Symbolising Quantified Arguments

1. (i) Symbolise the following argument, given the universe of discourse is $U =$ set of all animals.

Animals are either male or female.
Not all Cats are male,
Therefore, some cats are female.

(ii) $U =$ set of all animals.

Not all animals are cats,
An Animal must be a cat if it has a tail,
Therefore, not all animals have tails.

(iii) $U =$ set of all animals.

All animals are either male or female,
Tom is not female,
Therefore, Tom is male.

(iv) Symbolise the following argument, given the universe of discourse is $U =$ set of all elephants.

Elephants are either pink or grey,
All pink elephants can fly,
No elephants can fly,
Therefore, all elephants are grey.

(v) Repeat part (i) but with the Universe replaced by $U =$ set of all animals.

Choosing an appropriate Universe will often simplify the symbolising of an argument. But remember to choose the Universe to contain all objects to which the argument refers.

(vi) What would be appropriate universe for each of the following?

(a) Animals are either male or female,

(b) All cats eat meat, .

(c) Some cats eat mice.

(d) Some mice are eaten by cats.

(e) Some mice are eaten by cats while some cats are nibbled by mice.

Note In (c), if U is the set of all cats, $Cx = x$ is a cat and $mx = x$ eats mice, the sentence becomes $\exists x : Cx \wedge mx$. But we could change the universe to U, the set of all cats and all mice (or even the set of all animals). This time we need a predicate with two variables, $e(x, y) = x$ eats y, along with $Cx = x$ is a cat and $Mx = x$ is a mouse. We then symbolise (c) as $\exists x, \exists y : Cx \wedge My \wedge e(x, y)$. How would (d) be rewritten with this larger universe?

(vi) $U =$ set of all animals.

If some cat is male then some dog is female.
Tom is a male cat.
Therefore, not all dogs are male.

Proving Quantified Arguments

1) Give proofs of validity for the following arguments.

(i) $\forall x : Mx \vee Fx,\ \neg(\forall x : Cx \to Mx)\ \vdash\ \exists x : Cx \wedge Fx.$

(ii) $\forall x : \neg p(x)\ \vdash\ \neg(\exists x : p(x))$

(iii) $\neg(\forall x : Cx),\ \forall x : Tx \to Cx\ \vdash\ \neg(\forall x : Tx),$

(iv) $\forall x : Px \vee Gx,\ \forall x : Px \to Fx,\ \neg(\exists x : Fx)\ \vdash\ \forall x : Gx.$

2) Give proofs of validity for the following.

(i) $\exists x : p(x) \vee q(x)\ \vdash\ (\exists x : p(x)) \vee (\exists x : q(x)),$

(ii) $(\exists x : p(x)) \vee (\exists x : q(x))\ \vdash\ \exists x : p(x) \vee q(x),$

Have used different symbols u_0 and u_1 since $p(x)$ and $q(x)$ might be true for different objects in the universe.

(iii) $(\forall x : p(x)) \vee (\forall x : q(x))\ \vdash\ \forall x : p(x) \vee q(x),$

(iv) $\forall x : p(x) \to q(x),\ \neg(\exists x : q(x))\ \vdash\ \neg(\exists x : p(x)),$

(v) $(\exists x : p(x)) \to (\exists x : q(x)),\ \exists x : p(x)\ \vdash\ \exists x : q(x)$

(vi) $\forall w : pw \to qw\ \vdash\ (\forall x : qx \to rx) \to (\forall y : py \to ry),$

Note, the bound variables do not have to be x, and don't have to be the same throughout an argument.

(vii) $\forall x : ax \to (bx \vee rx),\ \neg(\exists x : rx)\ \vdash\ \forall x : ax \to bx.$

(viii) $\neg(\exists x : p(x) \vee q(x))\ \vdash\ \forall x : \neg p(x).$

(ix)

$$\begin{aligned}\forall x, \exists y &: p(x,y) \rightarrow r(x,y), \ \exists x, \forall y : r(x,y) \rightarrow s(x,y), \\ \forall x, \forall y &: p(x,y) \ \vdash \ \exists x, \exists y : s(x,y)\end{aligned}$$

Invalid arguments (Not covered in 2004, but there should be enough material in the web notrd for thes to be attempted.)

Show that the following arguments are invalid.

1) $\forall x : p(x) \vee q(x) \ \vdash \ (\forall x : p(x)) \vee (\forall x : q(x))$

2) $\forall x : p(x) \ \vdash \ \exists x : p(x)$,

3) $\forall x : p(x) \rightarrow r(x) \vee s(x), \ \exists x : r(x) \ \vdash \ \exists x : s(x) \vee p(x)$.

4) $(\exists x : p(x)) \rightarrow (\exists x : q(x)), \ \forall x : q(x) \ \vdash \ \exists x : p(x)$

5) $(\forall x : p(x) \rightarrow q(x)) \rightarrow (\exists x : p(x)), \ (\forall x : q(x) \rightarrow p(x)) \rightarrow (\exists x : q(x))$
$\vdash \ \exists x : p(x) \wedge q(x)$,

6) $\forall x : p(x) \rightarrow r(x) \vee s(x), \ \neg r u_0, \ \exists x : p(x) \ \vdash \ su$

7) Let $U =$ set of all animals.

(i) The only animals with tails are cats.
Tom is a cat.
Therefore, Tom has a tail.

(ii) All animals have tails if they are cats,
Jerry is not a cat,
Therefore, Jerry does not have a tail.

(iii) If cats exist then dogs exist.
Not all animals are dogs,
Therefore, some animals are not cats.