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38th Conference on Stochastic Processes and their Applications
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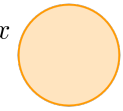
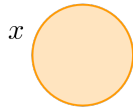
Growth-fragmentation models, random and deterministic

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Joint work with Jean Bertoin

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A finite-activity model



- $\langle \mu_t, f \rangle = \int f(x) \mu_t(dx) = \mathbb{E} \left[\sum_{\substack{u \text{ particle} \\ \text{alive at } t}} f(\text{size}(u)) \right]$
- $K(dy) = \lambda \nu(dy)$

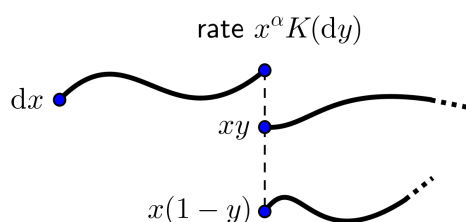
The pure fragmentation equation

$$\partial_t \langle \mu_t, f \rangle = \left\langle \mu_t, \int_{[\frac{1}{2}, 1)} \{f(xy) + f(x(1-y)) - f(x)\} K(dy) \right\rangle,$$
$$f \in C_c^\infty(0, \infty),$$
$$\mu_0 = \delta_1$$

The growth-fragmentation equation

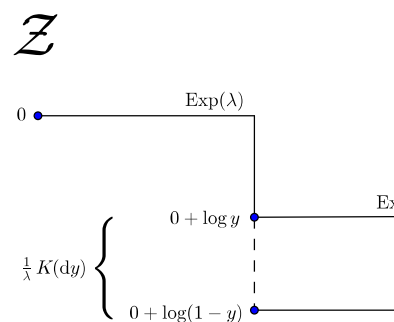
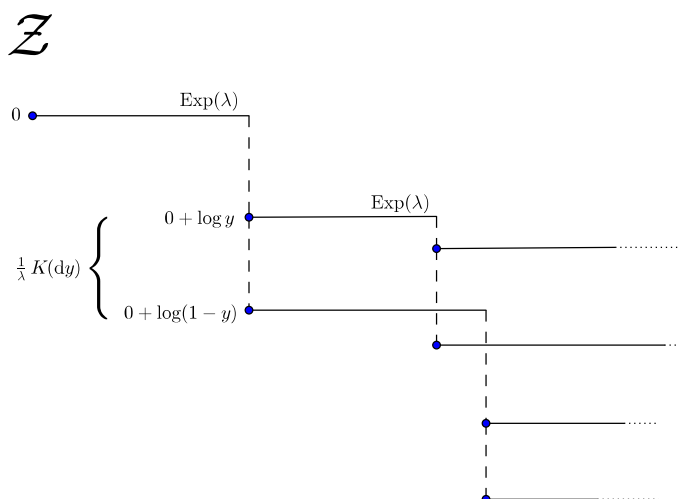
$$\partial_t \langle \mu_t, f \rangle = \left\langle \mu_t, x^\alpha \left[axf'(x) + \int_{[\frac{1}{2}, 1)} \{f(xy) + f(x(1-y)) - f(x) + (1-y)xf'(x)\} K(dy) \right] \right\rangle,$$

- $a \in \mathbb{R}$
- Require only $\int (1-y)^2 K(dy) < \infty$ (asymmetric children)
- $\alpha \in \mathbb{R}$



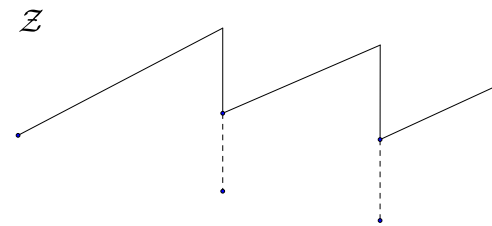
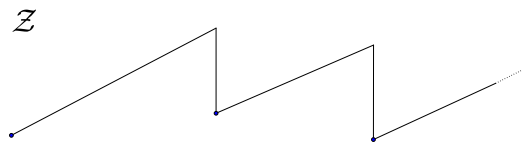
Fragmentation processes, $\alpha = 0$

The finite-activity model, on log-scale...
...is a compound Poisson process with immigration



Compensated fragmentation processes, $\int (1-y)^2 K(dy) < \infty$

- We can build the general (log-scale) model.
- Pick a Lévy process with negative jumps
- At each jump of size $\log y$, immigrate a new particle at relative position $\log(1-y)$



If $\int (1-y) K(dy) < \infty$, it is an ‘exchangeable fragmentation’ with growth/erosion.

Solutions

Theorem ($\alpha = 0$)

The measure

$$\langle \mu_t, f \rangle = \mathbb{E}_{\delta_1} \left[\sum_{\substack{u \text{ particle} \\ \text{alive at } t}} f(\text{size}(u)) \right]$$

is the *unique solution* with $\mu_0 = \delta_1$.

Theorem ($\alpha \neq 0$)

Under a Malthusian condition,

- An analogous solution (μ_t) with $\mu_0 = \delta_1$ exists.
- There is another solution (γ_t) , with $\gamma_0 = 0$ but $\gamma_t \neq 0$ for all $t > 0$.

Other work and open questions

- Asymptotics
- Non-existence (absent Malthusian condition)
- Spines of the fragmentation

- Minimal solutions
- Biased mass functions
- Process variant of 'starting from zero'
- Many other questions about compensated fragmentations

Further reading



J. Bertoin

Compensated fragmentation processes and limits of dilated fragmentations

[hal-00966190v2](#)



J. Bertoin, A. R. Watson

Probabilistic aspects of critical growth-fragmentation equations

[arXiv:1506.09187 \[math.PR\]](#)

Thank you!