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Asymptotic Rejection of a Class of Periodic Disturbances in Nonlinear Output Feedback Systems*

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Abstract

Asymptotic rejection of periodic disturbances whose basic wave patterns can be described by odd functions is considered. This class of disturbances covers asymmetric wave forms in the half period such as sawtooth wave form, as well as disturbances with symmetric half-period wave forms such as sinusoidal disturbances and triangular disturbances. The systems considered in this paper can be transformed to the nonlinear output feedback form. The amplitude and phase of the disturbances are unknown. The newly proposed half-period integration technique is used to estimate the unknown disturbances in the systems, together with observer design techniques to deal with nonlinearity. The proposed control design with the disturbance estimation asymptotically rejects the unknown disturbance, and ensures the overall stability of the system.

1 Introduction

One of the common deterministic disturbances considered for asymptotic rejection in dynamic systems is sinusoidal disturbance [1, 2, 3, 4], and very often the internal

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model principle is used to generate the desired feedforward control input to reject the unknown disturbances. A related problem is formulated as output regulation, where the output measurement contains the unknown disturbance [5, 6, 7]. For sinusoidal disturbances, the disturbance can be easily modelled as an output of a dynamic model and therefore a corresponding internal model can be designed [5, 6, 7, 8]. However, many periodic signals are not sinusoidal, and therefore can not be modelled as an output of a linear exosystem. Recently, a half-period integration method is proposed to characterise general periodic disturbances, and is used for asymptotic rejection of a class of general disturbances which have symmetric wave form in the half of the period, such as symmetric triangular wave, square wave. The half-period integration based disturbance rejection is demonstrated in a class of nonlinear output feedback systems which can be transformed to the output feedback form.

In this paper, the half-period integration technique is further explored for asymptotic rejection of more general disturbances than the disturbances with symmetric half-period wave form. We only require that the basic wave form of the disturbances is an odd function, without the requirement of a symmetric wave form in the half period. A set of results for odd-function periodic disturbances are obtained and they are applied in control design for asymptotic disturbance rejection in nonlinear output feedback systems. With the information of the basic wave form, the phase and amplitude can be estimated by the proposed design. With the estimated disturbance, control design is then proposed for disturbance rejection with stability. The nice property of the estimate ensures the asymptotic rejection of general periodical disturbances under the proposed control for nonlinear systems in the output feedback form. Two examples are included to demonstrate the proposed estimation and control algorithm for sawtooth disturbances whose half period wave pattern is asymmetric.

2 Problem Formulation

Consider a single-input-single-output nonlinear system which can be transformed into the output feedback form

$$\dot{\mathbf{x}} \quad = \quad \mathbf{A}_c\mathbf{x} + \boldsymbol{\psi}(y) + \mathbf{b}(u - w)$$

$$y = \mathbf{C}\mathbf{x} \quad (1)$$

with

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T, \mathbf{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_\rho \\ \vdots \\ b_n \end{bmatrix}$$

where $\mathbf{x} \in R^n$ is the state vector, $u \in R$ is the control, $\boldsymbol{\psi}$, is a known nonlinear smooth vector field in R^n with $\boldsymbol{\psi}(\mathbf{0}) = \mathbf{0}$, $w \in R$ is a periodical disturbance.

Assumption 1: The disturbance can be expressed as

$$w(t) = a w_b(t + \phi) \quad (2)$$

where the unknown constants a and ϕ are referred to as amplitude and phase, and $w_b(t)$ is a known function satisfying the following

A1.1 $w_b(t + T) = w_b(t)$ with T, the known period.

A1.2 $w_b(-t) = -w_b(t)$.

A1.3 There exists a δ, $0 < \delta < \frac{T}{4}$, such that for $t \in (0, \delta)$, $w_b(t) > K_b t^l$, and for $t \in (\frac{T}{2} - \delta, \frac{T}{2})$, $w_b(t) > K_b(\frac{T}{2} - t)^l$, with K_b and l being positive reals, and $w_b(t) \geq K_b \delta^l$ for $t \in [\delta, \frac{T}{2} - \delta]$.

A1.4 For $t \in [0, T)$, the function $w_b(t)$ is bounded, and has bounded derivatives except at a finite number of discontinuous points, where the left and right derivatives exist and are bounded.

From A1.1 and A1.2, we have $w_b(\frac{T}{2}) = w_b(\frac{T}{2} - T) = w_b(-\frac{T}{2}) = -w_b(\frac{T}{2})$. Hence we can conclude $w_b(\frac{T}{2}) = 0$.

Remark 1: The disturbances classified in Assumption 1 are more general than the disturbances considered in [9]. Assumption 1 removes the the condition $w_b(\frac{T}{4} - t) = w_b(\frac{T}{4} + t)$ which is required in [9]. It allows the periodic functions with asymmetric wave forms such as sawtooth waves in addition to the periodic functions with symmetric wave forms such as sinusoidal functions, square waves and triangular waves with zero means.

The problem considered in this paper is to design a dynamic feedback control law u so that the overall system is stable and the unknown disturbance $w(t)$ is asymptotically rejected in the sense that $\lim_{t\to\infty} y(t) = 0$. The asymptotic rejection algorithm proposed here adopts an indirect approach, ie, the disturbance is estimated and then the estimated disturbance is used for control design for disturbance rejection. For the control design of nonlinear systems, the following assumption is also needed.

Assumption 2: The system has stable zero dynamics, ie, the zeros of polynomial $\mathbf{B}(s) = \sum_{i=\rho}^{n} b_i s^{n-i}$ have negative real parts.

Remark 2: When $\boldsymbol{\psi}(y)$ in (1) is a linear function of y, the system is linear, and the zeros $\mathbf{B}(s)$ are the zeros of the linear system transfer function. For nonlinear systems, there are very few results in literature for control design with unstable zero dynamics. Even for the disturbance-free systems in output feedback form, there is no general control design method to ensure the global stability when there are more than one unstable zeros in the zero dynamics. For such a system, a semi-global result is presented in [10]. The assumption on the stable zero dynamics or minimum phase of the system does place a restriction. It remains as a challenging problem to remove this assumption. An example is included for disturbance rejection of nonminimum phase linear systems under state feedback.

Remark 3: We only consider the matched disturbances or input disturbances in (1). Many unmatched disturbances and even some cases of output regulation can be converted to the form as shown in (1) with w being interpreted as the desired feed-forward input. Therefore the problem considered in this paper can also be interpreted as the problems for general disturbance rejection and output regulation of which the desired input satisfies the conditions specified in Assumption 1, with the output measurement that does not contain the disturbance.

3 Disturbance Properties under Half-Period Integration

In this section, the periodic property and wave pattern properties described in Assumption 1 will be exploited to design estimation algorithms for a and ϕ. To explore the disturbance properties, we use the half-period integration operator $\mathcal{I}$ and the quarter-period delay operator $\mathcal{D}$,

$$\mathcal{I} \circ f(t) \;:=\; \mathcal{I}(f(t)) = \int_{t-\frac{T}{2}}^{t} f(s)ds$$

$$\mathcal{D}\circ f(t) \;:=\; \mathcal{D}(f(t)) = f(t-\frac{T}{4}) \tag{3}$$

For the convenience of notations, we often write $\mathcal{I}\circ f$ and $\mathcal{D}\circ f$ as $\mathcal{I}f$ and $\mathcal{D}f$ when no confusions are caused. It is easy to see the following properties of the introduced operators such as

$$\mathcal{I}\frac{df(t)}{dt} = f(t) - f(t-\frac{T}{2}) \tag{4}$$

for a C^1 function f, and

$$\mathcal{D}^k w(t) = \mathcal{D}^{\bar{k}} w(t) \tag{5}$$

with $\bar{k} = \mod(k,4)$, for a periodic function $w(t)$ with period T. The operations of $\mathcal{D}$ and $\mathcal{I}$ can be swapped in sequence, ie, $\mathcal{D}\circ\mathcal{I}\circ f = \mathcal{I}\circ\mathcal{D}\circ f$. An important property is described in the following lemma.

Lemma 3.1: If a function $f(t)$ satisfies the conditions specified in Assumption 1, so does the function $g(t)$ defined by $g = \mathcal{D}^3\circ\mathcal{I}\circ f$.

Proof: For A1.1, it can be obtained that

$$\begin{aligned} g(t+T) &= \mathcal{D}^3 \circ \int_{t+T-\frac{T}{2}}^{t+T} f(s)ds \\ &= \mathcal{D}^3 \circ \int_{t-\frac{T}{2}}^{t} f(s+T)ds \\ &= \mathcal{D}^3 \circ \int_{t-\frac{T}{2}}^{t} f(s)ds \\ &= g(t) \end{aligned} \tag{6}$$

For A1.2, it follows that

$$\begin{aligned} g(-t) &= \int_{-t-\frac{3}{4}T-\frac{T}{2}}^{-t-\frac{3}{4}T} f(s)ds = -\int_{t+\frac{5}{4}T}^{t+\frac{3}{4}T} f(-s)ds \\ &= \int_{t+\frac{5}{4}T}^{t+\frac{3}{4}T} f(s)ds = \int_{t}^{t-\frac{T}{2}} f(s+\frac{5}{4}T)ds \\ &= -\int_{t-\frac{T}{2}}^{t} f(s+\frac{5}{4}T)ds = -\int_{t-\frac{T}{2}}^{t} f(s-\frac{3}{4}T)ds \\ &= -g(t) \end{aligned} \tag{7}$$

For A1.3, it is easy to obtain that

$$g(t) = \int_{t-\frac{T}{4}}^{t+\frac{T}{4}} f(s)ds \tag{8}$$

For $0 < t < \frac{T}{4}$, it can be obtained that

$$\begin{aligned} g(t) &= \int_{t-\frac{T}{4}}^{\frac{T}{4}-t} f(s)ds + \int_{\frac{T}{4}-t}^{t+\frac{T}{4}} f(s)ds \\ &= \int_{\frac{T}{4}-t}^{t+\frac{T}{4}} f(s)ds \end{aligned} \tag{9}$$

Let $\delta' = \frac{T}{4} - \delta$ and consider $t \in (0, \delta')$. We have

$$(\frac{T}{4} - t, t + \frac{T}{4}) \subset (\delta, \frac{T}{2} - \delta), \quad \forall t \in (0, \delta') \tag{10}$$

Hence, from (9), we have, for $t \in (0, \delta')$, that

$$\begin{aligned} g(t) &\geq K_b \delta^l [(t + \frac{T}{4}) - (\frac{T}{4} - t)] \\ &= 2K_b \delta^l t := K_b' t^{l'} \end{aligned} \tag{11}$$

with $K' = 2K_b\delta^l$ and $l' = 1$.

Similarly, for $\frac{T}{2} - \delta < t < \frac{T}{2}$, it can be obtained that

$$g(t) = \int_{t-\frac{T}{4}}^{\frac{3T}{4}-t} f(s)ds \tag{12}$$

We also have, $\forall t \in (\frac{T}{2} - \delta', \frac{T}{2})$,

$$(t - \frac{T}{4}, \frac{3T}{4} - t) \subset (\delta, \frac{1}{2}T - \delta) \tag{13}$$

Therefore, we have the same result as shown in the second line of (11) for $\frac{T}{2} - \delta < t < \frac{T}{2}$, and this completes the proof.

Remark 4: Lemma 3.1 reveals an invariant property of periodic disturbances under half-period integration and delay operations whose basic wave form is an odd function. This invariant property plays an important part in the estimation and rejection of the class of periodic disturbances considered in this paper. In [9], the invariant property is obtained under the additional assumption that the basic wave form for half a period is symmetric, ie, $w_b(T/4-t) = w_b(T/4+t)$, and this restriction is removed in this paper. This removal allows us to asymptotically reject a more general class of disturbances than the method presented in [9].

The result in Lemma 3.2 [9] for the disturbance passing through a stable linear system without zero dynamics can be obtained in the same way as in [9].

4 Disturbance estimation

In order to extract the contribution in system state due to the disturbances, we use the same filter as in [9] :

$$\dot{\mathbf{p}} = (\mathbf{A}_c + \mathbf{kC})\mathbf{p} + \boldsymbol{\phi}(y) + \mathbf{b}u - \mathbf{k}y \tag{14}$$

where $\mathbf{p} \in R^n$, $\mathbf{k} \in R^n$ is chosen so that

$$\begin{aligned} K(s) &:= s^n - \sum_{i=1}^{n} k_i s^{n-i} \\ &= B(s)(s^\rho + \lambda_1 s^{\rho-1} + \ldots + \lambda_\rho)/b_\rho \end{aligned} \tag{15}$$

with λ_i being positive real design parameters such that $(s^\rho + \lambda_1 s^{\rho-1} + \ldots + \lambda_\rho)$ is Hurwitz.

An estimate of w is given by

$$\hat{w}(t) = \hat{a} w_b(\hat{\phi}_1) \tag{16}$$

where

$$\hat{a} = \frac{\mathcal{I} \circ |\bar{w}(t)|}{\mathcal{I} \circ |w_{b,\rho}(t)|} \tag{17}$$

$$\hat{\phi}_1(t) = \frac{1}{2}(\mathcal{I} \circ \mathrm{sign}(\bar{w}(t)) + \frac{T}{2})\mathrm{sign}(\bar{w}(t)) \tag{18}$$

with

$$\bar{w}(t) = \mathcal{Q} \circ (p_1 - y) \tag{19}$$

$$\mathcal{Q} = \mathcal{D}^{\bar{\rho}} \sum_{i}^{\rho} \lambda_i \mathcal{I}^i (1 - \mathcal{D}^2)^{\rho - i} \tag{20}$$

and $\bar{\rho} = \mathrm{mod}(3\rho, 4)$

Theorem 4.1: If the disturbance in (1) satisfies the conditions specified in Assumptions 1 and 2, then the estimate given in (16) converges to the actual disturbance in L_p, ie., $w - \hat{w} \in L_p$ for $p = 1, 2$ and ∞.

Remark 5: Note that $\mathrm{sign}(w_b(t))$ is the same for disturbances with or without half-period symmetric wave pattern, and that we have established that the invariant property under the operation of $\mathcal{D}^3\mathcal{I}$, and $\mathrm{sign}(\mathcal{D}^3\mathcal{I}w_b(t))$ will remain the same. The result shown in Assumption 2 [9] can be obtained based A1.4. Hence, the proof follows in a similar way to the proof of Theorem 4.1 in [9].

For the disturbances with straight lines in the wave form, the implementation of the estimation algorithm may be simplified. The simplified results are given for square waves and triangular waves in [9] as those wave forms are symmetric. We show the result for sawtooth waves which has an asymmetric wave form.

Corollary 4.2: If the basic wave form for the disturbance in (1) is the sawtooth wave form described by

$$w_b(t) = \begin{cases} 1 - \frac{2}{T}t & \text{for } 0 < t < T \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

tehn the disturbance $w(t) = aw_b(t+\phi)$ can be estimated by

$$\hat{w}(t) = \frac{\hat{a}}{T}(\frac{T}{2} - \mathcal{I}\text{sign}(\varpi(t))\text{sign}(\varpi(t)) \tag{22}$$

with the property that $w - \hat{w} \in L_p$ for $p = 1, 2$ and ∞.

Proof: The sawtooth wave form in (21) satisfies Assumption 1. Define $\bar{q}_1 = \mathcal{Q}q_1$ where q_1 is the first element of $\mathbf{q}$ defined by

$$\dot{\mathbf{q}} \;=\; (\mathbf{A}_c + \mathbf{kC})\mathbf{q} + \mathbf{b}w \tag{23}$$

with $\mathbf{q} \in R^n$ denoting the steady state only. It can be shown that

$$\text{sign}(\bar{q}_1(t)) = \begin{cases} 1 & \text{if } -\phi < t < \frac{T}{2} - \phi \\ -1 & \text{if } \frac{T}{2} - \phi < t < T - \phi \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

and

$$\begin{aligned} & \tfrac{1}{T}\mathcal{I} \circ \text{sign}(\bar{q}_1(t)) \\ &= \begin{cases} \frac{2}{T}(t+\phi) - \frac{1}{2} & \text{if } -\phi < t < \frac{T}{2} - \phi \\ \frac{3}{2} - \frac{2}{T}(t+\phi) & \text{if } \frac{T}{2} - \phi < t < T - \phi \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{25}$$

Hence we have

$$(\frac{1}{2} - \frac{1}{T}\mathcal{I}\text{sign}(\bar{q}_1(t))\text{sign}(\bar{q}_1(t)) = w_b(t+\phi) \tag{26}$$

Following a similar way to the proof Theorem 4.1 in [9], it can be shown that $(\text{sign}(\bar{q}_1) - \text{sign}(\varpi)) \in L_p$ and $a - \hat{a} \in L_p$, for $p = 1, 2$ and ∞. Then it can be concluded that $w - \hat{w} \in L_p$.

5 Disturbance Rejection with Stabilization

After the estimation of unknown disturbances, the control design follows in the same way as the control design for asymptotic rejection of disturbances with symmetric wave forms [9]. We include the final control design for completeness of presentation. A state observer is designed as

$$\dot{\hat{\mathbf{x}}} = (\mathbf{A}_c + \mathbf{kC})\hat{\mathbf{x}} + \boldsymbol{\psi}(y) + \mathbf{b}(u - \hat{w}) - \mathbf{k}y \tag{27}$$

Control design can then be carried out using backstepping based on (27). Finally the control input is given by

$$u = \hat{w} + \frac{\alpha_\rho - \hat{x}_{\rho+1}}{b_\rho} \tag{28}$$

where

$$\begin{aligned}\alpha_i &= z_{i-1} - c_i z_i - d_i(\frac{\partial \alpha_{i-1}}{\partial y})^2 z_i - k_i\hat{x}_1 + k_i y - \psi_i(y) \\ &\quad + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j}\dot{\hat{x}}_j + \frac{\partial \alpha_{i-1}}{\partial y}(\hat{x}_2 + \psi_1(y))\end{aligned} \tag{29}$$

for $i = 1, \ldots, \rho$, with c_i, d_i, $i = 1, \ldots \rho$, being positive real design parameters, $z_1 := y = x_1$, $z_i = \hat{x}_i - \alpha_{i-1}$, $i = 2, \ldots \rho$, $z_0 = z_{\rho+1} = 0$ and $\frac{\partial \alpha_0}{\partial y} = 0$.

Remark 6: The values of parameters $1/c_i$ affect the response speed of the system in a similar way as the time constant, as the dynamics of z_i can be approximated as $\dot{z}_i = -c_i z_i + r_i$ with r_i as an input. The influence of κ_i is more involved [11].

The stability result is summarized in the following theorem.

Theorem 5.1: For a system (1) satisfying Assumptions 1 and 2, the control input u given in (28) with the estimated disturbance $\hat{w}$ ensures the asymptotic rejection of the unknown disturbance, ie., $\lim_{t\to\infty} y(t) = 0$, and the boundedness of the other variables in the system.

Remark 7: If $\boldsymbol{\psi}(y) = \mathbf{f}y$ in (1) with $\mathbf{f} \in R^n$, ie, the system is linear, the control is designed as $u_l = \mathbf{k}_l^T\hat{\mathbf{x}} + \hat{w}$ where $\mathbf{k}_l$ is chosen so that $\mathbf{A}_c + \mathbf{fC} + \mathbf{bk}_l^T$ is Hurwitz.

6 Examples

Consider a nonlinear system in output feedback form

$$\dot{x}_1 = x_2 - y^3 + (u - w)$$

$$
\begin{aligned}
\dot{x}_2 &= (u - w) \\
y &= x_1
\end{aligned} \tag{30}
$$

where w is a disturbance with the sawtooth shown in (21) as the basic wave form. It is easy to see that the system (30) are in the format of (1) with $\boldsymbol{\phi}(y) = [y^3 \ 0]^T$ and $\mathbf{b} = [1 \ 1]^T$. The system is minimum phase, and therefore Assumption 3 is satisfied. Following the control design outlined in Sections 4 and 5, the control input is designed as

$$
u = \hat{w} - c_1 y - d_1 y - y^3 - \hat{x}_2 \tag{31}
$$

In the simulation study, the parameters are set as $k_1 = -2$, $k_2 = -1$, $c_1 = d_1 = \lambda_1 = 1$, $T = 2$, and the amplitude $a = 1$. The control input and the system output are shown in Figure 1, in which the output converges to zero with the input to asymptotically cancel the disturbance. Figure 2 shows the disturbance and its estimate.

In the second example, we consider a linear system

$$
\begin{aligned}
\dot{x}_1 &= x_2 + x_1 \\
\dot{x}_2 &= x_3 - x_1 + (u - w) \\
\dot{x}_3 &= 2x_1 - (u - w) \\
y &= x_1
\end{aligned} \tag{32}
$$

where w again is a the sawtooth disturbance, and all the states are measurable. With respect to the system output y, the system is nonminimum phase with an unstable zero at $s = 1$. The proposed disturbance rejection and control method cannot be directly applied. We can apply the fundamental design principle of the proposed method to this example under the full state feedback control design. In this case, we apply the half-period integration with delay to the second equation in (32), and therefore we define

$$
\bar{w} = \mathcal{D}^3 \mathcal{I}(x_3 - x_1 + u) - \mathcal{D}^3 x_2 + \mathcal{D}^5 x_2 \tag{33}
$$

Then the estimate given in (22) is a L_p convergent to the disturbance. The final control design for simulation is

$$
u = -49x_1 - 32x_2 - 25x_3 + \hat{w} \tag{34}
$$

where the first three terms are obtained based on state feedback control design for the disturbance-free case of (32). The simulation results are shown in Figure 3 with $T = 1$.

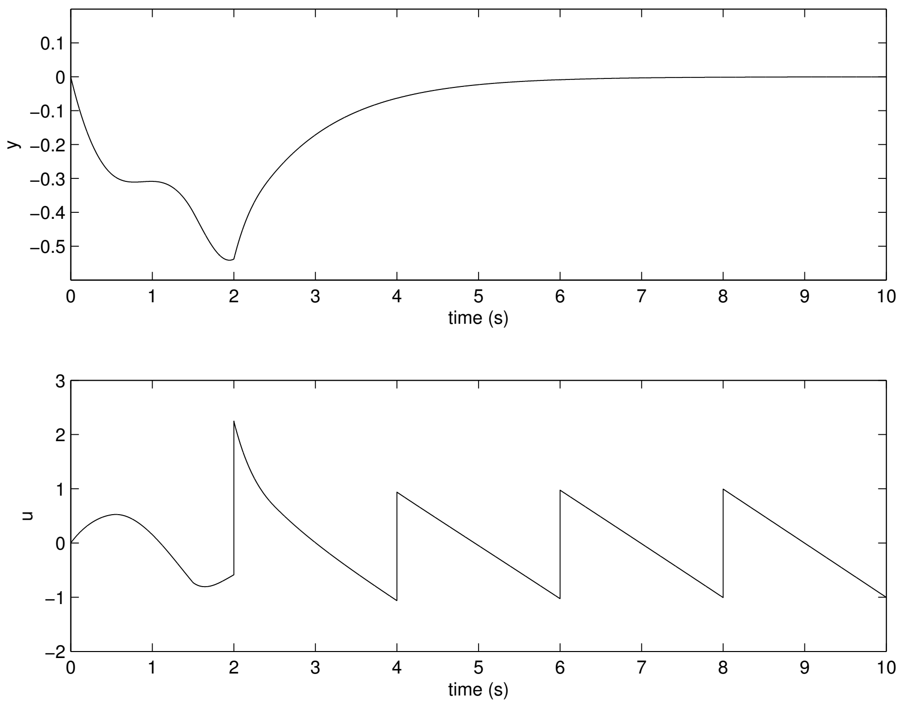

Figure 1: The system input and output under sawtooth disturbance

7 Conclusions

In this paper, we have proposed disturbance estimation and control design for asymptotic rejection of disturbances whose basic wave forms are odd functions and may not be symmetric in the half period. The invariant properties under the half-period integration with delay have been investigated for the class of periodic disturbances, and they have been used for disturbance estimation. Based on the proposed disturbance estimation method, the control design is demonstrated for asymptotic disturbance rejection with stability for nonlinear systems in the output feedback form. The results shown in this paper extend the class of periodic disturbances which can be asymptotically rejected to the periodic disturbances without the half-period symmetry.

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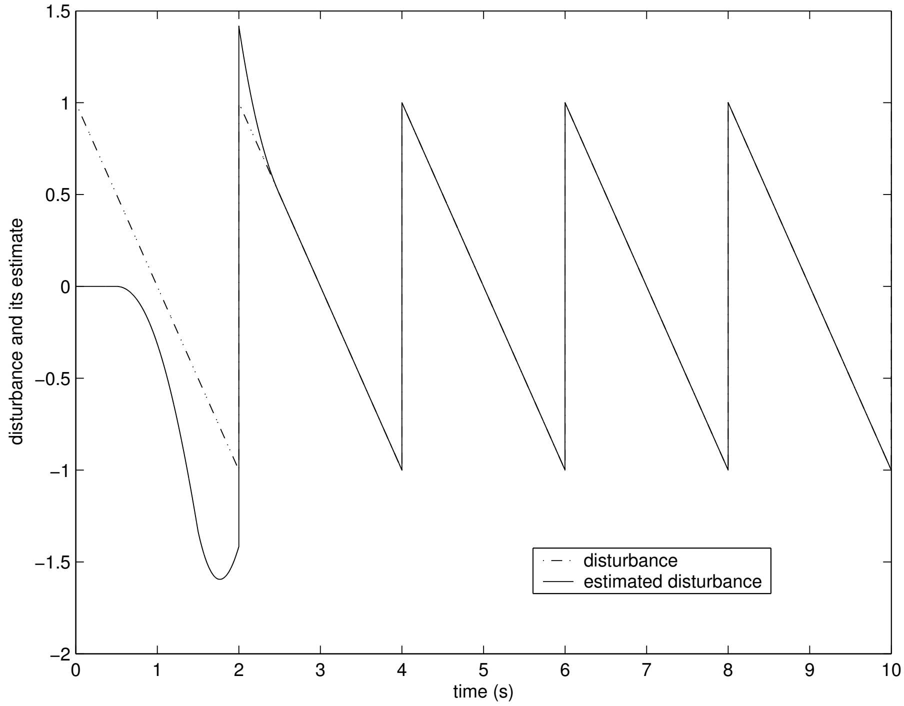

Figure 2: The disturbance and its estimate

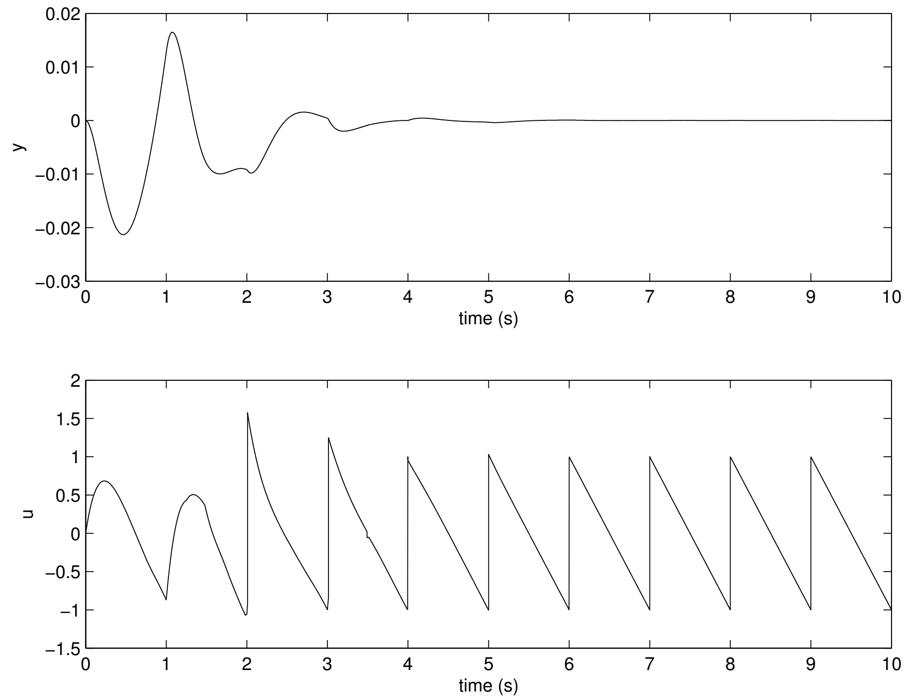

Figure 3: The system input and output under sawtooth disturbance