrow (C \wedge D), (C \vee E) \rightarrow F \vdash D \wedge F,$
- (vi) $A, B, (A \vee C) \rightarrow (B \rightarrow C) \vdash B \wedge C.$

4) Prove, using $\vee E$ along with the other rules of inference, that the following arguments are valid. (Remember, there are always *two* subproofs when $\vee E$ is used.)

- (i) $(r \rightarrow t) \vee (r \rightarrow s), r \vdash t \vee s,$
(Hint: finish both subproofs with $\vee I$)
- (ii) $(a \wedge s) \rightarrow t, s, (t \vee b) \rightarrow c, s \rightarrow (a \vee b) \vdash c,$

- (iii) $a \vee b, s \vee (\neg a), s \vee (\neg b) \vdash s,$
 (iv) $(t \wedge p) \rightarrow s, (\neg s) \rightarrow (\neg q), (p \vee q) \wedge t \vdash s.$
 (Hint: start with $\wedge E$)

5) Prove, using C.P., that the following arguments are valid.

- (i) $p \rightarrow q, (q \wedge r) \rightarrow s, r \vdash p \rightarrow s,$
 (ii) $q \rightarrow p, q \vee r \vdash (\neg p) \rightarrow r,$
 (iii) $(p \wedge q) \rightarrow r, p \vdash q \rightarrow r,$
 (iv) $(p \wedge q) \rightarrow r, p, (q \rightarrow r) \rightarrow t \vdash t,$
 (v) $(\neg p) \vee (\neg q) \vdash (p \rightarrow (\neg q)) \wedge (q \rightarrow (\neg p)),$
 Hint: Apply C.P. twice, once with p and then with $q.$
 (vi) $(p \wedge q) \rightarrow r \vdash p \rightarrow (q \rightarrow r),$

Hint apply C.P. twice.

6) Prove, using R.A.A., that the following arguments are valid.

- (i) $s \rightarrow \neg t, p \rightarrow (s \wedge t) \vdash \neg p,$
 (ii) $s \rightarrow \neg t, p \rightarrow (s \wedge t), p \vee q \vdash q,$
 Hint: Try to use part (i). This shows that we can use RAA without negating the conclusion of the argument.
 (iii) $(p \wedge (\neg q)) \rightarrow q \vdash p \rightarrow q,$
 Hint: Try C.P. first and then RAA
 (iv) $s \rightarrow p, t \rightarrow s \vdash \neg((\neg p) \wedge t),$
 (v) $\neg((\neg p) \vee q) \vdash p.$

In Questions 7 – 10 symbolize the arguments using the notation given. Then use truth tables to determine the validity or invalidity of each of the arguments. Finally for the arguments that are valid give a proof of validity using the rules of inference.

7) Let

- A = Alice is elected Prime Minister,
 B = Betty is elected deputy Prime Minister,
 C = Carol is elected Chancellor.

If Alice is elected Prime Minister, then either Betty is elected deputy Prime Minister or Carol is elected Chancellor. Betty is elected deputy Prime Minister. Therefore, if Alice is elected Prime Minister, then Carol is not elected Chancellor.

8) Let

A = The light is on,

B = The switch is down,

C = The door is open.

If the switch is down then the light is on. If the switch is not down then the door is open. If the door is open then the light open. Therefore, the light is on.

(Hint: use RAA)

9) Let

A = The seeds are planted in April,

B = The flowers bloom in July,

C = The seed catalogue is correct.

If the seed catalogue is correct, then if the seeds are planted in April then the flowers bloom in July. The flowers do not bloom in July. Therefore, if the seed catalogue is correct then the seeds are not planted in April.

10) Let

E = Ed win first prize,

F = Fred wins second prize,

G = George is disappointed.

If Ed wins the first prize, then either Fred wins the second prize or George is disappointed. Fred does not win the second prize. Therefore, if George is disappointed, then Ed does not win first prize.