Question Sheet 1

1) For each of the following decide whether it is a proposition or not, and if it is, indicate whether it is true or not.

(i) 15 is a positive number,

(ii) The Earth is flat,

(iii) $x^2 \geq 0$,

(iv) $x^2 \geq 0$ for every real number x,

(v) Shakespeare wrote the play Hamlet,

(vi) Hamlet is the best play ever written,

(vii) The next sentence is true,

(viii) The previous sentence is false

2) Let p, q and r denote the propositions

p: The bats are flying around,

q: The vampires are out of their coffins,

r: It is daytime.

Write the following in symbolic form

(i) It is nighttime and the vampires are in their coffins,

(ii) The bats are flying around and either the vampires are out of their coffins or it is daytime,

(iii) It is not the case that either it is daytime or the vampires are in their coffins,

(iv) Either it is daytime and the bats are flying around or the bats are not flying around and the vampires are out of their coffins.

3) With p, q and r as in question 2, write out the following propositions in words.

(i) $q \wedge (p \vee (\neg r))$,

(ii) $\neg((\neg r) \wedge (\neg p))$

4) If A and B are false and C and D are true what are the truth-values of the following propositions?

(i) $(\neg A) \wedge (C \vee (\neg B))$,

(ii) $(C \wedge (\neg D)) \vee (A \vee (\neg(\neg(B)))$.

5) Write out the truth tables for the following propositional forms

(i) $(\neg p) \vee (q \wedge (\neg r))$,

(ii) $(p \vee (\neg q)) \wedge ((\neg p) \wedge q)$,

(iii) $((\neg p) \vee (\neg q)) \vee (p \wedge q)$,

(iv) $[(p \wedge (\neg q)) \vee (q \wedge (\neg r))] \vee (r \wedge (\neg p))$.

Which of these are tautologies and which are contradictions?

6) Prove, using truth tables

(i) $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$,

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$,

(iii) $p \vee O \equiv p$,

(iv) $p \wedge I \equiv p$,

(v) $p \vee (\neg p) \equiv I$,

(vi) $p \wedge (\neg p) \equiv O$.

7) NAND gates are commonly used in electronics. If the inputs to a NAND gate are p and q, the output is given by $p \dagger q = \neg(p \wedge q)$. (There is no standard notation for NAND so the † is just an *ad hoc* notation. From this you can see that NAND simply means "Not AND".)

(i) Give the truth table for $p \dagger q$.

(ii) Show, using truth tables, that $\neg p \equiv p \dagger p$.

(iii) Start from $p \wedge q \equiv \neg(\neg(p \wedge q))$, the definition of † and then (ii), to show that $p \wedge q \equiv (p \dagger q) \dagger (p \dagger q)$.

(iv) Find a way to write $p \vee q$ which only uses †.

8) Prove, using the Boolean laws of logic,

(i) $q \vee ((\neg p) \wedge (\neg q)) \equiv \neg(p \wedge (\neg q))$,

(ii) $(P \vee Q \vee R) \wedge (P \vee (\neg Q) \vee R) \equiv P \vee R$,

(iii) $p \wedge p \equiv p$,

Hint: $p \equiv p \wedge I$ and use another law on I.

(iv) $p \vee p \equiv p$,

Hint: $p \equiv p \vee O$ and use another law on O.

(v) $(r \wedge t) \wedge (t \vee r) \equiv r \wedge t$.

Hint: use a distributive law on the left hand side and then apply results from earlier parts of this question.

(vi) $p \vee I \equiv I$.

Hint: start with $I \equiv p \vee (\neg p)$ and use another law on $\neg p$ to introduce I on the right-hand side.

(vii) $p \wedge O \equiv O$.

(viii) $\neg I \equiv O$ and $\neg O \equiv I$.