Tutorial 4

Question 19. Determine the z-transfer functions of the following plants (Hint: A ZOH should be added in each case):

1) $G_1(s) = \frac{1}{s(s+5)}$
2) $G_2(s) = \frac{s+2}{(s+1)(s+3)}$
3) $G_3(s) = \frac{10e^{-2Ts}}{s+5}$

Question 20. A plant is described by the transfer function

$$\frac{Y(s)}{U(s)} = \frac{5}{s(s+5)} \tag{29}$$

and the systems input and output are sampled with a sampling interval $T = 0.1$ second.

1) Obtain the z transfer function between the input and the output.

2) Obtain the difference equation relating $y(k)$ and $u(k)$.

3) Determine the system output (first five steps) under a unit step using the difference equation obtained in 2) .

Question 21. A first order system $\frac{Y(s)}{U(s)} = \frac{10}{s+5}$ is sampled at every T seconds. The control law for the system is designed as $u(kT) = -Ky(kT)$ where K is the controller gain.

1) Determine the range of the sampling interval T such that the closed-loop system is stable with $K = 10$.

2) Determine the range of controller gain K such that the closed-loop system is stable with $T = 0.1$ second.

Question 22. Consider a discrete-time system

$$\frac{Y(z)}{U(z)} = \frac{0.4z + 0.2}{z^2 - 1.4z + 0.4} \tag{30}$$

with feedback control $u(k) = -Ky(k)$.

1) Obtain the closed-loop transfer function and the characteristic equation of the closed-loop system, and then determine the stability of the system with $K = 1$ and $K = 10$ respectively.

2) Suggest a method to determine the range of the controller gain K such that the closed system is stable.