The University of Manchester

MATH32051

Hyperbolic Geometry

In-class Examination
40 minutes

1st November 2019

15:00 – 15:40

This test will count 20% towards your final mark.

Answer ALL of the questions in the answer books provided. A total of 26 marks are available.

No prepared notes of any kind or mathematical tables are to be brought into the examination room.

Electronic calculators may be used, provided that they cannot store text.

Q1 (i) State, without proof, the Gauss-Bonnet Theorem for a hyperbolic triangle. [2 marks]

(ii) Consider the tiling of the Poincaré disc by identical regular hyperbolic triangles in Figure 1 below. Calculate the internal angle of the triangle. Hence calculate the hyperbolic area of the triangle. [4 marks]

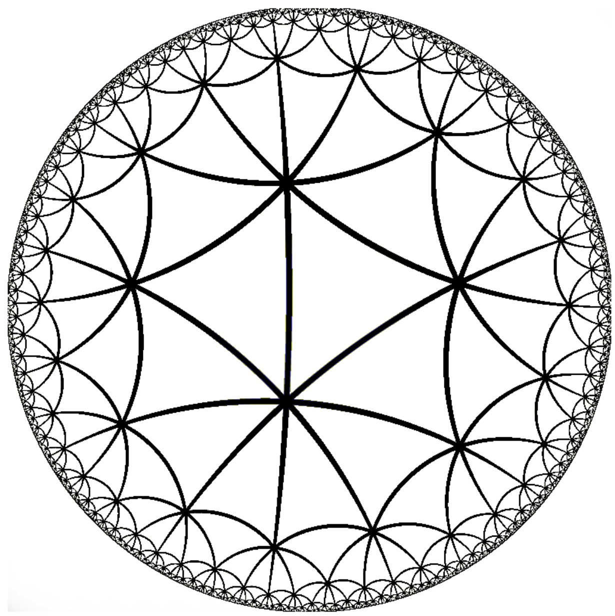

Figure 1: See Q1(ii).

(iii) Recall that a triangle Δ is said to be *ideal* if all the vertices of Δ are on the boundary.

Sketch a tiling of the Poincaré disc by ideal triangles. [4 marks]

Q2 (i) Define $\tau(\gamma)$ for a Möbius transformation γ of $\mathbb{H}$.

Briefly explain (without proof) how to use $\tau(\gamma)$ to classify γ as either a hyperbolic, parabolic or elliptic Möbius transformation of $\mathbb{H}$. [4 marks]

(ii) Consider the Möbius transformation

$$\gamma(z) = \frac{2z - 3}{5z - 3}.$$

Calculate $\tau(\gamma)$ and hence decide whether γ is hyperbolic, parabolic or elliptic. Check your answer by explicitly calculating the fixed points of γ. [4 marks]

(iii) Suppose that γ_1 is a Möbius transformation of $\mathbb{H}$ and is not the identity. Suppose that γ_1 has fixed points at 0 and ∞. Show that γ_1 is a dilation.

Let γ be a Möbius transformation of $\mathbb{H}$ with two fixed points at $a, b \in \mathbb{R}$. Prove that γ is conjugate to a dilation. [8 marks]

END of exam paper