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Adaptive Disturbance Rejection of Nonlinear Systems in An Extended Output Feedback Form*

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Abstract

This paper deals with asymptotic rejection of unknown sinusoidal disturbances for uncertain nonlinear systems in an extended output feedback form which allows the vector field coupled with the system input to have different nonlinear functions of the system output as its elements. A new internal model design is proposed to deal with nonlinear functions of the system output which are coupled with the input and the unknown disturbance. Adaptive control techniques are then used to deal with the uncertainty in the system. The proposed adaptive disturbance rejection algorithm with the new internal model design ensures the asymptotic rejection of the unknown sinusoidal disturbance and the boundedness of all the variables.

1 Introduction

Asymptotic rejection of sinusoidal disturbances in dynamics systems has been addressed in [1, 2, 3, 4] for linear systems. Adaptive disturbance rejection for nonlinear systems in the strict feedback form via state feedback is reported in [5] and for nonlinear systems in the output feedback form in [6]. The state feedback control

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proposed in [5] uses an adaptive internal model to deal with sinusoidal disturbances with unknown frequencies. Since then, adaptive rejection of unknown sinusoidal disturbances is solved in [7] for nonlinear systems in the output feedback form with known system parameters and in [8] for uncertain nonlinear systems in the output feedback form with output tracking. Adaptive rejection of unknown sinusoidal disturbances for nonlinear systems in the output feedback form with complete unknown parameters is achieved in [9] where no knowledge of any parameters in the system and the exosystem is assumed known. An indirect approach is recently proposed for nonlinear systems in the output feedback form with possible nonminimum phase or unstable zero dynamics [10]. Adaptive rejection of unknown sinusoidal disturbances is reported in [11] for nonlinear systems in the output feedback form with parametric uncertainty. Some recent results on stabilization and output regulation of nonlinear systems in output feedback form are also reported in [12, 13, 14].

In this paper, we consider adaptive disturbance rejection of uncertain nonlinear systems in an extended nonlinear output feedback form. The output feedback form considered in [15, 16] restricts the vector field associated with the input to be a constant vector multiplied by a nonlinear function of the system output. In the extended output feedback form, the elements in the vector field can be different nonlinear functions of the system output. The vector field coupled with the system input defines the system's zero dynamics. For the standard output feedback form, the zero dynamics is characterized by a companion matrix associated with the constant vector field, while for the extended nonlinear output feedback form, it is much more involved with functions of the system output. Comparing with the standard output feedback form in [15, 16], the output feedback form considered in [7, 9, 11] is even more restrictive, which requires the vector field to be a constant vector, no even multiplied by a nonlinear function of the system output. This restriction enables the input and the matched disturbances to directly enter the system, and it is needed for the convenience of adaptive internal model design, which forms an important part of the adaptive control system for rejecting sinusoidal disturbances with unknown frequencies. In fact the constant vector field associated with the input is also required for the results of global and semi-global output regulation in literature (e.g., [17]), for the same reason.

Adaptive control of nonlinear systems in the extended nonlinear output feedback form is considered in [18] where a state transform involving integrated nonlinear functions of the system output is introduced. With that state transform, an adaptive control algorithm is proposed to ensure the stability of nonlinear systems in the extended output feedback form with unknown parameters. In this paper, we exploit the same kind of state transform as in [18] together with the design principle of reduced-order observers for the internal model design. The proposed internal model involves a nonlinear function obtained by integrating a nonlinear function of the system output. This internal model does not provide direct state observation of the disturbance system or the exosystem, as the observer error dynamically involves an unknown system state and the unknown parameters. Adaptive control techniques are then applied to the control design, together with the proposed internal model, to ensure the asymptotic rejection of sinusoidal disturbances with unknown frequencies, for nonlinear systems in the extended output feedback form.

2 Problem formulation

We consider a single-input-single-output nonlinear system which can be transformed into the following form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_c x + \boldsymbol{\phi}_0(y) + \sum_{i=1}^{l} \boldsymbol{\phi}_i(y)\theta_i + \mathbf{g}(y)(u-v) \\ &:= \mathbf{A}_c x + \boldsymbol{\phi}_0(y) + \boldsymbol{\Phi}(y)\boldsymbol{\theta} + \mathbf{g}(y)(u-v) \\ y &= \mathbf{c}^T\mathbf{x} \end{aligned} \tag{1}$$

with

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \mathbf{g}(y) = \begin{bmatrix} g_1(y) \\ \vdots \\ g_n(y) \end{bmatrix}$$

where $\mathbf{x} \in R^n$ is the state vector, $u \in R$ is the control, $\boldsymbol{\phi}_i$, $i = 0, 1 \dots, l$, are known smooth nonlinear vector fields in R^n with $\boldsymbol{\phi}_i(0) = 0$, $\boldsymbol{\theta} \in R^l$ is the unknown constant vector of parameters, v is a matched disturbance generated by an unknown exosystem

$$\begin{aligned} \dot{\mathbf{w}} &= \mathbf{S}\mathbf{w} \\ v &= \mathbf{h}^T\mathbf{w} \end{aligned} \tag{2}$$

with $\mathbf{w} \in R^s$, $\mathbf{S} \in R^{s\times s}$, and $\mathbf{h} \in R^s$.

Remark 1 The well known output feedback form considered in [15, 16] restricts $\mathbf{g}(y)$ to the case that $g_1(y) = g_2(y) = \ldots = g_n(y)$, and the output feedback form considered in [7, 9, 11] requires $\mathbf{g}(y)$ to be a constant vector. The class of nonlinear systems considered in this paper allows different functions of $g_i(y)$, and therefore it is referred to as an extended output feedback form. The class of nonlinear systems in the extended output feedback form has more involved zero dynamics.

The problem considered in this paper is to design an adaptive feedback control algorithm to ensure the global stability of the system and asymptotic disturbance rejection of the system in the sense that all the variables remain bounded and that the system state and output converge to zero asymptotically. To solve the problem, a number of standing assumptions are needed.

Assumption 1 $\forall y \in R$, $|g_1(y)| \geq \delta$ with δ being a positive real constant, and furthermore, there exist positive definite matrices $\mathbf{P}, \mathbf{Q} \in R^{(n-1)\times(n-1)}$ such that

$$\mathbf{P}\mathbf{B}(y) + \mathbf{B}^T(y)\mathbf{P} \leq -\mathbf{Q} \tag{3}$$

where

$$\mathbf{B}(y) = \begin{bmatrix} -b_1(y) & 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_2(y) & 0 & 0 & \ldots & 1 \\ -b_{n-1}(y) & 0 & 0 & \ldots & 0 \end{bmatrix}$$

with $b_i(y) = g_{i+1}(y)/g_1(y)$ for $i = 1, \ldots, n-1$.

Assumption 2 The eigenvalues of $\mathbf{S}$ are with zero real parts and are distinct, and the pair $\{\mathbf{S}, \mathbf{h}^T\}$ is observable.

Remark 2 Assumption 1 implies that the system has stable zero dynamics. When $\mathbf{g}(y)$ is a constant vector, this assumption is equivalent to the minimum phase assumption which is often used for adaptive control and disturbances rejection in literature. Assumption 1 can be relaxed for adaptive control of the nonlinear systems considered in this paper if there are no disturbances [18].

Remark 3 Assumption 2 ensures that the exosystem is neutrally stable and the state of the exosystem is observable from the system output. This assumption is commonly used for asymptotic rejection of unknown sinusoidal disturbances.

3 State Transform

Define the internal state $\mathbf{z} \in R^{n-1}$ by

$$\mathbf{z} = \begin{pmatrix} x_2 \\ \vdots \\ x_n \end{pmatrix} - \begin{pmatrix} p_1(y) \\ \vdots \\ p_{n-1}(y) \end{pmatrix} \tag{4}$$

where

$$p_i(y) = \int_0^y b_i(\mu)d\mu$$

The dynamics of z can be obtained as

$$\dot{\mathbf{z}} = \mathbf{B}(y)\mathbf{z} + \boldsymbol{\phi}_z(y) + \mathbf{\Phi}_z(y)\boldsymbol{\theta} \tag{5}$$

where

$$\boldsymbol{\phi}_z(y) = \boldsymbol{\phi}_{0,2:n}(y) - \mathbf{b}(y)\phi_{0,1}(y) + \mathbf{B}(y)\mathbf{p}(y)$$
$$\mathbf{\Phi}_z(y) = \mathbf{\Phi}(y)_{(2:n)} - \mathbf{b}(y)\mathbf{\Phi}_{(1)}(y)$$

with the subscript (2:n) denoting the vector or matrix formed from the 2nd to nth row of the original vector or matrix, and the subscript (1) denoting the first row. Therefore the system in $(y,\, z)$-coordinate is described by

$$\begin{cases} \dot{\mathbf{z}} &= \mathbf{B}(y)\mathbf{z} + \boldsymbol{\phi}_z(y) + \mathbf{\Phi}_z(y)\boldsymbol{\theta} \\ \dot{y} &= z_1 + p_1(y) + \phi_{0,1}(y) + \mathbf{\Phi}_{(1)}(y)\boldsymbol{\theta} + g_1(y)(u - v) \end{cases} \tag{6}$$

Remark 4 After the state transform, the adaptive control problem can be easily solved by refining the new control input $u' = g_1(y)u$, if there is no disturbance, ie, $v = 0$. In fact this is a standard practice in adaptive control of nonlinear systems in the output feedback form [15, 16]. The disturbance v makes the problem more complicated. In this case, if we redefine the new control in the same way, it will leave $g_1(y)v$ as an unmatched disturbance term, and it cannot be tackled by the existing methods in literature for disturbance rejection of unknown sinusoidal disturbances. The existing methods with adaptive internal models can only deal with disturbance rejection or output regulation of nonlinear systems in the standard output feedback form with the further restriction that $g_1(y)$ is a constant [7, 17, 11].

4 Internal Model Design

If $g_1(y)$ is a constant, as in the nonlinear systems considered in [7, 9, 11], the disturbance v is directly observed through $\dot{y}$ via a constant coefficient and an internal model can be designed by following the principle of reduced order observer design. This design principle has been used in the internal model design for disturbance rejection and output regulation [7, 9, 11, 17]. The difficulty now is how to deal with this function of the system output. Instead of directly using the system output in the reduced order observer formulation for the internal model design, we propose a solution here with a nonlinear function of the output y, given by

$$q(y) = \int_0^y \frac{1}{g_1(\mu)} d\mu \tag{7}$$

With $q(y)$, we design the internal model as

$$\dot{\boldsymbol{\xi}} = \mathbf{F}(\boldsymbol{\xi} - q(y)\mathbf{k}) + \mathbf{k}[u + \frac{1}{g_1(y)}(p_1(y) + \phi_{0,1}(y))] \tag{8}$$

where $\{\mathbf{F}, \mathbf{k}\}$ can be any controllable pair with $\mathbf{F} \in R^{s\times s}$ and Hurwitz, and $\mathbf{k} \in R^s$.

Remark 5 The internal model (8) is different from the one used in literature for disturbance rejection and output regulation. The nonlinear function $q(y)$ plays an important part in dealing with the interaction of the system output and the unknown input disturbances. Here the design principle of reduced order observers is exploited for the internal model design, but the internal model is not a reduced order observer for the state variables of the exosystem.

Remark 6 The internal model introduced above can also be applied to disturbance rejection in the standard output feedback form, and extends the algorithms presented in [7, 9, 11] to the case that a nonlinear function of the system output is coupled with the input.

Define an auxiliary error

$$\mathbf{e} = \mathbf{M}\mathbf{w} - \boldsymbol{\xi} + q(y)\mathbf{k} \tag{9}$$

where $\mathbf{M} \in R^{s\times s}$ is the unique and nonsingular solution of the following equation

$$\mathbf{MS} - \mathbf{FM} = \mathbf{kh}^T \tag{10}$$

The uniqueness of M is guaranteed by the exclusive eigenvalues of $\mathbf{F}$ and $\mathbf{S}$ and the invertibility of $\mathbf{M}$ is as a result of controllability of $\{\mathbf{F}, \mathbf{k}\}$ and observability of $\{\mathbf{S}, \mathbf{h}^T\}$.

To obtain the dynamics of $\mathbf{e}$, we have

$$\begin{aligned}
\dot{\mathbf{e}} &= \mathbf{MSw} - \mathbf{F}(\boldsymbol{\xi} - q(y)\mathbf{k}) - \mathbf{k}[u + \frac{1}{g_1(y)}(p_1(y) + \phi_{0,1}(y))] \\
&\quad + \frac{\mathbf{k}}{g_1(y)}[z_1 + p_1(y) + \phi_{0,1}(y) + \boldsymbol{\Phi}_{(1)}(y)\boldsymbol{\theta} + g_1(y)(u - \mathbf{h}^T\mathbf{w})] \\
&= (\mathbf{FM} + \mathbf{kh}^T)\mathbf{w} - \mathbf{F}(\boldsymbol{\xi} - q(y)\mathbf{k}) + \frac{\mathbf{k}}{g_1(y)}(z_1 + \boldsymbol{\Phi}_{(1)}(y)\boldsymbol{\theta} - g_1(y)\mathbf{h}^T\mathbf{w}) \\
&= \mathbf{Fe} + \frac{\mathbf{k}}{g_1(y)}(z_1 + \boldsymbol{\Phi}_{(1)}(y)\boldsymbol{\theta}) \qquad (11)
\end{aligned}$$

Remark 7 If z_1 and $\boldsymbol{\theta}$ in (11) are known, we can change the design of the internal model (8) to make the dynamics of the auxiliary error as $\dot{\mathbf{e}} = \mathbf{Fe}$, and then the internal model serves as a reduced order observer for the disturbances. The unknown terms in (11) will be tackled in the subsequent design.

5 Adaptive Control Design

Based on the auxiliary error, we can express the matched disturbance as

$$v = \boldsymbol{\psi}^T(\mathbf{e} + \boldsymbol{\xi} - q(y)\mathbf{k}) \qquad (12)$$

where $\boldsymbol{\psi} = \mathbf{M}^{-T}\mathbf{h}$. Hence, the dynamics of the system output is given by

$$\dot{y} = z_1 + p_1(y) + \phi_{0,1}(y) + \boldsymbol{\Phi}_{(1)}(y)\boldsymbol{\theta} + g_1(y)u - g_1(y)(\mathbf{e} + \boldsymbol{\xi} - q(y)\mathbf{k})^T\boldsymbol{\psi} \quad (13)$$

The unknown parameter vectors $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$ in the dynamics of y will be tackled by adaptive control terms associated with their estimates $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\psi}}$. They also appear in the dynamics of $\mathbf{z}$ and $\mathbf{e}$ as shown in (5) and (11), and their influence on the overall system is tackled by an adaptive control term associated with an estimate $\hat{\vartheta}$ of ϑ which is given by

$$\vartheta = 2\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\psi}\|^2\|\boldsymbol{\theta}\|^2 + \frac{8\|\mathbf{P}\|^2}{\lambda^2\delta^2}(4\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\psi}\|^2 + \delta^2)(1 + \|\boldsymbol{\theta}\|^2) \qquad (14)$$

where λ denotes the minimum eigenvalue of $\mathbf{Q}$.

Finally, we design the control input as

$$\begin{aligned} u &= \frac{1}{g_1(y)}[-(\frac{1}{2}+d+\frac{1}{2}g_1^2(y))y - p_1(y) - \phi_{0,1}(y) - \hat{\vartheta}r(y) \\ &\quad - \boldsymbol{\Phi}_{(1)}(y)\hat{\boldsymbol{\theta}} + g_1(y)(\boldsymbol{\xi} - q(y)\mathbf{k})^T\hat{\boldsymbol{\psi}}] \end{aligned} \tag{15}$$

where d is a positive real constant design parameter, and $r(y)$ is a function of y given by

$$r(y) = \frac{1}{y}(\frac{\|\boldsymbol{\Phi}_{(1)}(y)\|^2}{g_1(y)} + \|\boldsymbol{\Phi}_z(y)\|^2 + \|\boldsymbol{\phi}_z(y)\|^2) \tag{16}$$

Note that r is a continuous function of y. This property is guaranteed by the fact that $\boldsymbol{\Phi}_{(1)}(y)$, $\boldsymbol{\Phi}_z(y)$ and $\boldsymbol{\phi}_z(y)$ are smooth functions of y and the fact $\boldsymbol{\Phi}_{(1)}(0) = 0$, $\boldsymbol{\Phi}_z(0) = 0$ and $\boldsymbol{\phi}_z(0) = 0$. The adaptive laws are designed as

$$\dot{\hat{\boldsymbol{\theta}}} = y\boldsymbol{\Gamma}_\theta\boldsymbol{\Phi}_{(1)}^T(y) \tag{17}$$

$$\dot{\hat{\boldsymbol{\psi}}} = -yg_1(y)\boldsymbol{\Gamma}_\psi(\boldsymbol{\xi} - q(y)\mathbf{k}) \tag{18}$$

$$\dot{\hat{\vartheta}} = \gamma y r(y) \tag{19}$$

where $\boldsymbol{\Gamma}_\theta \in R^{l\times l}$ and $\boldsymbol{\Gamma}_\psi \in R^{s\times s}$ are positive definite matrices, and γ is a positive real constant.

6 Stability Analysis

We start the analysis from the subsystems of $\mathbf{z}$, $\mathbf{e}$ and y, and then establish the stability of the overall adaptive control system. For the $\mathbf{z}$-subsystem, define

$$V_z = \mathbf{z}^T\mathbf{P}\mathbf{z} \tag{20}$$

From (5), we have

$$\begin{aligned} \dot{V}_z &= -\mathbf{z}^T\mathbf{Q}\mathbf{z} + 2\mathbf{z}^T\mathbf{P}\boldsymbol{\phi}_z(y) + 2\mathbf{z}^T\mathbf{P}\boldsymbol{\Phi}_z(y)\boldsymbol{\theta} \\ &\le -\frac{\lambda}{2}\|\mathbf{z}\|^2 + \frac{4}{\lambda}\|\mathbf{P}\|^2\|\boldsymbol{\phi}_z(y)\|^2 + \frac{4}{\lambda}\|\mathbf{P}\|^2\|\boldsymbol{\theta}\|^2\|\boldsymbol{\Phi}_z(y)\|^2 \end{aligned} \tag{21}$$

Define

$$V_e = \mathbf{e}^T\mathbf{P}_e\mathbf{e} \tag{22}$$

where $\mathbf{P}_e$ is the positive definite matrix which satisfies

$$\mathbf{P}_e\mathbf{F} + \mathbf{F}^T\mathbf{P}_e = -2\mathbf{I} \tag{23}$$

From (11), we have

$$\begin{aligned}\dot{V}_e &= -2\mathbf{e}^T\mathbf{e} + 2\mathbf{e}^T\mathbf{P}_e\mathbf{k}\frac{z_1}{g_1(y)} + 2\mathbf{e}^T\mathbf{P}_e\frac{\mathbf{k}}{g_1(y)}\mathbf{\Phi}_{(1)}(y)\boldsymbol{\theta} \\ &\leq -\|\mathbf{e}\|^2 + 2\frac{\|\mathbf{P}_e\mathbf{k}\|^2}{g_1^2(y)}z_1^2 + 2\|\mathbf{P}_e\frac{\mathbf{k}}{g_1(y)}\mathbf{\Phi}_{(1)}(y)\boldsymbol{\theta}\|^2 \\ &\leq -\|\mathbf{e}\|^2 + 2\frac{\|\mathbf{P}_e\mathbf{k}\|^2}{\delta^2}z_1^2 + 2\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\theta}\|^2\|\frac{\mathbf{\Phi}_{(1)}(y)}{g_1(y)}\|^2 \end{aligned} \tag{24}$$

Let us consider the dynamics of y. Substituting (15) into (13), we obtain

$$\begin{aligned}\dot{y} &= z_1 - (\frac{1}{2} + d + \frac{1}{2}g_1^2(y))y - \hat{\vartheta}r(y) + \mathbf{\Phi}_{(1)}(y)\tilde{\boldsymbol{\theta}} \\ &\quad -g_1(y)\mathbf{e}^T\boldsymbol{\psi} - g_1(y)(\boldsymbol{\xi} - q(y)\mathbf{k})^T\tilde{\boldsymbol{\psi}} \end{aligned} \tag{25}$$

where $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ and $\tilde{\boldsymbol{\psi}} = \boldsymbol{\psi} - \hat{\boldsymbol{\psi}}$. Define

$$V_y = \frac{1}{2}(y^2 + \gamma^{-1}\tilde{\vartheta}^2 + \tilde{\boldsymbol{\theta}}^{\boldsymbol{T}}\mathbf{\Gamma}_\theta^{-1}\tilde{\boldsymbol{\theta}} + \tilde{\boldsymbol{\psi}}^T\mathbf{\Gamma}_\psi^{-1}\tilde{\boldsymbol{\psi}}) \tag{26}$$

where $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$. From the adaptive laws (19), (17), (18) and dynamics of y in (25), we have

$$\begin{aligned}\dot{V}_y &= yz_1 - (\frac{1}{2} + d + \frac{1}{2}g_1^2(y))y^2 - \vartheta yr(y) - yg_1(y)\mathbf{e}^T\boldsymbol{\psi} \\ &\leq -dy^2 + \frac{1}{2}z_1^2 - \vartheta yr(y) + \frac{1}{2}\|\boldsymbol{\psi}\|^2\|\mathbf{e}\|^2 \end{aligned} \tag{27}$$

Finally, we define

$$V = V_y + \|\boldsymbol{\psi}\|^2V_e + 2\frac{4\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\psi}\|^2 + \delta^2}{\lambda\delta^2}V_z \tag{28}$$

Combining the results in (27), (24), and (21), we have

$$\begin{aligned}\dot{V} &\leq -dy^2 - \frac{1}{2}\|\boldsymbol{\psi}\|^2\|\mathbf{e}\|^2 - \frac{4\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\psi}\|^2 + \delta^2}{2\delta^2}\|\mathbf{z}\|^2 - \vartheta yr(y) \\ &\quad +2\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\psi}\|^2\|\boldsymbol{\theta}\|^2\|\frac{\mathbf{\Phi}_{(1)}(y)}{g_1(y)}\|^2 \\ &\quad +\frac{8\|\mathbf{P}\|^2}{\lambda^2\delta^2}(4\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\psi}\|^2 + \delta^2)(1 + \|\boldsymbol{\theta}\|^2)(\|\boldsymbol{\phi}_z(y)\|^2 + \|\mathbf{\Phi}_z(y)\|^2) \end{aligned} \tag{29}$$

Note from (14) and (16) that the last two terms in the above are dominated by $\vartheta r(y)$ and hence we have

$$\dot{V} \leq -dy^2 - \frac{1}{2}\|\boldsymbol{\psi}\|^2\|\mathbf{e}\|^2 - \frac{4\|\mathbf{P}_e\mathbf{k}\|^2\|\boldsymbol{\psi}\|^2 + \delta^2}{2\delta^2}\|\mathbf{z}\|^2 \tag{30}$$

Therefore, we can conclude that $\|\mathbf{z}\| \in L_2 \cap L_\infty$, $\|\mathbf{e}\| \in L_2 \cap L_\infty$ and $y \in L_2 \cap L_\infty$, and that the variables $\hat{\boldsymbol{\psi}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\vartheta}}$ are all bounded. The boundedness of $\boldsymbol{\xi}$ follows the boundedness of $\mathbf{e}$ and $\mathbf{w}$. Hence we conclude that all the variables in the closed loop adaptive control system are bounded. Since the derivatives of $\mathbf{z}$, $\mathbf{e}$ and y are all bounded, we can further conclude, by invoking Babalat's lemma together with $\|\mathbf{z}\| \in L_2 \cap L_\infty$, $\|\mathbf{e}\| \in L_2 \cap L_\infty$ and $y \in L_2 \cap L_\infty$, that $\lim_{t\to\infty} \mathbf{z}(t) = 0$, $\lim_{t\to\infty} \mathbf{e}(t) = 0$, and finally $\lim_{t\to\infty} y(t) = 0$.

We summarize the above analysis in the following theorem.

Theorem 1 *The adaptive control system that consists of the internal model (8), and the adaptive laws (17), (18) and (19) and the control input (15), ensures the boundedness of all the variables in the closed loop system, and asymptotically rejects the unknow disturbance in the system (1) in the sense that the system state and output converge to zero asymptotically.*

7 An Example

Consider a second order nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 + (e^y - 1)\theta + \sqrt{1+y^2}(u+v) \\ \dot{x}_2 &= y^2\theta + (1+y^2)(u+v) \\ y &= x_1 \end{aligned} \quad (31)$$

where v is generated by

$$\begin{aligned} \dot{w}_1 &= \omega w_2 \\ \dot{w}_2 &= -\omega w_1 \\ v &= \mathbf{h}^T\mathbf{w} \end{aligned} \quad (32)$$

with $\mathbf{h} \in R^2$ and $h \neq 0$, ω is unknown. This second order system is in the format of (1) with $\phi_0(y) = 0$, $\mathbf{\Phi}(y) = [e^y - 1, y^2]^T$, $g_1(y) = \sqrt{1+y^2}$ and $g_2(y) = 1 + y^2$. It can be obtained that $B(y) = -b_1(y) = g_2(y)/g_1(y) = -\sqrt{1+y^2}$ and Assumption 1 is satisfied with $\delta = 1$, $P = 1$ and $Q = 2$, from, $\forall y \in R$,

$$\sqrt{1+y^2} \geq 1 \quad (33)$$

$$-\sqrt{1+y^2} - \sqrt{1+y^2} \leq -2 \quad (34)$$

and hence $\lambda = 2$. The exosystem satisfies Assumption 2 with $s = 2$, the eigenvalues of $\mathbf{S}$ at $\pm j\omega$ and $\{\mathbf{S}, \mathbf{h}^T\}$ observable. Note that only information of $\mathbf{S}$ used in the control design is $s = 2$, the dimension of the exosystem.

With

$$p_1(y) = \int_0^y \sqrt{1+\mu^2}d\mu = \frac{1}{2}(y\sqrt{1+y^2} + \sinh^{-1}(y)) \tag{35}$$

we have the transformed system as

$$\begin{aligned} \dot{z} &= -b_1(y)z - b_1(y)p_1(y) + (\Phi_2(y) - b_1(y)\Phi_1(y))\theta \\ \dot{y} &= z_1 + p_1(y) + \Phi_1(y)\theta + g_1(y)(u - v) \end{aligned} \tag{36}$$

For the internal model design, we have

$$q(y) = \int_0^y \frac{1}{\sqrt{1+\mu^2}}d\mu = \sinh^{-1} y \tag{37}$$

and the internal model design directly follows (8) with $\phi_{0,1}(y) = 0$. The adaptive control input is designed as

$$u = \frac{1}{g_1(y)}[-(\frac{1}{2} + d)y - p_1(y) - \hat{\vartheta}r(y) - \Phi_1(y)\hat{\theta}] - \frac{1}{2}g_1(y)y + (\xi - q(y)k)^T\hat{\psi} \tag{38}$$

with

$$r(y) = \frac{1}{y}[\frac{\Phi_1^2(y)}{g_1(y)} + (\Phi_2(y) - b_1(y)\Phi_1(y))^2 + b_1^2(y)p_1^2(y)] \tag{39}$$

The adaptive laws directly follow (17), (18) and (19).

In the simulation study, all the initial values were set to zero except $\mathbf{x}(0) = [1, 0]^T$ and $\mathbf{w}(0) = [1, 0]^T$. For the exosystem, we set $\mathbf{h} = [1, 0]^T$, and $\omega = 1$ rad/s for $0 \le t < 35$ and $\omega = 1.5$ rad/s afterward. For the internal model, we set $\mathbf{F} = [-1, 1; -2, 0]$ and $\mathbf{k} = [0, 1]^T$. The unknown parameter was set as $\theta = 1$, the control parameter $d = 1$. The adaptive gains were set as $\Gamma_\theta = 1$, $\mathbf{\Gamma}_\psi = 10\mathbf{I}$, and $\gamma = 1$. The state variables and the control input are shown together in Figure 1, and $\hat{\boldsymbol{\psi}}$ in Figure 2. In Figure 1, all the state variables converge asymptotically to zero. The estimate $\hat{\boldsymbol{\psi}}$ converges respectively to $[0, 1]^T$ and $[-1.25, 1]^T$ which are the correct values of $\boldsymbol{\psi}$ for frequencies 1 rad/s and 2 rad/s.

8 Conclusions

An adaptive control algorithm has been proposed for the asymptotic rejection of unknown sinusoidal disturbances in uncertain nonlinear systems which can be transformed to the extended nonlinear output feedback form. The proposed control design enlarges the class of nonlinear systems of which unknown sinusoidal disturbances can be asymptotically rejected. The success of the adaptive disturbance rejection relies on the state transformation and the new internal model which is able to tackle the nonlinear function coupled with the system input. The proposed internal model can also be used to deal with disturbance rejection of the nonlinear output feedback systems in the standard output feedback form, which will result in an extension to the results achieved in [7, 9, 11] for high relative degree systems.

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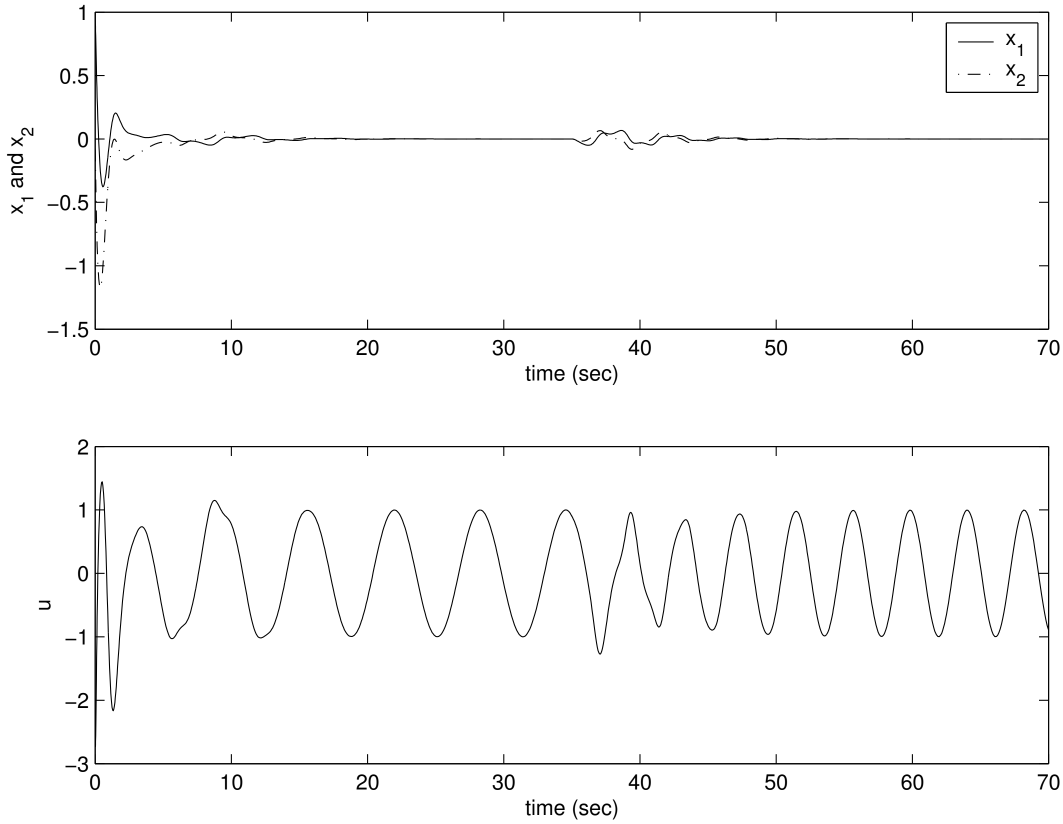

Figure 1: State variables x_1 and x_2 and input u.

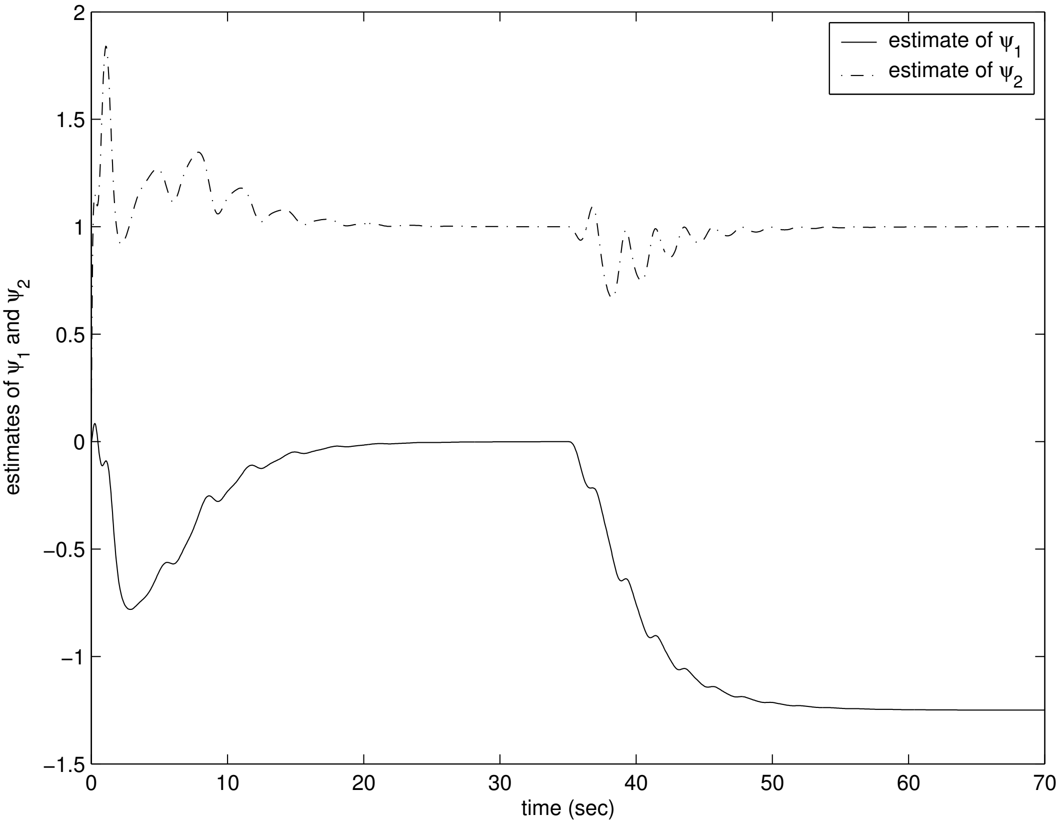

Figure 2: Adaptive estimates $\hat{\psi}_1$ and $\hat{\psi}_2$.