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Global Adaptive Output Regulation of A Class of Nonlinear Systems with Nonlinear Exosystems ⋆

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Abstract

This paper deals with global output regulation with nonlinear exosystems for a class of uncertain nonlinear output feedback systems. The circle criterion is exploited for the internal model design to accommodate the nonlinearities in the exosystems, and the explicit conditions are given for the exosystems such that proposed internal model design can be applied. The uncertainties of the output feedback systems are in the form of unknown constant parameters, and adaptive control techniques are used to ensure the global stability of the proposed control design for output regulation.

Key words: Nonlinear Systems, Output Regulation, Adaptive Control.

1 Introduction

Output regulation concerns with stabilization of dynamic systems as well as rejecting the disturbances and/or tracking the desired trajectories. The output regulation problem of linear systems is well posed and solved in (Davison, 1976; Francis, 1977) . For output regulation of nonlinear systems, the necessary and sufficient conditions for the existence of a local full information solution are specified as that the linearized system is stabilizable and there exists a certain controlled invariant manifold in (Isidori and Byrnes, 1990; Huang and Rugh, 1990). Global output regulation for output feedback system is reported in (Serrani and Isidori, 2000) and global adaptive output regulation for the output feedback systems is shown in (Ding, 2001).

A common assumption in early results of global output regulation via measurement feedback (Serrani and Isidori, 2000; Ding, 2001) requires that the exosystem must be linear. Recently, some progresses are reported on output regulation with nonlinear exosystems (Priscoli, 2004; Ding, 2006; Byrns and Isidori, 2004; Chen and Huang, 2005). Semigobal output regulation with nonlinear exosystems have been solved by using high gain internal models (Priscoli, 2004; Byrns and Isidori, 2004). A general framework based on steady state generators is proposed for output regulation with linear exosystems (Huang and Chen, 2004), and this concept of steady state generators is subsequently used to provide a solution to output regulation with nonlinear exosystems via state feedback (Chen and Huang, 2005). Output regulation with nonlinear exosystems via measurement feedback is dealt with in (Ding, 2006) for nonlinear systems in the output feedback form, and the nonlinear internal model is constructed based on high gain design and the Hermite-Birkhoff interpolation.

In this paper, we consider output regulation with nonlinear exosystems via measurement feedback. To tackle the nonlinearity in the exosystem, we exploit the circle criterion (Khalil, 2002) for the internal model design to produce the desired feedforward control term. In fact, the circle criterion has been exploited recently for observer design of nonlinear systems and observer based nonlinear control design (Arcak and Kokotovic, 2001; Arcak, 2005). Applying the circle criterion for internal model design is a natural extension of this trend. Of course there are differences in designing an observer for state estimation and an internal model for output regu-

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lation, with the later one being more challenging. On the other hand, the exosystems are of special characteristics, which lead to specific conditions to be identified for the internal model design using the circle criterion. A general condition will also be specified for nonlinear terms in the dynamic system, which allows more general nonlinear functions than polynomials. Section 2 addresses the class of nonlinear systems that we will consider and some notations. Section 3 introduces some transformation to clarify the systems considered. The construction of nonlinear internal model is shown in Section 5 and an example is included in Section 6 to demonstrate the proposed design.

2 Notations and Basic Concepts

Consider the following single-input-single-output nonlinear system which can be transformed into the output feedback form

$$\begin{cases} \dot{x} = A_c x + \phi(y)a + D(w) + bu \\ y = Cx \\ e = y - q(w) \end{cases} \tag{1}$$

with

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_\rho \\ \vdots \\ b_n \end{bmatrix}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}$, e is the measurement output, $a \in \mathbb{R}^q$ and $b \in \mathbb{R}^n$ are vectors of unknown parameters, $D : \mathbb{R}^m \to \mathbb{R}^n, \phi : \mathbb{R} \to \mathbb{R}^{n\times q}$ with $\phi(0) = 0$ and $|\phi(y_1) - \phi(y_2)| \le \Delta_1(|y_1|)\delta_1(|y_1 - y_2|)$ and $\delta_1(\cdot) \in \mathcal{K}$ and $\Delta_1(\cdot)$ is nondecreasing and the function $\delta_1(\cdot)$ is a known smooth function, $w \in \mathbb{R}^m$ are disturbances, and they are generated from an nonlinear exosystem

$$\dot{w} = s(w). \tag{2}$$

Remark 1 *The assumption about the function ϕ is satisfied for many kind of functions, for example, polynomial functions.*

Remark 2 *The coordinate-free geometric conditions for the existence of state transformation for transforming a nonlinear system into (1) are specified in (Marino and Tomei, 1993). $b_\rho \neq 0$ indicates the nonlinear system before the transformation has a constant relative degree of ρ.*

Assumption 1. The system is of minimum phase, i.e., the polynomial $\mathcal{B}(s) = \sum_{i=\rho}^{n} b_i s^{n-i}$ is Hurwitz, and the high frequency gain b_ρ is known.

Assumption 2. The flows of vector field $s(w)$ are bounded and converge to periodic solutions.

Remark 3 *The periodic solutions of the exosystems include harmonic functions, and other periodic functions such as limit cycles of nonlinear dynamic systems. Assumption 2 may be relaxed to include more general exosystems. This relaxation requires the results on invariant manifolds which are discussed in (Pavlov, van de Wouw and Nijmeijer, 2005)*

The adaptive output regulation problem that we are going to solve is to find a finite dimensional system

$$\begin{cases} \dot{\mu} = \nu(\mu, e(t)), \mu \in \mathbb{R}^s \\ u = u(\mu, e(t)) \end{cases}$$

such that for every $x(0) \in \mathbb{R}^n, w(0) \in \Omega \subset \mathbb{R}^m, x(t), \mu(t)$ and $u(t)$ are bounded $\forall t \ge 0$, and $\lim_{t\to\infty} e(t) = 0$.

Motivated by the result in (Isidori and Byrnes, 1990), the following assumption is proposed in order to solve the adaptive output regulation problem.

Assumption 3. There exist $\varpi(w) \in \mathbb{R}^n$ and $\iota(w)$ with $\varpi_1(w) = q(w)$ for each a, b such that

$$\frac{\partial \varpi}{\partial w} s(w) = A_c \varpi + \phi(q(w))a + D(w) + b\iota(w)$$

3 State Transformation

For the system (1) with relative degree $\rho > 1$, the following filter is introduced (Marino and Tomei, 1993)

$$\begin{cases} \dot{\xi}_1 = -\lambda_1 \xi_1 + \xi_2 \\ \vdots \\ \dot{\xi}_{\rho-1} = -\lambda_{\rho-1}\xi_{\rho-1} + u \end{cases} \tag{3}$$

where $\lambda_i > 0$ for $i = 1, \cdots, \rho - 1$ are the design parameters. Define the filtered transformation

$$\bar{z} = x - [\bar{d}_1, \cdots, \bar{d}_{\rho-1}]\xi$$

where $\xi = [\xi_1, \cdots, \xi_{\rho-1}]^T$, $\bar{d}_i \in \mathbb{R}^n$ for $i = 1, \cdots, \rho - 1$ and they are recursively generated by $\bar{d}_{\rho-1} = b$ and $\bar{d}_i = [A_c + \lambda_{i+1} I]\bar{d}_{i+1}$ for $i = \rho - 2, \cdots, 1$. The system (1) is then transformed to

$$\begin{cases} \dot{\bar{z}} = A_c \bar{z} + \phi(y)a + D(w) + d\xi_1 \\ y = C\bar{z} \end{cases} \tag{4}$$

where $d = [A_c + \lambda_1 I]\bar{d}_1$. It can be shown that $d_1 = b_\rho$ and

$$\mathcal{D}(s) := \sum_{i=1}^{n} d_i s^{n-i} = \mathcal{B}(s)\prod_{i=1}^{\rho-1}(s+\lambda_i)$$

With ξ_1 as the input, the system (4) is with relative degree one and minimum phase. We introduce another state transformation to extract the internal dynamics of (4) with $z \in \mathbb{R}^{n-1}$ given by

$$z_j = \bar{z}_{j+1} - \frac{d_{j+1}}{d_1}y \qquad j = 1, \cdots, n-1.$$

With the coordinates (z, y), (4) is rewritten as

$$\begin{aligned}\dot{z}_i &= -\tfrac{d_{i+1}}{d_1}z_1 + z_{i+1} + (\tfrac{d_{i+2}}{d_1} - \tfrac{d_{i+1}d_2}{d_1^2})y \\ &\quad +(\phi_{i+1}(y) - \tfrac{d_{i+1}}{d_1}\phi_1(y))a + D_{i+1}(w) \\ &\quad -\tfrac{d_{i+1}}{d_1}D_1(w), i = 1, \cdots, n-2, \\ \dot{z}_{n-1} &= -\tfrac{d_n}{d_1}z_1 - \tfrac{d_n d_2}{d_1^2}y + (\phi_n(y) - \tfrac{d_n}{d_1}\phi_1(y))a \\ &\quad +D_n(w) - \tfrac{d_n}{d_1}D_1(w), \\ \dot{y} &= z_1 + \tfrac{d_2}{d_1}y + \phi_1(y)a + D_1(w) + b_\rho\xi_1\end{aligned} \tag{5}$$

Lemma 1 *Under Assumption 3 there exists $\pi(w) \in \mathbb{R}^{n-1}$ along the trajectories of exosystem satisfying*

$$\begin{aligned}\tfrac{d\pi_i(w(t))}{dt} &= -\tfrac{d_{i+1}}{d_1}\pi_1(w(t)) + \pi_{i+1}(w(t)) + q(w(t)) \\ &\quad \times(\tfrac{d_{i+2}}{d_1} - \tfrac{d_{i+1}d_2}{d_1^2}) + D_{i+1}(w(t)) - D_1(w(t)) \\ &\quad \times\tfrac{d_{i+1}}{d_1} + (\phi_{i+1}(q(w(t))) - \tfrac{d_{i+1}}{d_1}\phi_1(q(w(t))))a, \\ &\quad i = 1, \cdots, n-2,\end{aligned}$$

$$\begin{aligned}\tfrac{d\pi_{n-1}(w(t))}{dt} &= -\tfrac{d_n}{d_1}\pi_1(w(t)) - \tfrac{d_n d_2}{d_1^2}q(w(t)) \\ &\quad +(\phi_n(q(w(t))) - \tfrac{d_n}{d_1}\phi_1(q(w(t))))a \\ &\quad +D_n(w(t)) - \tfrac{d_n}{d_1}D_1(w(t)),\end{aligned}$$

Proof. From the last row of (3) since it is an asymptotically stable linear system there is a static response for every external input $u(w(t))$, which is from the Assumption 3, i.e., there exists a function $\chi_{\rho-1}(w)$ such that

$$\frac{d\chi_{\rho-1}(w(t))}{dt} = -\lambda_{\rho-1}\chi_{\rho-1}(w(t)) + \iota(w(t)).$$

Recursively, if there exists $\chi_i(w)$ such that

$$\frac{d\chi_i(w(t))}{dt} = -\lambda_i\chi_i(w(t)) + \chi_{i+1}(w(t)),$$

then there exists $\chi_{i-1}(w)$ such that

$$\frac{d\chi_{i-1}(w(t))}{dt} = -\lambda_{i-1}\chi_{i-1}(w(t)) + \chi_i(w(t)).$$

Define

$$\begin{pmatrix}\pi(w) \\ q(w)\end{pmatrix} = \mathcal{A}_c(\varpi(w) - [\bar{d}_1, \cdots, \bar{d}_{\rho-1}]\chi),$$

where $\chi = (\chi_1, \cdots, \chi_{\rho-1})^T$ and

$$\mathcal{A}_c = \begin{pmatrix} -\frac{d_2}{d_1} & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{d_n}{d_1} & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0\end{pmatrix}.$$

It can be seen that $\pi(w)$ satisfies the dynamics of z along the trajectories of (2) as shown in (5), and hence the lemma is proved.

Based on the above lemma, we have

$$\frac{\partial q(w)}{\partial w}s(w) = \pi_1(w) + \frac{d_2}{d_1}q(w) + \phi_1(q(w))a + D_1(w) + b_\rho\alpha(w).$$

where $\alpha(w) = \chi_1(w)$. With ξ_1 being viewed as the input, $\alpha(w)$ is the feedforward term used for output regulation to tackle the disturbances, and it is given by

$$\alpha = b_\rho^{-1}(\frac{\partial q(w)}{\partial w}s(w) - \pi_1(w) - \frac{d_2}{d_1}q(w) - \phi_1(q(w))a - D_1(w))$$

We now introduce the last transformation based on the invariant manifold with

$$\tilde{z} = z - \pi(w(t)).$$

Finally we have the model for the control design

$$\begin{aligned}\dot{\tilde{z}}_i &= -\tfrac{d_{i+1}}{d_1}\tilde{z}_1 + \tilde{z}_{i+1} + (\tfrac{d_{i+2}}{d_1} - \tfrac{d_{i+1}d_2}{d_1^2})e \\ &\quad +(\phi_{i+1}(y) - \phi_{i+1}(q(w)))a \\ &\quad -\tfrac{d_{i+1}}{d_1}(\phi_1(y) - \phi_1(q(w)))a, i = 1, \cdots, n-2, \\ \dot{\tilde{z}}_{n-1} &= -\tfrac{d_n}{d_1}\tilde{z}_1 - \tfrac{d_n d_2}{d_1^2}e + (\phi_n(y) - \phi_n(q(w)))a \\ &\quad -\tfrac{d_n}{d_1}(\phi_1(y) - \phi_1(q(w)))a, \\ \dot{e} &= \tilde{z}_1 + \tfrac{d_2}{d_1}e + (\phi_1(y) - \phi_1(q(w)))a + b_\rho(\xi_1 - \alpha(w))\end{aligned}$$

i.e., the system can be represented as

$$\begin{cases} \dot{\tilde{z}} = \mathcal{A}\tilde{z} + \Xi e + \Omega(y, w, d)a, \\ \dot{e} = \tilde{z}_1 + \frac{d_2}{d_1}e + (\phi_1(y) - \phi_1(q(w)))a + b_\rho(\xi_1 - \alpha(w)) \end{cases} \quad (6)$$

where

$$\mathcal{A} = \begin{bmatrix} -\frac{d_2}{d_1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{d_n}{d_1} & 0 & \cdots & 0 \end{bmatrix},$$

$$\Xi = (\frac{d_3}{d_1} - \frac{d_2^2}{d_1^2}, \cdots, \frac{d_n}{d_1} - \frac{d_{n-1}d_2}{d_1^2}, -\frac{d_n d_2}{d_1^2})^T,$$

$$\Omega(y, w, d) = \begin{pmatrix} \phi_2(y) - \phi_2(q(w)) - \frac{d_2}{d_1}(\phi_1(y) - \phi_1(q(w))) \\ \vdots \\ \phi_n(y) - \phi_n(q(w)) - \frac{d_n}{d_1}(\phi_1(y) - \phi_1(q(w))) \end{pmatrix}$$

Lemma 2 *There exist a known function $\zeta(\cdot)$ which is nondecreasing and an unknown constant Δ, which is dependent on the initial state w_0 of exosystem, such that*

$$\begin{cases} |\Omega(y, w, d)| \le \Delta|e|\zeta(|e|) \\ |\phi_1(y) - \phi_1(q(w))| \le \Delta|e|\zeta(|e|) \end{cases}$$

Proof. From the assumption of ϕ we can see that

$$|\phi(y) - \phi(q(w))| \le \Delta_1(|q(w)|)\delta_1(|e|).$$

Since the trajectories of exosystem are bounded and $\delta_1(\cdot)$ is smooth there exist smooth nondecreasing known function $\zeta(\cdot)$ and a nondecreasing known function $\Delta_2(|w_0|)$, such that

$$\begin{cases} \delta_1(|e|) \le |e|\zeta(|e|), \\ \Delta_1(|q(w)|) \le \Delta_2(|w_0|). \end{cases}$$

From previous discussion the result of the Lemma is obtained.

Let $V_z = \tilde{z}^T P_A \tilde{z}$, where

$$P_A\mathcal{A} + \mathcal{A}^T P_A = -I.$$

Then using $2ab \le ca^2 + c^{-1}b^2$ and $\zeta^2(|e|) \le \zeta^2(1 + e^2)$ there exist unknown positive real constants Λ_1, Λ_2 such that

$$\begin{aligned} \dot{V}_z &= -\tilde{z}^T\tilde{z} + 2\tilde{z}^T P_A(\Xi e + \Omega(y, w, d)a) \\ &\le -\tfrac{3}{4}\tilde{z}^T\tilde{z} + \Lambda_1 e^2 + \Lambda_2 e^2\zeta^2(1 + e^2) \end{aligned} \quad (7)$$

noting that

$$\begin{aligned} 2\tilde{z}^T P_A \Xi e &\le \tfrac{1}{8}\tilde{z}^T\tilde{z} + 8e^T\Xi^T P_A^2 \Xi e \\ &\le \tfrac{1}{8}\tilde{z}^T\tilde{z} + \Lambda_1 e^2 \end{aligned}$$

and

$$\begin{aligned} 2\tilde{z}^T P_A \Omega(y, w, d)a &\le \tfrac{1}{8}\tilde{z}^T\tilde{z} + 8a^T\Omega^T P_A^2 \Omega a \\ &\le \tfrac{1}{8}\tilde{z}^T\tilde{z} + \Lambda_2^1|\Omega|^2 \\ &\le \tfrac{1}{8}\tilde{z}^T\tilde{z} + \Lambda_2^1\Delta^2|e|^2\zeta^2(|e|) \\ &\le \tfrac{1}{8}\tilde{z}^T\tilde{z} + \Lambda_2 e^2\zeta^2(1 + e^2), \end{aligned}$$

where Λ_2^1 is an unknown positive real constant.

4 Internal Model

To solve the problem, we need an assumption on the structure of the exosystem.

Assumption 4: For the system

$$\begin{cases} \dot{w} = s(w) \\ \alpha = \alpha(w), \end{cases}$$

there exists an immersion system

$$\begin{cases} \dot{\eta} = F\eta + G\gamma(J\eta) \\ \alpha = H\eta \end{cases}$$

where $\eta \in \mathbb{R}^r$ and $H = [1, 0, \cdots, 0]$, and (H, F) is observable, and $(v_1 - v_2)^T(\gamma(v_1) - \gamma(v_2)) \ge 0$ and G, J are some appropriate dimensional matrices.

Remark 4 *It is well known in the literature, for example, in (Huang and Chen, 2004), that one crucial problem for output regulation problem is the existence of an appropriately defined dynamic system, often referred to as an internal model, to produce the desired steady state output. Conditions have been identified for the existence of an internal model, sometimes even a nonlinear internal model as shown in (Huang and Chen, 2004), when the exosystems are linear. It is not clear at the moment what general conditions can be specified to guarantee the existence of such an internal model for nonlinear systems with nonlinear exosystems, and this is subject to further research. In this paper, Assumption 4 assumes the existence of an immersion system which implies the existence of the internal model with nonlinear exosystem. Furthermore, the conditions can be directly checked for the internal model proposed in this paper.*

Remark 5 *Assumption 4 extends the condition used in (Ding, 2006), with $G\gamma(J\eta) = \phi(\alpha)$ and $\{F, H\}$ in observer canonical form.*

Since the feedforward term α is unknown, we design the following internal model, which produces the estimated feedforward term α,

$$\dot{\hat{\eta}} = (F - KH)(\hat{\eta} - b_\rho^{-1}Ke) + G\gamma(J(\hat{\eta} - b_\rho^{-1}Ke)) + K\xi_1$$

where $K \in \mathbb{R}^r$ is chosen such that $F_0 = F - KH$ is Hurwitz and there exist a positive definite matrix P_F and a semi-positive definite matrix Q satisfying

$$\begin{cases} P_F F_0 + F_0^T P_F = -Q \\ P_F G + J^T = 0 \\ \eta^T Q \eta \geq \gamma_0 |\eta_1|^2, \qquad \gamma_0 > 0, \quad \eta \in \mathbb{R}^r \\ \text{span}(P_F K) \subseteq \text{span}(Q) \end{cases} \tag{8}$$

Remark 6 *Note the condition specified in (8) is weaker than the condition that there exist $P_F > 0$ and $Q > 0$ satisfying*

$$\begin{cases} P_F F_0 + F_0^T P_F = -Q \\ P_F G + J^T = 0, \end{cases} \tag{9}$$

which can be checked by LMI. This will be seen in the example. In particular, if G and J^T are two column vectors, (F_0, G) controllable, (J, F_0) observable and $Re[-J(j\omega I - F_0)^{-1}G] > 0$, $\forall \omega \in \mathbb{R}$, then there exists a solution of (9) from a known Meyer-Kalman-Yacubovich Theorem.

If we define the auxiliary error

$$\tilde{\eta} = \eta - \hat{\eta} + b_\rho^{-1} K e,$$

it can be shown that

$$\begin{aligned} \dot{\tilde{\eta}} \; = & \; F_0 \tilde{\eta} + G(\gamma(J\eta) - \gamma(J(\hat{\eta} - b_\rho^{-1} K e))) \\ & + b_\rho^{-1} K (\tilde{z}_1 + \tfrac{d_2}{d_1} e + (\phi_1(y) - \phi_1(q(w)))a) \end{aligned}$$

Let $V_\eta = \tilde{\eta} P_F \tilde{\eta}$. Then following the spirit of (7) there exist unknown positive real constants Θ_1 and Θ_2 such that

$$\begin{aligned} \dot{V}_\eta \; = & \; -\tilde{\eta}^T Q \tilde{\eta} + 2\tilde{\eta}^T P_F b_\rho^{-1} K (\tilde{z}_1 + \tfrac{d_2}{d_1} e) \\ & + 2\tilde{\eta}^T P_F b_\rho^{-1} K (\phi_1(y) - \phi_1(q(w)))a \\ & + 2\tilde{\eta}^T P_F G(\gamma(J\eta) - \gamma(J(\hat{\eta} - b_\rho^{-1} K e)) \\ \leq & \; -\tfrac{3}{4}\gamma_0 |\tilde{\eta}_1|^2 + \tfrac{12}{\gamma_0} b_\rho^{-2} \tilde{z}_1^2 + \Theta_1 e^2 + \Theta_2 e^2 \zeta^2 (1 + e^2) \end{aligned} \tag{10}$$

5 Control Design

From (6) and $\alpha = \eta_1 = \hat{\eta}_1 + \tilde{\eta}_1 - b_\rho^{-1} K_1 e$ as in Assumption 4, we have

$$\begin{aligned} \dot{e} \; = & \; \tilde{z}_1 + \tfrac{d_2}{d_1} e + (\phi_1(y) - \phi_1(q(w)))a + \bar{\xi}_1 \\ & + b_\rho (\tilde{\xi}_1 - \tilde{\eta}_1 - \hat{\eta}_1 + b_\rho^{-1} K_1 e) \end{aligned}$$

where $\tilde{\xi}_1 = \xi_1 - \hat{\xi}_1$ and $\hat{\xi}_1 = b_\rho^{-1} \bar{\xi}_1$.

For the virtual control $\hat{\xi}_1$, we design $\bar{\xi}_1$ as, with $c_0 > 0$,

$$\bar{\xi}_1 = -c_0 e + b_\rho \hat{\eta}_1 - K_1 e - \hat{l} e (1 + \zeta^2 (1 + e^2))$$

where $\hat{l}$ is an adaptive coefficient. Then we have the resultant error dynamics

$$\begin{aligned} \dot{e} \; = & \; \tilde{z}_1 - c_0 e + \tfrac{d_2}{d_1} e - \hat{l} e (1 + \zeta^2 (1 + e^2)) \\ & + (\phi_1(y) - \phi_1(q(w)))a + b_\rho (\tilde{\xi}_1 - \tilde{\eta}_1) \end{aligned}$$

Then for $V_e = \frac{1}{2} e^2$ there exist unknown positive real constants Ψ_1 and Ψ_2 and a sufficiently large unknown positive constant β such that

$$\begin{aligned} \dot{V}_e \; = & \; -c_0 e^2 + e \tilde{z}_1 + \tfrac{d_2}{d_1} e^2 + e b_\rho (\tilde{\xi}_1 - \tilde{\eta}_1) \\ & + e(\phi_1(y) - \phi_1(q(w)))a - \hat{l} e^2 (1 + \zeta^2 (1 + e^2)) \\ \leq & \; -c_0 e^2 + \tfrac{1}{8} \beta \tilde{z}_1^2 + \tfrac{1}{4} \gamma_0 \tilde{\eta}_1^2 + \Psi_1 e^2 + \Psi_2 e^2 \zeta (1 + e^2) \\ & - \hat{l} e^2 (1 + \zeta^2 (1 + e^2)) + b_\rho e \tilde{\xi}_1 \end{aligned} \tag{11}$$

Let $V_0 = \beta V_z + V_\eta + V_e + \frac{1}{2}\gamma^{-1}(\hat{l} - l)^2$ where $\beta \geq \frac{96}{\gamma_0} b_\rho^{-2}$ is chosen and $l = \Psi_1 + \Psi_2 + \Theta_1 + \Theta_2 + \beta(\Lambda_1 + \Lambda_2)$, is an unknown constant. Let

$$\dot{\hat{l}} = \gamma e^2 (1 + \zeta^2 (1 + e^2)).$$

Then

$$\dot{V}_0 \leq -\frac{1}{2} \beta \tilde{z}^T \tilde{z} - \frac{1}{2} \gamma_0 |\tilde{\eta}_1|^2 - c_0 e^2 + b_\rho e \tilde{\xi}_1.$$

From this and (3) the real control u can be designed using the well known backstepping method.

Theorem 3 *If Assumptions 1-4 are satified and there exists $K \in \mathbb{R}^r$ such that $F_0 = F - KH$ is Hurwitz and there exist a positive definite matrix P_F and a semposi-tive definite matrix Q satisfying (8), then there exists a controller to solve the output regulation of (1).*

6 An Example

We use an example to illustrate the proposed control design, concentrating on the design of nonlinear internal model. Consider a first order system

$$\begin{cases} \dot{y} = 2y + \theta \sin y - y^3 - \theta \sin w_1 + w_2 + u \\ e = y - w_1 \end{cases}$$

where θ is an unknown parameter, the disturbance w is generated by

$$\begin{cases} \dot{w}_1 = w_1 + w_2 - w_1^3, \\ \dot{w}_2 = -w_1 - w_2^3. \end{cases}$$

It is easy to see that $V(w) = \frac{1}{2} w_1^2 + \frac{1}{2} w_2^2$ satisfies

$$\frac{dV}{dt} = w_1^2 - w_1^4 - w_2^4 \leq 0, \qquad \text{when} \qquad |w_1| \geq 1,$$

and that

$$\begin{cases} q(w) = w_1, \\ \pi = w_1 \\ \alpha(w) = -w_1. \end{cases}$$

From the exosystem and the desired feedforward input α, it can be seen that Assumption 4 is satisfied with $\eta = -w$ with

$$\begin{cases} F = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \\ \gamma_1(s) = \gamma_2(s) = s^3, \quad J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{cases}$$

Let $K = (2, 0)^T$. Then

$$F_0 = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix} \quad P_F = I, \quad Q = \mathrm{diag}(2, 0),$$

the internal model is as following

$$\begin{cases} \dot{\hat{\eta}}_1 = -(\hat{\eta}_1 - 2e) + \hat{\eta}_2 - (\hat{\eta}_1 - 2e)^3 + 2u, \\ \dot{\hat{\eta}}_2 = -(\hat{\eta}_1 - 2e) - \hat{\eta}_2^3. \end{cases}$$

The control input is given by

$$\begin{cases} u = -ce + \hat{\eta}_1 - \hat{l}e(1 + (e^2 + 1)^2), \\ \dot{\hat{l}} = \gamma e^2(1 + (e^2 + 1)^2). \end{cases}$$

For simulation study, we set $c = 1, \theta = 1, \gamma = 1$, and the initial state is $y(0) = 1, w_1(0) = 2, w_2(0) = 2$. The initial state of dynamic controller is zero. The system output and input are shown in Figure 1, while the feedforward term and its estimation is shown in Figure 2 and the portrait of the exosystem is shown in Figure 3. As shown in the figures, the internal model successfully reproduces the feedforward control needed after a transient period, and the system output measurement is regulated to zero, as required.

7 Conclusion

We have proposed a new control design method for output regulation with nonlinear exosystems. A set of conditions have been identified under which the circle criterion can be applied for the internal model design to deal with the nonlinearities in the exosystem. With the proposed internal model design, global output regulation has been solved for nonlinear output feedback systems. A further research is to look for some conditions under which the internal model will have the proposed form.

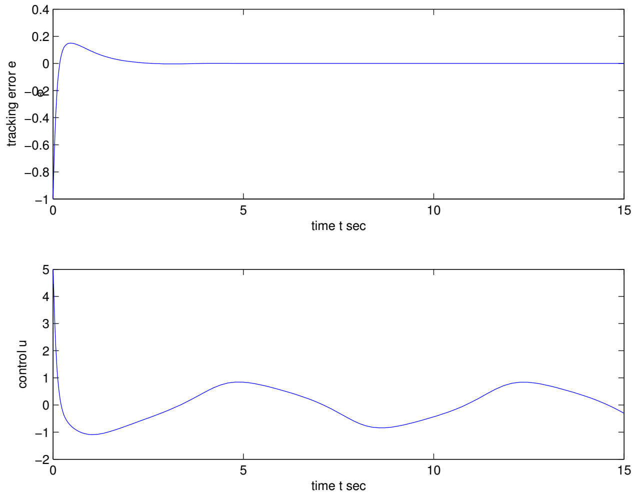

Fig. 1. The systems output e and input u.

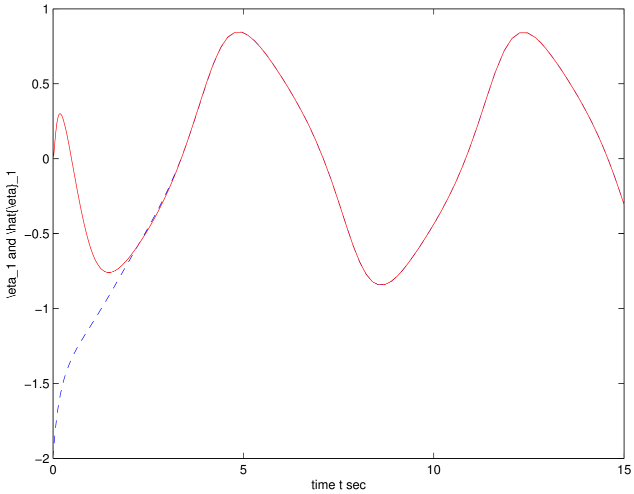

Fig. 2. The systems's feedforward control η_1 and its estimation $\hat{\eta}_1$.

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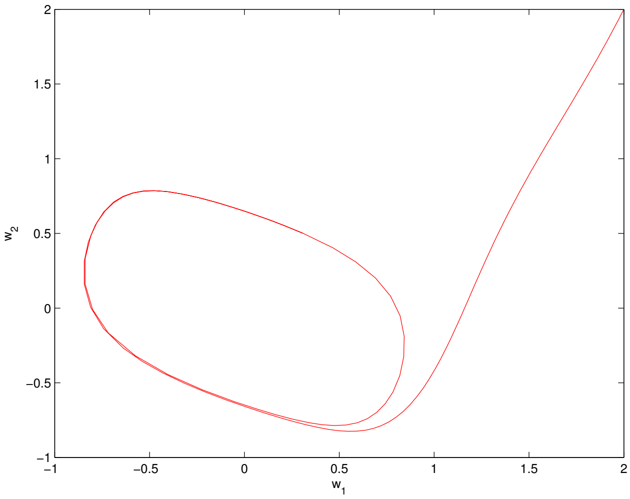

Fig. 3. The portrait of exosystems.

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