The University of Manchester

MATH20142

Complex Analysis
Coursework test

In-class Examination
40 minutes

11th March 2020

10:00am – 10:40am

This test will count 20% towards your final mark.

Answer ALL of the questions in the answer books provided. A total of 20 marks are available.

No prepared notes of any kind or mathematical tables are to be brought into the examination room.

Electronic calculators may be used, provided that they cannot store text.

Q1 (i) Determine the radius of convergence of the following power series. You may use any standard results from the course provided that you state them clearly.

$$\sum_{n=1}^{\infty} \frac{(4i)^n}{n} z^n.$$

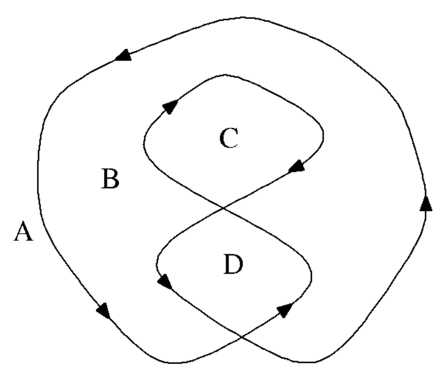
[4 marks]

(ii) Recall that

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}, \quad \sinh z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}.$$

What is the coefficient of the z^6 term in $\sin z \sinh z$? [4 marks]

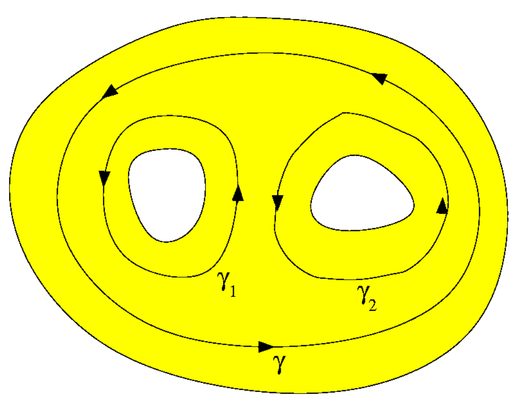
Q2 (i) Consider the closed path γ illustrated below. Calculate (by eye) the winding number of γ in the regions A, B, C, D.



[4 marks]

(ii) State, without proof, the Generalised Cauchy Theorem. [2 marks]

(iii) Suppose that f is holomorphic on the domain D (the shaded region in the figure below).



Suppose that $\int_{\gamma_1} f = 1 + 2i$ and $\int_{\gamma_2} f = 3 + 4i$. Use the Generalised Cauchy Theorem to calculate $\int_{\gamma} f$. [6 marks]

END of exam paper