Question Sheet 2

1) Let r, s and t denote the propositions

r: Judith goes out for a walk,

s: The moon is out,

t: It is snowing.

Write the following in symbolic form

(i) If the moon is out and it is not snowing then Judith goes out for a walk,

(ii) It is not snowing if Judith goes out for a walk,

(iii) Judith goes out for a walk only if either the moon is out or it is snowing,

(iv) It is snowing if, and only if, either Judith does not go out for a walk or the moon is not out.

2) With r, s and t as in question 8, write the following propositions in English.

(i) $(r \wedge s) \leftrightarrow (\neg t)$,
(Be careful. Also write out $(\neg t) \leftrightarrow (r \wedge s)$ in English and compare.)

(ii) $t \rightarrow (s \rightarrow r)$,

(iii) $(t \wedge (\neg s)) \rightarrow \neg r$.

3) If A and B are false and C and D are true then what are the truth-values of the following propositions?

(i) $(\neg B) \rightarrow (\neg(C \vee D))$,

(ii) $(A \leftrightarrow B) \rightarrow (C \leftrightarrow (\neg D))$,

(iii) $A \rightarrow (B \rightarrow (C \rightarrow D))$,

(iv) $((A \rightarrow B) \rightarrow C) \rightarrow D$.

4) Determine the truth-values of each of the following implications.

(i) If $3 + 4 = 12$ then $3 + 2 = 6$,

(ii) If $3 + 4 = 12$ then $3 + 2 = 5$,

(iii) If $3 + 4 = 7$ then $3 + 2 = 6$,

(iv) If $3 + 4 = 7$ then e is the fifth letter of the alphabet.

5) Which of the following propositional forms are tautologies?

(i) $(p \vee q) \to (q \vee p)$,
(ii) $p \to ((p \vee q) \vee r)$,
(iii) $p \to (q \to (q \to p))$,
(iv) $((p \to q) \leftrightarrow q) \to p$,
(v) $(p \wedge q) \to (p \vee r)$,
(vi) $(p \to q) \leftrightarrow (q \to p)$.

6) Show by truth tables that $p \wedge (q \vee r)$ and $(p \wedge q) \vee r$ are not equivalent and that $(p \wedge (q \vee r)) \to ((p \wedge q) \vee r)$ is a tautology.

7) Prove, using truth tables

(i) $p \to q \equiv (\neg q) \to (\neg p)$,
(ii) $\neg(p \to q) \equiv p \wedge (\neg q)$,
(iii) $p \to q \equiv (\neg p) \vee q$,
(iv) $p \to q \equiv (p \wedge (\neg q)) \to O$.
(v) Using part (iii), find a form equivalent to $p \leftrightarrow q$ which only uses the $\neg, \wedge$ and $\vee$ connectives.

(It is because of parts (iii) and (v) that the Boolean laws of logic can be written without $\to$ or $\leftrightarrow$.)

8) Prove, using question 7(iii) and other results from propositional logic, that

$$r \to (p \to q) \equiv (p \wedge r) \to q.$$

So, do *not* use truth tables.

9) The "exclusive or" connective $\veebar$ is defined such that $p \veebar q$ is true when either p or q are true *but not both.*

(i) Construct the truth table for $p \veebar q$,
(ii) In words we might say that "$p \veebar q$ is true" if, and only if, "$p \vee q$ is true and it is not the case that $p \wedge q$ is true". So we might think that

$$p \veebar q \equiv (p \vee q) \wedge (\neg(p \wedge q)).$$

Use truth tables to show this is so.

(iii) Use part (ii) along with the Boolean Laws and any results needed from Sheet 1, Question 8, to show that

$$p \veebar p \equiv O, \qquad p \veebar I \equiv \neg p \qquad \text{and} \qquad p \veebar O \equiv p.$$

Do *not* use truth tables to show these hold.

(iv) Are either of the following true for all p, q?

$$p \veebar (q \wedge r) \equiv (p \veebar q) \wedge (p \veebar r)\,,$$
$$p \wedge (q \veebar r) \equiv (p \wedge q) \veebar (p \wedge r)$$

Hint: Use truth tables.

Maybe you can now see why we use the "*inclusive or*" $\vee$ in logic.

(v) Start with part (ii) and use DeMorgan's laws along with the distributive laws to show that

$$p \veebar q \equiv (p \wedge (\neg q)) \vee ((\neg p) \wedge q)\,.$$

Use this result to prove the valid equivalence in part (iv) without truth tables. (Hint: always start with the most complicated side.)