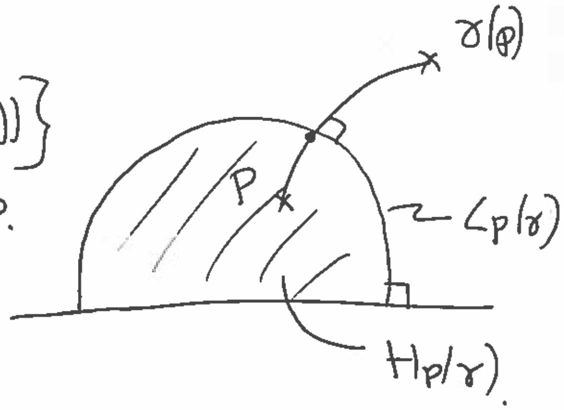
(4)

4. $L_p(\gamma)$ divides $\mathbb{H}$ into two half-planes. Let

$$H_p(\gamma) = \{ z \in \mathbb{H} \mid d_{\mathbb{H}}(z,p) < d_{\mathbb{H}}(z, \gamma(p)) \}$$

be the half-plane that contains p.



5. $$D(p) = \bigcap_{\gamma \in \Gamma \setminus \{id\}} H_p(\gamma).$$

<u>Theorem</u> Let Γ be a fuchsian group. Let p be as in the technical lemma. Then $D(p)$ is a fundamental domain. (We call $D(p)$ a <u>Dirichlet region</u>.)
Moreover, if $\text{Area}_{\mathbb{H}}\, D(p) < \infty$ then $D(p)$ is a convex hyperbolic polygon. (In this case, we call $D(p)$ a <u>Dirichlet polygon</u>.)

See clarification

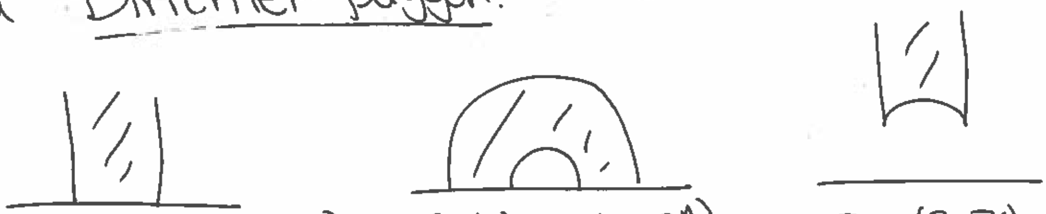
Clarification to Fri Wk 7 lecture

Thm Let Γ be a Fuchsian group. Let p be as in the technical lemma. Then $D(p)$ is a fundamental domain.

Definition We call $D(p)$ a <u>Dirichlet region</u>.

Remark $D(p)$ will be a convex set (it's the intersection of convex sets, so must be convex), but may or may not have finitely many sides.
If it does have finitely many sides then we call $D(p)$ a <u>Dirichlet polygon</u>.

Examples:



(integer translations) (dilations by 2^n) $PSL(2,\mathbb{Z})$

these have infinite area — this has finite area.

Remark There are many conditions which imply $D(p)$ has finitely many sides. For example:

$$\text{Area}_{\mathbb{H}}\, D(p) < \infty \implies D(p) \text{ has finitely many sides.}$$

But note that $D(p)$ can have finitely many sides (& so be a Dirichlet polygon) even if it has infinite hyperbolic area