

Question Sheet 10

1) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 2 & 0 \\ 0 & -1 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & 1 & -5 \\ 7 & 1 & 0 \end{pmatrix}$.

Where possible, evaluate the following

- (i) AB ,
- (ii) AC ,
- (iii) $AB + AC$,
- (iv) $B + C$,
- (v) $A(B + C)$, (now compare (iii) and (v))
- (vi) BA ,
- (vii) BC
- (viii) $A^2 (= AA)$,
- (ix) B^2 .

2) Find the inverses, where possible, of the following matrices.

- (i) $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$,
- (ii) $\begin{pmatrix} 6 & 10 \\ 1 & 2 \end{pmatrix}$,
- (iii) $\begin{pmatrix} 6 & -10 \\ -3 & 5 \end{pmatrix}$,
- (iv) $\begin{pmatrix} 6 & 4 & -5 \\ 11 & 3 & 1 \end{pmatrix}$,
- (v) $\begin{pmatrix} 1 & 4 & -1 \\ 0 & 3 & 5 \\ 2 & 3 & -10 \end{pmatrix}$.

3) Use the inverse from question 2(ii) to solve the following system of equations.

$$\begin{aligned} 6x_1 + 10x_2 &= 4 \\ x_1 + 2x_2 &= -3. \end{aligned}$$

4) Solve the following systems of equations using Gaussian elimination on matrices.

(i)

$$\begin{aligned}x - 10y &= 2 \\ 3x + 4y &= 6,\end{aligned}$$

(ii)

$$\begin{aligned}2x_1 + 2x_2 - x_3 &= 13 \\ 5x_1 + 3x_2 + 2x_3 &= 8 \\ 2x_1 + x_2 + x_3 &= 5,\end{aligned}$$

(iii)

$$\begin{aligned}-x + 2y + 2z &= 2 \\ x + 3y + z &= -2 \\ -x + y + z &= 4.\end{aligned}$$

(Remember, always check your answer by substituting back in.)

5) 512K of memory is to be partitioned into three segments. The second segment is twice the length of the third and 3K more than the first. What is the length of each segment?

6) Let $A = \begin{pmatrix} 1 & 4 & -1 \\ 0 & 3 & 5 \\ 2 & 3 & -10 \end{pmatrix}$ and $B = \begin{pmatrix} 10 & -7 & 0 & 1 \\ 2 & 0 & 1 & -5 \\ 4 & 3 & 0 & -5 \end{pmatrix}$.

If there exists a matrix X which satisfies $AX = B$ what is the size of X ?

Find X .